Complex Organizations, Leverage and Interest Rates⋆

Giovanna Nicodano[a], Luca Regis[b]

[a] *UniTo, ESOMAS Department and Collegio Carlo Alberto; CEPR and ECGI; giovanna.nicodano@unito.it.*
[b] *UniTo, ESOMAS Department and Collegio Carlo Alberto; luca.regis@unito.it.*

Abstract

Our trade-off model determines optimal leverage responses to interest rate changes within a sponsor–backed unit structure. This setup captures private equity organizations, parent–subsidiary groups, and securitization arrangements, in which a sponsor can provide contingent support to a leveraged affiliated unit. When interest rates fall below a cutoff, the sponsor optimally chooses zero leverage in order to maximize its ability to support the backed unit in distress states, while the backed unit increases borrowing. This effect strengthens with higher correlation between sponsor and backed-unit cash flows, which increases the likelihood that the sponsor can provide support in positive cash flow states.

In contrast, stand-alone firms display the traditional trade-off result: their optimal leverage declines monotonically as interest rates fall because the tax advantage of debt decreases. The backed unit therefore reacts differently to interest-rate changes. By relaxing its cash-flow constraint and raising lenders' losses upon default, sponsor support leads lenders to charge higher spreads, which in turn increase the tax shield and sustain higher leverage. Numerical analysis shows that these mechanisms increases the backed unit's default probability, credit spreads, and expected default costs in low-interest-rate environments, while both stand-alone firms and sponsors become less risky.

Our results provide a new explanation for the heterogeneous leverage responses to interest rates observed across organizational forms, such as the divergence between public firms and leveraged buyout targets. They also highlight a tax-shield channel of risk-taking that is distinct from mechanisms based on search for yield or time-inconsistent monetary policy. The model also suggests that complex organizations may contribute to financial instability when interest rates are sufficiently low, while also implying that aggregate default risk may be partially smoothed by the coexistence of stand-alone and sponsor-backed firms.

Keywords: Capital structure, monetary policy, tax-bankruptcy trade-off, LBOs, securitization, structured finance, multinationals

JEL Classification: G32, H32, L32

This version: March 20, 2026

⋆. We are grateful to the members of the 2022 Advisory Scientific Committee of the European Systemic Risk Board for stimulating this investigation, to Pierre Collin-Dufresne, Johan Hombert, Sebnem Kalemli-Ozcan, Petros Migiakis, Loriana Pelizzon, Javier Suarez, Ali Uppal, and participants to the 13th IBEO Conference Alghero, the 38th GDRE Conference, to seminars at the Central Bank of Ireland and the University of Bielefeld for useful comments. Funding from the Italian MIUR ("Excellent Departments" 2018–2022 and 2023-2027) is gratefully acknowledged.

1. Introduction

The leverage of complex organizations increases in response to a reduction in interest rates (Andersson et al. 2025; Powell 2019; Rosengren 2019). This inverse relationship is consistent with the cash-flow-based motive for borrowing in models of the transmission of monetary policy, as lower rates increase debt service capacity. In contrast, the trade-off theory of capital structure predicts optimal deleveraging when interest rates fall, due to a reduced tax shield (Fischer et al. 1989; Ju and Ou-Yang 2006; Duarte et al. 2022). While the empirical relevance of this mechanism for stand-alone firms remains debated, the tax–bankruptcy trade-off appears central for complex organizations.[2]

This paper studies how optimal debt responds to the risk-free rate in both the stand-alone unit of Leland 2007 and in a complex organization composed of two such units, in which one (the sponsor) provides contingent support to the other (the backed unit) to maximize joint value. Such support arrangements are prevalent in private equity funds (Bernstein et al. 2019; Hotchkiss et al. 2021; Haque et al. 2023), multinational corporations and business groups (Bodie and Merton 1992; Beaver et al. 2019; Shi et al. 2023), securitization structures (Gorton and Souleles 2006), complex banks (Segura and Zeng 2020), and shadow banks (Allen et al. 2023).

We show that debt responds heterogeneously to declines in interest rates. First, below a cutoff rate, the stand-alone unit continues to deleverage, while the sponsor optimally chooses zero leverage. Second, below the same cutoff, the backed unit increases its borrowing sufficiently that aggregate leverage at the organizational level may rise. Third, credit spreads, expected default costs, and lenders' losses upon default exhibit similar heterogeneous responses.

These results arise from the interaction between cash-flow constraints and selective sponsor's support, modeled as in Luciano and Nicodano (2014). Each unit defaults when its realized after-tax cash flow, augmented by any sponsor transfer, falls short of the face value of its zero-coupon debt at maturity. In a stand-alone unit, lower interest rates tighten this constraint by reducing the tax shield and hence after-tax cash flows available for debt repayment, increasing default risk. This incentive to deleverage is even stronger for the sponsor, which internalizes the benefit of avoiding bankruptcy costs through targeted transfers to the backed unit. As interest rates fall sufficiently, the sponsor specializes in providing contingent support. This support relaxes the backed unit's cash-

2. See Jiangli et al. 2007 for securitization and Hanlon and Heitzman 2022; Brok 2024 for multinational groups. In LBO targets, interest rates are an important determinant of leverage (Gompers et al. 2016), with both tax shields (Kaplan 1989; Acharya et al. 2013; Renneboog et al. 2007) and default rates in excess of comparable stand-alone firms (Hotchkiss et al. 2021).

flow constraint and reduces its default probability, while simultaneously lowering lenders' recovery rates upon default. Indeed, the sponsor provides support selectively, only when the backed unit has positive cash flows that fall short the service of debt. Anticipating higher losses given default, lenders demand higher credit spreads which enhance the tax shield.

The standard trade-off result of optimal debt reduction for the stand-alone unit therefore reverses in the backed unit. This occurs to such an extent that the total debt of the complex organization may dramatically increase. The complex organization may thus incur higher expected default costs when interest rates decline sufficiently. This outcome is likelier the higher the correlation between the cash flows of the sponsor and those of the backed unit. This correlation determines the likelihood that the sponsor will have sufficient excess cash flow to assist its insolvent backed unit in servicing its debt.

These results matter for policy reasons, as well. The high leverage of complex organizations, in low interest rates environments, raises concerns about their contribution to financial instability when interest rates eventually return to higher levels (see, e.g., Fahlenbrach et al. 2023, Andersson et al. 2025 and Buch 2025, for private equity; Powell 2019, and Rosengren 2019, for securitization). Our model however implies that default costs peak when interest rates fall to sufficiently low levels, rather than when they subsequently rebound. To better understand the quantitative impact of low rates on leverage and default costs, we perform two numerical analyses, usually keeping the same model parameters as in Leland (2007), and Nicodano and Regis (2019), letting the risk free rate fall from 5% to 1%.

With the first parametric combination, the sponsor chooses positive leverage because its trade-off ratio, the ratio between the tax and the bankruptcy cost rates, sufficiently exceeds the one in the backed unit. The face value of debt is higher in the complex organization than in equivalent stand-alone units (199 vs. 169). Total tax savings are higher (10.58 vs. 10.32) and default costs are lower (5.67 vs. 6.09) relative to the stand-alone case, leading to higher total value (170.59 vs. 169.88). At a lower risk-free interest rate of 3%, we observe that the total debt raised by the complex organization decreases. However, when the interest rate reaches 1%, the optimal debt ends up exceeding the initial level (232 vs. 199). In the stand-alone units, instead, the combined optimal debt falls monotonically from 170 to 121 as interest rates drop.

Moreover, the backed unit raises all the organization's debt when the interest rate falls below 3%. Thus, the sponsor optimally specializes in providing support. This occurs because a lower interest rate shifts the balance between two opposing incentives, namely increasing the sponsor's tax shield versus providing additional support to the backed unit, towards the latter. The default costs of the organization increase sharply as the interest rate falls below 3%. In particular, those of the backed unit are more than 11 times the default costs of an equivalent stand-alone unit (9.08 vs. 0.79) and the (5-year) default probability reaches 54% when the interest rate is 1%. Losses given default deteriorate as well, rising to 175.42 from 88.42 (75.61% vs. 71.31% in percentage terms) relative to the 5% interest rate case. These changes affect the endogenous (yearly) spread, which rises from 5.86% to 12.20%. These results speak to the financial stability concerns expressed by policy-makers with regard to highly leveraged vehicles in low interest rates scenario.

This is *a fortiori* true in the second numerical case, where units have equal parameters. At the initial level of the risk-free rate, the sponsor's optimal leverage is already zero - reminiscent of a private equity fund. In turn, the backed unit pays a much higher credit spread than the stand-alone unit (8.15 vs. 1.23), due not only to its higher optimal debt (220 vs. 57) but also the higher losses given default of its lenders (149 vs. 28.96). With a risk-free rate equal to 1%, the optimal debt further diverges between the backed and the stand-alone units (247 vs. 34.3).

Our paper adds to the literature on complex organizations (see e.g. Bolton and Oehmke (2019), Cestone and Fumagalli (2005), Kahn and Winton (2004), and Shi et al. (2023)) insights concerning their optimal capital structure and default probability. Regarding capital structure, it shows that sufficiently low interest rates strikingly increase the already high leverage and default probability of the backed unit, at an increasing spread. Earlier studies on the leverage of complex organizations focus on a fixed rate, showing that the sponsor's limited liability leads to optimal debt being unevenly distributed across units both in full information (Nicodano and Regis (2019)) and asymmetric information settings (Bianco and Nicodano (2006)). In turn, dynamic trade-off theory implies optimal firm deleveraging in response to an interest rate cut (Fischer et al. (1989), Ju and Ou-Yang (2006), and Duarte et al. (2022)). We show that this result applies to the stand-alone unit in Leland (2007) but not to its backed unit equivalent.[3]

The results in this paper matter beyond the boundaries of corporate finance, unveiling a tax-shield channel of risk-taking. The inverse relationship between debt and interest rates serves as

3. This heterogeneity may provide a theoretical rationale for the opposite relationship between leverage and interest rates observed in both LBO targets and public firms (Axelson et al. 2013) and private SMEs raising debt against earnings and public firms (Caglio et al. 2022).

a foundation of the macro-banking literature investigating the transmission of monetary policy. Within this literature, some papers investigate the reason why highly- leveraged, low-rated firms raise more debt in a low interest rates environment. Observing that traditional corporate finance models struggle in providing an explanation, Farhi and Tirole (2009) argue that lower interest rates stimulate loan demand by fostering expectations of continued accommodative monetary policy. They find that highly leveraged companies benefit the most from such policies. In our static model, where the change in the risk-free rate is one-shot, shows a new driver of higher demand for loans by highly leveraged units, namely their high tax-shield enabled by the sponsor's selective support. In turn, the wider spreads charged by lenders in low-rate environments compensate them for their higher and more frequent losses given default. These effects are unrelated to moral hazard, which is instead behind the investors' search for higher yield that increases fund supply to riskier firms in Martinez-Miera and Repullo (2017) and Becker and Ivashina (2015).

In our work, the sponsor's selective support ensures that the backed unit borrows more, when rates are lower, because of a looser default constraint in positive cash-flow states. This reminds of monetary policy transmission models which emphasize the relaxation of firms' cash-flow constraints due to lower interest rates (Adler 2024; Bräuning and Wang 2020; Drechsel 2023; Greenwald 2019; Lian and Ma 2021). We show that the combined effect of looser cash-flow constraint and higher tax shield leads to higher debt. Our model thus suggests that complex organizations may contribute to financial instability in low interest rate environments.[4] However, lenders are fully aware of both the borrowers' higher default probability and losses upon default.

The paper unfolds as follows. Section 2 studies both the stand alone and the complex organization's leverage choice. In Sections 3 and 4 we provide insights on numerical leverage, default probability and lenders' losses-upon-default adjustments following a change in interest rates. While Section 3 studies the case of a zero-leverage sponsor, Section 4 shifts attention to a leveraged sponsor. Section 5 concludes.

2. The Model

This section describes our set-up for both stand-alone units and the connected units in complex organizations.

4. This differs from leverage cycle models, where default probability increases with higher rates because the value of collateral falls (Geanakoplos 2022), since the units in our model borrow against earnings, only.

At time 0, a controlling entity owns two units, $i = S, B$. Each unit has a random exogenous operating cash flow, X_i, that is realized at time T. The symbol $G(\cdot)$ denotes the cumulative distribution function and with $f(\cdot)$ the density of X_i, identical for the two units; $g(\cdot,\cdot)$ is the joint distribution of X_S and X_B and ρ their correlation coefficient. At time 0, the controlling entity issues a zero-coupon debt with maturity T to finance the cash-flows in the two units. The face value F_i of the debt issued maximizes the total arbitrage-free value (ν_{SB}) of equity, E_i, and debt, D_i in the two units:

$$\nu_{SB} = \max_{F_S, F_B} \sum_{i=S,B} (E_i + D_i)\,. \tag{1}$$

Each unit pays a flat proportional income tax at an effective rate $0 < \tau_i < 1$ and loses a proportion, $0 < \alpha_i < 1$, of its cash flows in default. Interest on debts is entirely deductible from taxable income. This tax advantage of debt generates a trade-off. On the one hand, increasing leverage generates tax savings, while on the other it increases expected default costs because – everything else being equal – higher leverage increases default likelihood.

At time T, cash flows are realized and distributed in the following order. First, corporate income taxes are paid. Then, debt obligations are fulfilled, if possible. When a unit cannot meet its debt obligations, its income, net of both taxes and default costs, is distributed to the lenders. Shareholders will receive net residual cash flows, if any, after debt is fully reimbursed. Net cash flows are then defined as $X_i^n = (1-\alpha_i)X_i 1_{\{0 \leq X_i \leq X_i^Z\}} + \left((1-\alpha_i-\tau_i)X_i + \tau_i X_i^Z\right) 1_{\{X_i^Z \leq X_i \leq X_i^d\}} + \left((1-\tau_i)X_i + \tau_i X_i^Z\right) 1_{\{X_i \geq X_i^d\}}$. The expressions for the values of equity and debt, E_i and D_i, are in the Appendix.

In this set-up, maximizing the value of debt and equity for the owner is equivalent to minimizing the sum of expected taxes (T_i) and default costs (C_i):

$$\nu_{SB} = \min_{F_S, F_B} \sum_{i=S,B} T_i + C_i. \tag{2}$$

The expected tax burden of each unit is proportional to its expected operational cash flow X_i, net of the tax shield X_i^Z, defined as the interest deductions. Since debt is modeled as a zero-coupon bond, interests are equal to the difference between the face value of debt, F_i, and its market value D_i: $X_i^Z = F_i - D_i$. Default costs are proportional to income.

Below, we will model two different ways of owning the two units. In one case, units are connected through a conditional support guarantee. In the other case, they are separate.

2.1. The stand-alone Units

It is useful to start with the benchmark case of unconnected, stand-alone units. The expected tax burden in each stand-alone (SA) unit is equal to:

$$T_i^{SA}(F_i) = \tau_i \phi \mathbb{E}[(X_i - X_i^Z)^+], \tag{3}$$

where the expectation is computed under the risk neutral probability and $\phi = \frac{1}{(1+r)^T}$ is the discount factor for the time-T horizon at which the cash flows are realized. The superscripts and subscripts, i, indicate whether the stand-alone unit is endowed with the sponsor ($i = S$) or backed unit ($i = B$) parameters.

Each stand-alone unit defaults when its realized net cash flow is lower than the face value of debt; in other words, default occurs when cash flows are lower than the default threshold X_i^d defined as $X_i^d = F_i + \frac{\tau_i}{1-\tau_i} D_i$. Expected default costs, that are a dead-weight loss, are equal to:

$$C_i^{SA}(F_i) = \alpha_i \phi \mathbb{E}\left[X_i 1_{\{0<X_i<X_i^d\}}\right]. \tag{4}$$

They are proportional to the default cost parameter, α_i, and increase in realized cash flows, when the unit goes bankrupt. A rise in the nominal value of debt, F_i, increases the default threshold, X_i^d, thereby increasing the expected default costs.

When units are owned separately, the value of the objective function (2) is simply the sum of the values of the taxes and default costs in each unconnected unit. Notice that the optimal value of each unit is equal to:

$$V_i(F_i^*(\phi); \phi) = V_i(0; \phi) + TS_i(F_i^*(\phi); \phi) - C_i(F_i^*(\phi); \phi), \tag{5}$$

where $F_i^*(\phi)$ is the optimal face value of debt in unit i, $V_i(0; \phi)$ is the value of a zero-leverage unit, and $TS_i(F_i; \phi) = T_i^{SA}(0; \phi) - T_i^{SA}(F_i; \phi)$ is the present value of the tax savings from leverage, equal to the difference between the taxes paid by a zero-leverage unit and a unit which issues debt F_i. Equation (5) highlights that, among other parameters, the optimal solution depends on the discount factor ϕ, and thus on the risk-free rate.

It is worth recalling some properties of the components of the stand-alone unit's value. The tax shield is a convex function of F_i. In fact, increasing the face value of debt increases the tax shield, thereby reducing the tax burden. This occurs because the market value of debt, D_i, increases with

F_i at a decreasing rate (due to higher risk). The default threshold X_i^d is instead concave in the face value of debt, F_i. Luciano and Nicodano (2014) prove that a stand-alone unit has positive optimal debt if the sum of tax burden and default costs is convex in the face value of debt. Nicodano and Regis (2019) further show that it raises positive debt even with a zero risk-free rate because the endogenous spread generates a positive tax shield. The spread, defined as

$$s_i = \left(\frac{F_i}{D_i}\right)^{\frac{1}{T}} - \left(\frac{1}{\phi}\right)^{\frac{1}{T}}, \tag{6}$$

is indeed the part of the rate of return on debt due to its riskiness.

We can now analyze the optimal response to a reduction in the risk-free rate that increases the discount factor, ϕ. Throughout the analysis, while we let ϕ vary, we keep the cash flow distribution fixed. Indeed, we assume that valuation is always performed under the risk neutral probability and let the expected present value of cash flows vary with ϕ. Unfortunately, since the solution of problem (1) is not available in closed-form, even in the case of stand-alone units, we can not directly study the sensitivity of optimal debt to interest rate changes. However, we obtain some interesting results at fixed debt levels. We first prove a lemma on the spread.

Lemma 1. *The spread, s_i, falls as the interest rate, r, declines holding fixed the face value of debt, F_i.*

Proof. See the Appendix. ∎

The lemma establishes that, when interest rates decrease, the increase in the market value of debt, that reduces the yield, prevails over increase due to the lowering of the risk-less rate. An implication of the previous lemma is that the incentive to borrow for tax deductions falls in the risk-free rate. This is the signature feature of trade-off capital structure models.

We now turn to the tax burden and default costs. At a given debt level, F_i, the interest rate influences them, in equations (3) and (4), through two channels. First, they are both discounted expected values, and thus depend on the discount factor ϕ directly. Second, they depend on the tax shield, X_i^Z, and the default threshold, X_i^d, which are influenced by the market value of debt D_i, which in turn depends on ϕ. The proposition below summarizes these effects.

Proposition 1. *Assume the interest rate falls. Then, in a stand-alone company with fixed face value of debt F_i, (a) the tax shield falls and the no-default threshold increases; (b) if $\tau < 0.5$, the former effect dominates; (c) the expected values of both taxes and default costs increase.*

Proof. See the Appendix. ■

While results concerning taxes (in Part (a) and (c) of the proposition) are straightforward, because the tax shield falls together with both the interest rate and the spread, the ones concerning default costs are subtler. They stem from a reduction in net after-tax cash flows available to repay debt, due to the reduction in the tax shield. This indeed leads to an increase in the probability of default. Part (b) establishes that, for reasonable values of the tax rate ($\tau < \frac{1}{2}$), the loss of tax benefits from leverage is first order relative to the increase in default costs.

We are now able to determine the value changes in both debt and equity due to the interest rate reduction. First, a discount factor effect raises the value of both debt and equity. Second, the variations in both the tax shield and no default thresholds captured by Proposition 1 increase the default probability, reducing both market values. The first effect prevails, as the next Proposition proves.

Proposition 2. *Assume the interest rate falls. Then, in a stand-alone company, the market values of both debt and equity increase holding fixed the face value of debt.*

Proof. See the Appendix. ■

Since the market value of both debt and equity increase as the interest rate falls, the market value of the stand-alone company increases for any F_i.

We now turn to the analysis of the optimal level of debt for the stand-alone firm. While under normal circumstances F^*_{SA} is increasing in the interest rate, there are some peculiar conditions that may reverse the intuitive pattern. The proposition below shows that lower interest rates reduce stand-alone leverage when the marginal tax advantage of debt falls faster than the marginal expected distress cost.

Proposition 3. *Assume that $\frac{\partial^2 D}{dF d\phi} > 0$, i.e. that an interest rate drop increases the marginal effect of leverage on the value of debt, and convexity of (2). Then, the optimal face value of debt in a stand alone decreases as the interest rate falls, provided that the following condition holds:*

$$\frac{\alpha}{\tau} > \frac{(1 - \frac{\partial D}{dF})g(X^Z)\frac{\partial D}{d\phi} - \frac{\partial^2 D}{dF d\phi}(1 - G(X^Z))}{\frac{f(X^d)}{1-\tau}\left[\frac{\partial^2 D}{dF d\phi}X^d + \frac{\frac{\partial D}{d\phi}(1-\tau+\tau\frac{\partial D}{dF})}{1-\tau}\right] + \frac{dX^d}{dF}X^d f'(X^d)\frac{\partial X^d}{\partial\phi}} = \frac{\frac{\partial MTS}{\partial\phi}}{\frac{\partial MDC}{\partial\phi}},$$

where MTS stands for Marginal Tax Savings and MDC stands for Marginal Default Costs and all quantities are evaluated at the optimum. This condition is satisfied for $\alpha \to 1$ and $\tau \to 0$ and *for every $\alpha > 0, \tau > 0$ if marginal tax savings from leverage decrease and marginal default costs increase when interest rates decrease. In particular, this is always the case if:*

- $f'(X^d) \geq 0$ *and* $\frac{\partial^2 D}{dF d\phi}(1 - G(X^Z)) - (1 - \frac{\partial D}{dF})g(X^Z)\frac{\partial D}{d\phi} \geq 0.$

Proof. See the Appendix. ■

We obtain the condition in the proposition by comparing the effect of a decrease in interest rates on marginal tax benefits and marginal default costs from additional leverage. Indeed, the firm decreases the optimal leverage if the latter outweighs the former. The condition states that the optimal face value decreases if the ratio of default costs to the tax rate exceeds the ratio between the sensitivities to an interest rate drop of marginal tax benefits and marginal default costs.

The condition is always satisfied if marginal tax savings from leverage decrease and marginal default costs increase when interest rates fall. This is the case under normal circumstances, and it is likelier to be satisfied as $\alpha > 0$,$1 - \tau > 0$, X_0/σ are larger.

However, marginal tax benefits may increase when interest rates drop if $(1 - \frac{\partial D}{dF})g(X^Z)\frac{\partial D}{d\phi} \leq \frac{\partial^2 D}{dF d\phi}(1 - G(X^Z))$, i.e. if the valuation effect, that increases the no-tax area, is stronger than the marginal tax savings decrease effect. This is unlikely to be true, but may happen when X^Z is already very high, i.e. when the firm is highly levered. Marginal default costs decrease when interest rate drops only when $f'(X^d) < 0$ and $|f'(X^d)|$ is high enough. Again, this may be the case if debt is already very high (above the mode, in a uni-modal distribution). The assumption of a positive $\frac{\partial^2 D}{dF d\phi}$ may also fail, again under very particular circumstances.

2.2. The sponsor and its backed unit

We now consider two connected units, linked through a conditional guarantee, as in Luciano and Nicodano (2014). The guarantee allows the sponsor to transfer part of its net profits to an insolvent, but profitable, backed unit, when such transfer is able to prevent its insolvency. Formally, the sponsor transfers an amount $F_B - X_B^n$ to the backed unit, provided its net profits are large enough to repay its lenders first, so that both units are solvent: $(X_S^n - F_S \geq F_B - X_B^n)$. Importantly, the sponsor enjoys limited liability relative to the debt of its backed unit.

As discussed in Luciano and Nicodano (2014), the selective guarantee is always value-enhancing for a fixed risk-free rate. On the one hand, default costs fall, at a given level of debt, because the guarantee embeds the option of letting the subsidiary default when the parent would otherwise go bankrupt. On the other hand, the guarantee increases the tax burden by increasing the fair value of debt for lenders, leading to a lower tax shield. The first effect dominates the second.

Furthermore, Luciano and Nicodano (2014) also show conditions ensuring that the sponsor is optimally zero-leverage, for fixed risk-free rate and equal parameters in the two units. Indeed, there is a trade-off between increasing the sponsor's borrowing and tax shield, and increasing the backed unit's tax shield while reducing the sponsor's debt. The rescue by the sponsor is more likely the smaller is the sponsor debt, F_S, because, everything else fixed, the sponsor needs less cash flows in servicing its own debt.[5] It may therefore be optimal to forego the tax savings from leverage in the sponsor while concentrating them in the backed unit.

Below we show that a zero-leverage sponsor is optimal, for unrestricted parameters, when the risk-free rate falls sufficiently low:

Proposition 4. *Assume a convex value function in* (2)*. Then, there exists an interest rate level* $\bar{r}(\tau_S, \tau_B, \alpha_B, G, g)$ *below which the optimal sponsor's debt is zero,* $F_S^* = 0$*. Everything else fixed,* $\bar{r}$ *decreases with* τ_S*, for* $\tau_S < 0.5$*.*

Proof. See Appendix. ■

When interest rates are sufficiently low, it becomes optimal for the sponsor to remain unlevered in order to maximize its capacity to support the backed unit in distress states. The cutoff interest rate is lower if the sponsor's tax rate is higher, making its own tax shield more valuable. The rationale is that a lower risk-free rate on the one hand reduces the sponsor's tax shield, as in a stand-alone unit. Differently from the latter case, a lower rate increases the value of providing support. The latter effect dominates when r is low enough. Numerical analysis of the value of the cutoff rate $\bar{r}$ also reveals that it decreases in α_B and increases in τ_B, when the incentive to borrow from the backed unit are larger.

As a consequence, the leverage of the backed unit and of the complex organization as a whole may increase when the interest rate falls, as we show numerically in the following section.

3. Optimal Leverage and Credit Risk sensitivity to the Risk-Free Rate

In this section, we numerically analyze the changes in the optimal capital structure associated with variation in the risk-free rate. We also analyze the changes in the endogenous default probability, spread and loss given default. We compare stand-alone units to the connected units belonging to a complex organization, assuming they display the same parameters.

5. The rescue event, $h(X_B)$ is formally defined in the Appendix.

Table 1: Base-case parameters		
Symbol	**Parameter**	**Value**
τ	Tax rate	20%
α	Default costs rate	23%
r	Annual risk-free rate	5%
$\phi = \frac{1}{(1+r)^5}$	Discount factor	0.78
$X(0)$	Cash flow present value	100
V_U	Zero-leverage unit after-tax present value	80.05
T	Time horizon	5
σ	Cash flow volatility at T	$22\sqrt{5}$
ρ	Cash flow correlation	0.2

Table 1: This table displays the base-case parameters, following Leland (2007). The zero-leverage value, (V_U), is the present value of positive cash flows net of taxes. The annual cash-flow volatility, 22, is scaled to the 5-year horizon.

Table 1 displays the base-case calibration parameters, as in Leland (2007), which refer to a typical BBB company.

We set the tax rate and the proportional bankruptcy costs to $\tau_i = 20\%$ and $\alpha_i = 23\%$, $i = S, B$ respectively, and then proceed to parametric changes. We fix the marginal distributions of cash flows at maturity (5 years) to a normal distribution with mean $100 * (1.05)^5$ and standard deviation $\sigma = 22 * \sqrt{5}$ and we maintain a joint normality assumption for connected units, letting correlation vary.

We start by assessing the capital structure changes in the stand-alone unit, when the risk-free rate falls from 5% to 1%.

3.1. The Stand-Alone Company

Our first observation is that, in the stand-alone case, the decrease in interest rates reduces the incentives towards leverage (see Table 2).

Indeed, the optimal face value of debt decreases by almost 40%, from 57.1 to 34.3. Also the market value of debt drops by almost 25%, from 42.2 to 31.8. While the value of a hypothetical zero-leverage company increases sharply (from 80.05 to 97.20) when the interest rate drops, due to the lower discount rate, the value of leverage, i.e. the difference between the optimally leveraged and the zero-leverage unit value, drops dramatically, from 1.42 to 0.25. The reduction in the value of leverage is explained by the lower tax shield, which falls from 14.89 to 2.46. As a consequence, the tax savings from leverage fall, from 2.32 to 0.47. Symmetrically, taxes increase from 17.70 to

Table 2: Optimal Stand-Alone Organization			
Variable	**Formula**	Interest rate 5%	Interest rate 1%
Optimal zero-coupon bond principal	F^*	57.10	34.30
Optimal levered value	$V^* = D^* + E^*$	81.47	97.45
Value of optimal debt	D^*	42.21	31.84
Value of optimal equity	E^*	39.26	65.61
Tax shield	$X_Z^* = F - D$	14.89	2.46
No-default threshold	$X^d = F + \frac{\tau}{1-\tau} D$	67.65	42.26
Zero-leverage unit value	V_U	80.05	97.20
Value of optimal leverage	$V^* - V_U$	1.42	0.25
Debt yield	$y^* = (F^*/D^*)^{\frac{1}{5}} - 1$	6.23%	1.50%
Spread	$s^* = \left(\frac{F}{D}\right)^{\frac{1}{5}} - \left(\frac{1}{\phi}\right)^{\frac{1}{5}}$	1.23%	0.50%
Taxes	$T^* = \tau\phi\mathbb{E}[(X - X^Z)^+]$	17.70	23.84
Tax savings	$TS^* = T^{(0)} - T^{(F^*)}$	2.32	0.47
Default costs	$C^* = \alpha\phi\mathbb{E}\left[X\mathbf{1}_{\{0<X<X^d\}}\right]$	0.89	0.22
5-year default probability	$DP^* = \int_{-\infty}^{X^d} f(x)\,dx$	11.14%	4.13%
Loss given default	$LGD^* = \frac{F^* - D^* \frac{1}{\phi}}{DP^*}$	28.96	20.16

Table 2: This table displays the optimal structure of a stand-alone unit with parameters as in Table 1, for two levels of interest rates, 5% and 1%. Yields and spreads are annualized. The default probability is defined as the probability that the unit is not able to repay its lenders after $T = 5$ years, when the cash flows are realized. The cash flow distribution is fixed at T: $X \sim N(127.63, 49.19)$.

23.84.

Since the optimal debt is smaller, the default threshold shrinks, albeit less sharply than the tax shield, from 67.65 to 42.26. Default costs fall accordingly, from 0.89 to 0.21. Interestingly, default costs drop also relative to both the optimal value and the present value of expected cash flow. The lower riskiness of the optimal stand-alone company as interest rates fall is driven by a much smaller default probability. In the base-case, it is 11.14% at the 5-year horizon. This is largely due to leverage, as the probability of default for the zero-leverage unit is 0.47%, only. When interest rate drops to 1%, leverage decreases and the default probability decreases accordingly, down to 4.13%. The endogenous (annualized) spread[6] reflects such change, decreasing from 1.23% to 0.50%.

Lenders' losses given default fall in absolute terms as the interest rate decreases, from 28.96 to 20.16, due to the lower face value of debt. As a percentage of the principal, however, they increase:

6. The spread s is the difference between the yield, defined as $y = (F/D)^{\frac{1}{5}}$ and the interest rate r.

only about 41% of the principal is recovered by lenders in default when the interest rate is 1% vs. 50% when it is 5%. Such increase in losses given default, which are defined as $LGD = \frac{F-D\frac{1}{\phi}}{DP}$, derives from the increase in the discount factor in the calculation of expected discounted losses (at the numerator) and by the decrease in the default probability (at the denominator). In summary, defaults become less frequent—which reduces expected default costs—but each default is more severe relative to the outstanding debt. As the default threshold moves closer to zero with lower borrowing, defaults tend to occur in states where after-tax profits are negative or very small, so lenders recover little or nothing.

The decrease in optimal leverage in response to interest rate cuts is a consistent pattern across parametric changes. It occurs when cash flow volatility is higher (44%) or lower (15%), when the tax rate is higher (24%) or lower (16%) and when the proportional default cost parameter is higher (26%) or lower (20%), consistent with the insight deriving from Proposition 1. The decrease in leverage due to a fall in interest rates is milder the higher the incentive toward leverage, i.e. the higher the volatility and the tax rate and the lower the default cost rate.

We can summarize these results as follows, assuming that everything else is unchanged including the distribution of future cash flows:

Observation 1. *When the risk-free interest rate decreases, the optimal leverage of a stand-alone company falls together with its expected default costs, default probability and losses given default.*

3.2. A Zero-Leverage sponsor: the Private Equity Case

In this section we turn to the case where a sponsor may support the service of debt of a profitable but insolvent backed unit. We start by addressing the case where both are endowed with the base-case parameters presented in Table 1. Table 3 reports the optimal capital structure and relevant figures for different levels of correlation between cash flows between the connected units. However, we will focus our discussion on the case of a weak (0.2) positive correlation between unit cash flows. This is also the correlation maintained in Figure 1 and Figure 2, that display the implied changes in connected units vs. two equivalent stand-alone units when the risk-free rate varies.

When the risk-free rate is 5%, Table 3 shows that the backed unit is optimally a very risky entity, despite the presence of a supporting sponsor. The sponsor does not borrow while the backed company has an optimal face value of debt which is almost twice that of two stand-alone units (220 versus 114.2), as in Luciano and Nicodano (2014). Such polarized capital structure is typical of private equity fund-LBO target arrangements (Cohn et al. 2014). Thanks to zero leverage, the sponsor maximizes the support provided to the highly leveraged unit, which is in turn able to maximally exploit its tax shield. The tax shield indeed reaches 103 in the subsidiary unit, up from 14.9 in each stand-alone unit. As a consequence, tax savings from leverage in the backed unit are far higher than in two stand-alone (14.62 vs. 4.64). Expected default costs are four times larger than in a stand-alone, reaching 8.13 (vs. 1.78). Such extreme exploitation of the tax-bankruptcy trade-off is allowed for by the conditional guarantee provided by the sponsor, that limits default costs at a given faced value of debt relative to the stand-alone unit. As a consequence, the sponsor/backed unit organization is more valuable than two equivalent stand-alone (166.59 vs. 162.94).

Indeed, its default threshold (i.e. the cash flow level below which default occurs) grows to a startling 242, up from 67.7 of each stand-alone unit. Its default probability is thus much higher and the losses given default are far larger than those of each stand-alone unit (47% vs. 11% and 148.98 vs. 28.96, respectively). Recall that lenders' losses given default are larger because the sponsor supports the backed company only if the latter has positive cash flows. As a consequence, the endogenous spread, which reflects the credit risk compensation demanded by the lenders, rises to 8.45%, up from 1.23% in the stand-alone unit. It is the large spread that leads to the high tax savings we just illustrated.

Let us now move to Figure 1 and assess changes in response to a drop in the risk-free rate. Despite the already high leverage, the backed unit's face value of debt further increases to 247 when the risk-free rate is 1%. At every interest-rate level considered, it remains optimal for the sponsor to choose zero leverage. Thus, as interest rates fall, the complex organization exploits the tax–bankruptcy trade-off even more aggressively through the backed unit, while the stand-alone firm optimally reduces its leverage. We now study how this result derives from endogenous debt pricing and costly default.

First, tax savings increase along with debt. Second, the default probability and the deadweight costs of default increase, driving up the spread. More precisely, the tax savings increase as the interest rate falls, from 14.6 to 19.1, and the default costs almost double, rising from 8.1 to 14.5. The ratio of default costs to total group value rises from 4.88% to 7.29%, outpacing the growth of tax savings to group value, which instead rises from 8.78% to 9.59%. While the value of leverage, i.e. the difference between the optimal and the zero-leverage value, falls from 6.49 to 4.54 due to the drop in interest rates, the complex organization remains 2.3% more valuable than two stand-alone units (198.96 vs. 194.42).

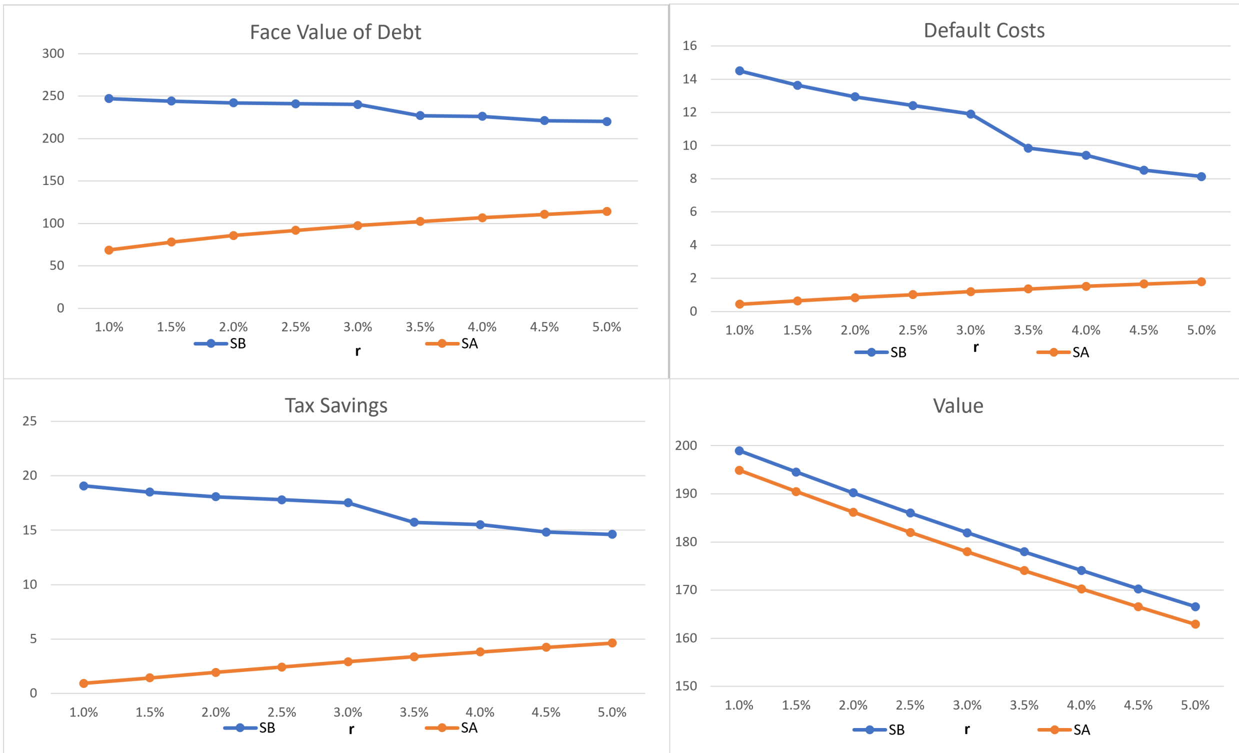

Figure 1: The four panels represent the optimal zero-coupon debt principal, default costs, tax savings and total value of the complex organization organization (in blue) and the equivalent stand-alone arrangement (in orange) for different interest rate levels, ranging from 1% to 5%. The sponsor is optimally zero-leverage for all interest rate levels considered. The parameters are collected in Table 1. The cash flows of the units are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter 0.2.

The value consequences of a drop in interest rates are as follows. While debt market value – concentrated in the backed company – increases by 14%, from 117.06 to 133.43, the equity value (concentrated in the sponsor) increases much more, by almost 33%. Such increase in equity value, due to the discounting effect of lower interest rates, contributes to explain the reason why the market value of leverage in the complex organization does not increase.

The credit risk of the backed unit increases as the risk-free rate decreases. Indeed, as portrayed in Figure 2, the default probability increases to 63.3% when the risk-free interest rate is 1%, up from 47.38% when $r = 5\%$. The spread increases accordingly, topping 12.11%, as does the loss given default, from 148.98 to 168.68 (or from 67.72% to 68.29% in percentage terms). Again, the backed unit behaves very differently relative to the stand-alone, whose default probability and spread decrease when the interest rate decreases. Let us finally note that the zero-leverage sponsor is insolvent very rarely in all interest rate scenarios. This happens only when its cash flow realization is negative, an event that is independent from the level of interest rates in our setting. This is why

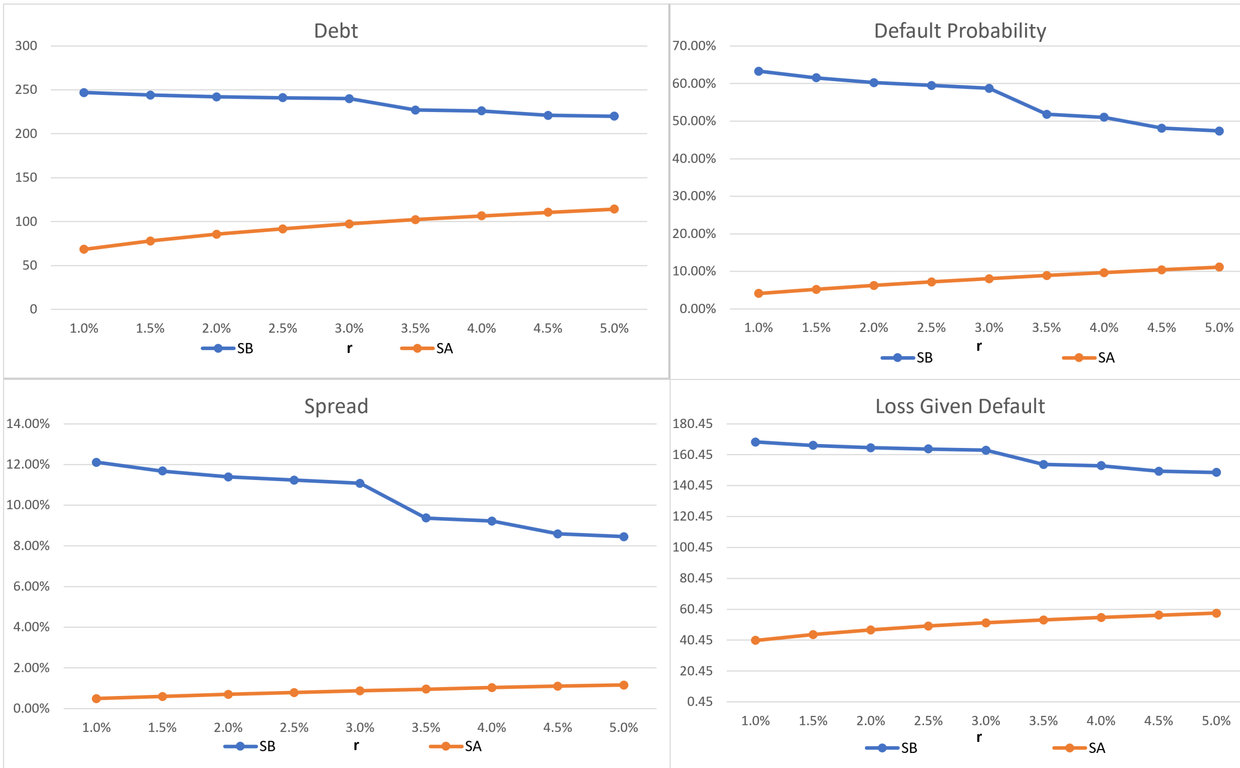

Figure 2: This figure contrasts the optimal face value of debt, 5-year default probability, annualized credit spread and loss given default of a company backed by a bankruptcy-remote sponsor (in blue) and the equivalent stand-alone (in orange) for different interest rate levels, ranging from 1% to 5%. The parameters are collected in Table 1. The cash flows of the units are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter 0.2.

the joint default probability is unaltered when the interest rate changes.

Figure 1 and Figure 2 show that the patterns we just discussed hold uniformly as the level of the risk free rate falls from 5% to 1%. Furthermore, the above results qualitatively hold true for different correlation levels, as reported in Table 3.

In particular, higher cash flow correlation makes support more valuable because it allows for higher tax savings. To obtain such tax savings, the sponsor has to be able to provide funds when the supported unit, while insolvent, has positive cash flows. Both the optimal face value of debt of the backed unit and its riskiness, as captured by the spread, increase with correlation and top 257 and 12.99%, respectively, when cash flow correlation is equal to 0.8 and the interest rate is 1%.

The statement below summarizes these patterns, assuming that only the risk-free rate varies:

Observation 2. *In a unit backed by a zero-leverage sponsor, the optimal debt increases when the risk-free interest rate decreases. Both its spread and tax savings, along with its default probability and its losses given default, increase. These changes are larger the higher is the cash flow correlation*

Table 3: Optimal Value and Debt — Sponsor / Backed Unit						
	Correlation					
	-0.8		0.2		0.8	
Variable	Interest Rate		Interest Rate		Interest Rate	
	5%	1%	5%	1%	5%	1%
Face Value of Debt	183 (0;183)	201 (0;201)	220 (0;220)	247 (0;247)	227 (0;227)	257 (0;257)
Market Debt Value	133.58 (0;133.58)	153.38 (0;153.38)	117.06 (0;117.06)	133.43 (0;133.43)	115.53 (0;115.53)	133.55 (0;133.55)
Equity Value	32.74 (32.74;0)	42.88 (42.88;0)	49.52 (49.52;0)	65.53 (65.53;0)	51.84 (51.84;0)	66.70 (66.70;0)
Total Value	166.32 (32.74;133.58)	196.26 (42.88;153.38)	166.59 (49.52;117.06)	198.96 (65.53;133.43)	167.36 (51.83;115.53)	200.24 (66.70;133.54)
Value of Leverage	6.23	1.86	6.50	4.56	7.27	5.86
Tax Savings	7.57 (0;7.57)	8.87 (0;8.87)	14.62 (0;14.62)	19.07 (0;19.07)	15.50 (0;15.50)	20.16 (0;20.16)
Taxes	32.45 (20.01;12.44)	39.73 (24.30;15.43)	25.40 (20.01;5.39)	29.53 (24.30;5.23)	24.52 (20.01;4.51)	28.44 (24.30;4.14)
Default Costs	1.91 (0;1.91)	7.29 (0;7.29)	8.13 (0;8.13)	14.50 (0;14.50)	8.24 (0;8.24)	14.29 (0;14.29)
Yield	(N/A;6.50%)	(N/A;5.56%)	(N/A;13.45%)	(N/A;13.11%)	(N/A;14.46%)	(N/A;13.99%)
Spread	(N/A;1.50%)	(N/A;4.56%)	(N/A;8.45%)	(N/A;12.11%)	(N/A;9.46%)	(N/A;12.99%)
Default Probability	(0%;11.05%)	(0%;30.76%)	(0%;47.38%)	(0%;63.30%)	(0%;50.43%)	(0%;64.73%)
Joint Default Probability	0.01%	0.22%	0.47%	0.47%	0.47%	0.47%
Loss Given Default	(0;113.28)	(0;129.38)	(0;148.98)	(0;168.68)	(0;157.77)	(0;180.21)

Table 3: This table displays the optimal structure of a backed unit with its sponsor when both units have the parameters reported in Table 1, for two levels of interest rates, 5% and 1%. Sponsor and backed unit figures are reported in parentheses, respectively, while the total figure is outside the parenthesis. Cash flows are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter ranging from -0.8 to 0.8. Yields and default probabilities are annualized. Default probabilities refer to the likelihood that lenders are not repaid in full when cash flows are realized at $T = 5$.

between the sponsor and its backed unit.

These findings are broadly consistent with the observed divergent response, by public companies and comparable LBO targets, to lower interest rates. Axelson et al. (2013) find that the ratio of debt to EBITDA is higher in LBO targets but not in matched public companies when interest rates fall. The spreads they find depend on the type of debt. The median is equal to 262bp and 937bp for senior and junior bank loans respectively, reaching up to 916bp and 1048bp for senior and subordinated bonds respectively. The spread implied by our model, when 1% (5%) is the level of interest rates, is equal to 441bp (130bp) when cash flow correlation between the fund and the LBO target is -0.8, reaching 1130bp (762bp) when cash flow correlation is 0.2. Our numerical exercise also shows that these spreads allow to reduce the tax burden of the LBO target to up to one fifth of the taxes paid by a similar zero-leverage company. [7]

Our results bear implications for empirical work. They imply that information on the structure of organizations is essential to understand leverage choices, as the response to interest rates is opposite for a stand-alone and a backed unit with the same characteristics. Furthermore, while we know that information about the complex structure improves on default prediction (Beaver et al. 2019), our model suggests a flip in the sign of the prediction depending on the presence of the sponsor.

Finally, the analysis sheds light on regulators' concerns of potential financial instability observing the rise – from below 4 to above 5 - in the multiple of average of average total debt to EBITDA for leveraged transactions priced at or above LIBOR + 225bp, as interest rates were falling (Rosengren 2019). Our results indicate that when leverage in structured finance increases in response to lower interest rates, both default probabilities and, to a lesser extent, lenders' losses given default also rise. However, our numerical exercise is not a full-fledged calibration. That is, it allows for neither increases in mean cash flows triggered by a lower risk-free rate, nor lower cash-flow volatility and higher productivity relative to stand-alone companies observed in private equity research. [8]. Furthermore, we see that the default rates of stand-alone and backed units move in opposite directions. This suggests that an economy composed of both complex organizations and stand-alone units displays smoother changes in aggregate default costs as interest rates vary relative to one composed just of the latter.

4. A Leveraged Sponsor: the Parent-Subsidiary and Securitization Cases

The previous section analyses a sponsor that is optimally zero-leverage both before and after the interest rate reduction. However, leveraged parent units in multinationals and other corporate groups, which also act as sponsor by backing their subsidiaries' debt, are common (Bianco and Nicodano 2006; Brok 2024; Anantavrasilp et al. 2020). Similarly, (shadow) banks that sponsor securitization vehicles retain part of the loans in their books (Allen et al. 2023; Jiangli et al. 2007; Gorton and Souleles 2006). This section turns to the case of a sponsor that is optimally leveraged before the interest rate reduction. This obtains provided it enjoys higher marginal tax rates and/or a lower proportional bankruptcy cost parameter than its backed unit (see Nicodano and Regis

7. As in Haque (2022), the backed unit (the LBO target) is not over-leveraged, even in a low interest rate environment, despite displaying much higher credit risk costs than its stand-alone counterpart.

8. see Nicodano et al. 2026 for a study targeted to the replication of observed private equity valuation and debt multiples.

(2019)).

Let us then consider the case when the tax rate of the sponsor ($\tau_S = 24\%$) exceeds that in the backed unit ($\tau_B = 16\%$).[9] Since the incentive to raise debt is in general stronger the higher is cash flow volatility, we also set $\sigma_S = \sigma_B = \sigma = 44 * \sqrt{5}$. We first focus on the base-case correlation ($\rho = 0.2$). Figure 3 reports the optimal debt, market leverage, tax savings and default costs of both the complex organization and the equivalent two stand-alone units as the interest rate varies between 1% and 5%. Figure 4 displays the optimal debt, default probability, spread and loss given default of the sponsor and the backed unit, comparing them with their equivalent stand-alone values. Table 4 reports the numerical values of the optimal characteristics.

At the initial level of interest rates ($r = 5\%$), the total face value of optimal debt exceeds that in two equivalent stand-alone units (199 vs. 169). The complex organization raises more debt from the backed unit than from its sponsor (124 vs. 75), despite the higher sponsor's tax rate which makes its debt tax shield more valuable than that of its backed unit. To preserve its ability to provide support, the sponsor optimally borrows less than its equivalent stand-alone (75 vs. 103) and bears much lower default costs. In turn, the backed unit is not as leveraged as in the case of the previous section 3.2, since its tax shield is now relatively less valuable than the sponsor's, but still borrows more than its stand-alone counterpart (124 vs. 68). Overall, tax savings are higher relative to the stand-alone case (10.58 vs. 10.32), and default costs are mitigated (5.67 vs. 6.09) thanks to the sponsor's support, leading to higher total value (170.59 vs. 169.88). The sponsor and the backed unit appear to be similarly risky, with a (5-year ahead) default probability of around 35% and 37%, respectively. While the sponsor displays a lower (5.86% vs. 7.25%) (annualized) spread than its stand-alone equivalent, the opposite happens for the backed unit (5.86% vs. 4.74%).

As the interest rate decreases to 1%, we highlight two main effects. First, the total face value of debt raised by the complex organization has a non-monotone behavior. It first decreases, as r falls from 5% to 3%, and then increases relative to its initial 5% level. As Figure 3 captures, this is in sharp contrast with what happens to the two equivalent stand-alone units, whose combined optimal debt falls monotonically from 170 to 121 as interest rates drop. Second, when the interest rate falls sufficiently, all debt is optimally raised at the backed unit level. Indeed, Proposition 4 indicates that the sponsor may optimally specialize in providing support. This occurs because a

9. Such lower tax rate is also consistent with securitizations SPVs and affiliates incorporating in jurisdictions with more favorable tax treatment, such as Cayman and Ireland.

lower interest rate shifts the balance between two opposing incentives—increasing the sponsor's own tax shield versus providing additional support to the backed unit—in favor of the latter. As the sponsor becomes zero leverage, the backed unit maximally exploits the tax shield, allowing the organization to become more valuable at the cost of increasing its riskiness.

The default costs of the organization increase sharply as the interest rate falls to 1%. In particular, those of the backed unit are more than 11 times the default costs of an equivalent stand-alone unit (9.08 vs. 0.79). The (5-year) default probability reaches 54%, up from 37.8% in the baseline 5% interest rate. Losses given default also deteriorate, rising to 175.42 from 88.42 (75.61% vs. 71.31% in percentage terms). These changes affect the endogenous (yearly) spread, which rises from 5.86% to 12.20%. However, the default probabilities of the two units may move in two opposite directions. In the sponsor, as debt decreases the default probability drops when the interest rate moves from 5% to 1%. Below 3%, when the sponsor becomes a zero-leverage entity, it defaults only when its realized cash flows are negative (9.73% probability), bearing no losses due to its limited liability.[10]

Thus, for our selected parameters, we observe a transformation to a zero-leverage sponsor and a highly leveraged backed unit. This transformation may also be interpreted as a sale of a subsidiary from a business group or multinational to a private equity fund that increases the acquired unit's leverage. The complex organization in this paper indeed resembles a private equity arrangement when the sponsor, like a private equity fund, is optimally zero-leverage. It may instead remind of a parent-subsidiary group when the sponsor is leveraged. Our model then implies that zero-leverage (leveraged) sponsors are value maximizing when the level of interest rates is low (high) enough. This suggests that public-to-private transactions of backed units increase when interest rates fall. [11]

In Table 4, we let the cash flow correlation of the two units vary. The table shows that the level of cash flow correlation has an impact on whether the transformation occurs. In particular, the zero-leverage sponsor is optimal when correlation is relatively high, that is the complex organization is not diversified. This happens because the value of the conditional bailout guarantee is more effective in saving default costs the higher the cash flow correlation. Consider the case of perfect negative

10. As the risk-free rate approaches 3% from above, lenders' losses given default get closer to 100% since the default threshold approaches zero, implying that there is hardly any recovery due to negligible or negative after-tax profits.

11. While lower interest rates have accompanied higher LBO activity (as in Kaplan and Strömberg (2009)), there may be other factors behind such association (Ivashina 2022) beyond the demand-side factors we stress.

correlation. The sponsor is then able to provide support when the backed unit has negative cash flow and dissipative default costs are zero, or *viceversa* the sponsor is unable to provide support when the insolvent backed company has positive cash flows and default costs.

This leads us to the following observation, under the usual *ceteris paribus* assumption:

Observation 3. *When the risk-free rate falls, a parent-subsidiary structure with balanced debt may transform into a zero-leverage sponsor and a highly leveraged backed company, even if the tax rate (proportional bankruptcy cost) of the parent exceeds (is lower than) the subsidiary's. This is more likely to happen when the units in the transformed entity will display higher cash flow correlation.*

A caveat is in order. Subsidiaries of both multinational corporations and business groups distribute dividends to their parent companies (see Almeida and Wolfenzon 2006; Desai et al. 2004; Anantavrasilp et al. 2020), in contrast to the typical securitization vehicle. This is a type of contingent support that our model does not incorporate. However, Nicodano and Regis 2019 show that such payments enhance the sponsor's incentive to borrow. Accordingly, we expect the threshold interest rate, $\bar{r}$, to be lower for sponsors receiving larger dividend distributions from their subsidiaries.

In Table 4, as the risk-free rate drops to the 1% level, we find that both the total debt and the default costs of the complex organization increase for high enough correlation ($|\rho| > -0.2$). In more detail, when the interest rate is 5% default costs are smaller in the complex organization than in the two equivalent stand-alone units (6.09), as in the case presented in Figure 3, unless correlation is very high (0.8). On the contrary, when the interest rate drops to 1%, this happens only for negative correlation levels (-0.8 and -0.2), when the capital structure in the complex organization remains balanced across sponsor and backed units. This implies a possible divergence between the organization that is privately optimal (the one that maximizes value) and the one that is socially optimal (the one that minimizes default cost) depending on the level of the interest rate. In the 5% interest rate scenario, the privately optimal and the socially optimal organization is the complex one for almost all correlation levels. On the contrary, in the low interest rate scenario, the complex organization is privately optimal but no longer welfare-optimal, as stand-alone units display lower default costs.

The exposure to default for the complex organization as a whole, and for its components units, varies with interest rates. Default probabilities increase in the backed unit for the correlation levels for which, following a drop in interest rates, the transformation occurs. The 5-year-ahead default

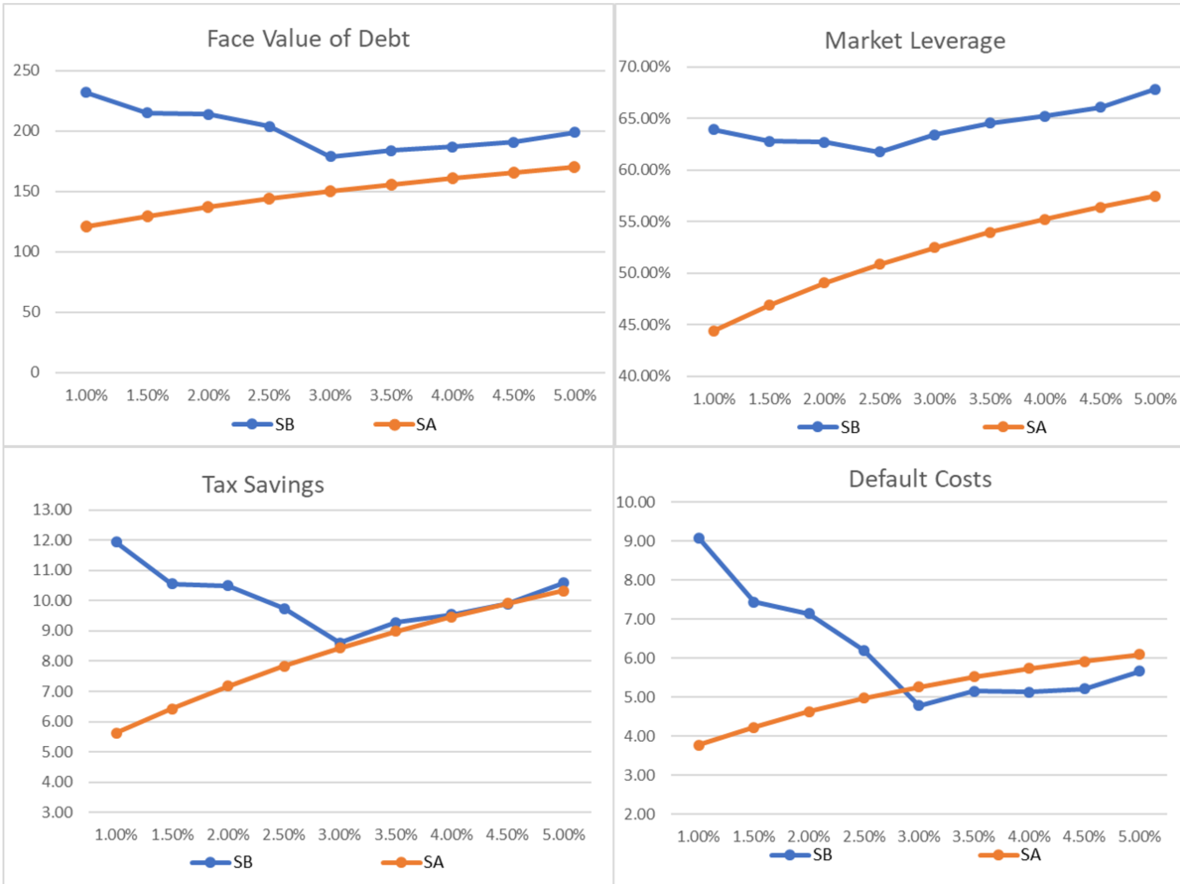

Figure 3: This figure portrays the optimal debt (face value and market value), default costs and tax savings of the sponsor/backed unit arrangement (in blue) when interest rate ranges from 1% to 5% and compares the figures with those of equivalent stand-alone units (orange). In the upper left panel, the red line depicts the optimal debt of the sponsor.

probability tops an impressive 57.44% when $\rho = 0.8$, and the annualized spread consequently reaches 12.85%. On the contrary, the probability of joint default decreases. This is due to both the limited liability of the sponsor and its optimal zero-leverage, ensuring that the sponsor defaults only when its cash flows are negative. Similarly, losses given default in the backed unit (sponsor) increase (decrease) for all correlation levels when interest rates drop from 5% to 1%. Thus, the investors in the sponsor units are better protected from default in low interest rate environments.

Again, some model-based insights seem broadly consistent with observation. The first observation derives from a study of Small and Medium Enterprises (SMEs), covering approximately 70 percent of total corporate and industrial loans made to U.S. firms from 2012 to 2019 (Caglio et al. 2022). The impact of monetary policy in the full sample is driven by private companies, which are mostly Small and Medium Enterprises (SMEs). SMEs with higher leverage borrow more, at a higher cost, during monetary expansions. This result is driven by their higher demand for credit

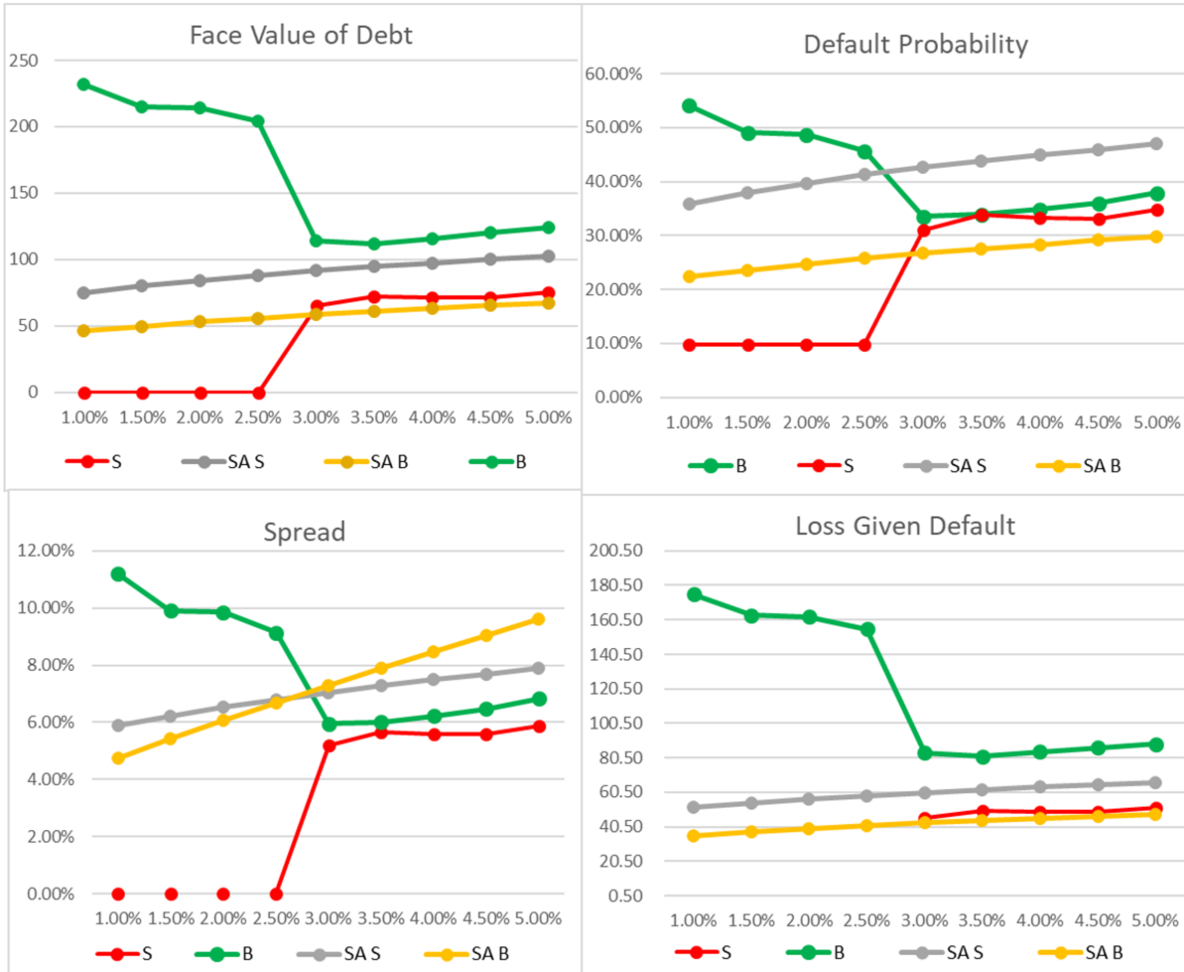

Figure 4: This figure portrays the debt (face value), default probabilities, spreads and loss given defaults of the sponsor (red), the backed unit (green) and compares their figures with those of equivalent stand-alone units (in grey and yellow, respectively). Spreads are annualized, the default probabilities are the probabilities that lenders are not repaid in full when the cash flows are realized at $T = 5$ years.

due to looser cash flow constraints, while their lenders do not increase risk-taking. The higher cost of borrowing these leveraged SMEs pay relative to others is a result of their higher credit demand. On the contrary, highly leveraged public firms borrow less against their collateral during monetary expansions.

The last observation regards Sponsor/SPV arrangements in securitization, that embed a non-contractual support mechanism, display thinly capitalized SPVs and leveraged sponsors (Gorton and Souleles 2006) and, depending on the jurisdiction of incorporation, may enjoy more favorable tax rates and proportional bankruptcy costs than their sponsors. Our results provide a rationale for the disproportionate increase in lending through securitization vehicles as interest rates fell in the first two decades of this century (Powell 2019).[12]

12. They may also explain the reason why securitization amplifies the reduction in credit standards associated with low short-term interest rates in Maddaloni and Peydró 2011.

Table 4: Optimal Value and Debt — Transformation of a Complex Organization

Variable	Correlation -0.8, Interest Rate 5%	-0.8, 1%	-0.2, 5%	-0.2, 1%	0, 5%	0, 1%	0.2, 5%	0.2, 1%	0.8, 5%	0.8, 1%
Face Value of Debt	187	171	186	158	190	192	199	232	239	251
	(73;114)	(54;117)	(82;104)	(58;100)	(80;110)	(0;192)	(75;124)	(0;232)	(29;210)	(0;251)
Market Value of Debt	116.79	133.82	111.76	120.42	112.86	124.74	115.70	130.49	116.64	131.22
	(43.83;72.96)	(41.21;92.61)	(47.92;63.85)	(43.79;76.62)	(47.04;65.82)	(0;124.74)	(44.79;70.91)	(0;130.49)	(19.48;97.16)	(0;131.22)
Equity Value	54.33	70.27	58.94	83.20	57.79	79.01	54.89	73.60	54.54	74.53
	(28.58;25.75)	(41.62;28.65)	(29.14;29.80)	(46.33;36.87)	(30.24;27.56)	(71.05;7.96)	(32.16;22.72)	(70.12;3.48)	(49.39;5.14)	(74.53;0)
Total Value	171.12	204.09	170.70	203.62	170.66	203.75	170.59	204.10	171.18	205.75
	(72.41;98.71)	(82.83;121.26)	(77.06;93.64)	(90.13;113.49)	(77.28;93.38)	(71.05;132.70)	(76.95;93.64)	(70.12;133.97)	(68.88;102.30)	(74.53;131.22)
Value of Leverage	5.47	2.94	5.05	2.47	5.01	2.60	4.96	2.98	5.53	4.66
Taxes	32.20	44.42	31.51	44.27	31.28	41.83	30.83	38.35	29.15	36.77
	(20.06;12.14)	(27.57;16.85)	(19.28;12.23)	(27.29;16.98)	(19.46;11.82)	(30.17;11.66)	(19.89;10.94)	(30.17;8.17)	(23.25;5.90)	(30.17;6.59)
Tax Savings	9.22	5.86	9.90	6.02	10.13	8.45	10.58	11.94	12.26	13.52
	(4.79;4.42)	(2.60;3.26)	(5.66;4.33)	(2.89;3.13)	(5.39;4.74)	(0;8.45)	(4.96;5.62)	(0;11.94)	(1.60;10.66)	(0;13.52)
Default Costs	3.78	3.10	4.86	3.58	5.15	6.01	5.67	9.08	6.76	8.97
	(2.12;1.66)	(1.39;1.71)	(2.77;2.09)	(1.64;1.94)	(2.62;2.53)	(0;6.01)	(2.26;3.41)	(0;9.08)	(0.26;6.50)	(0;8.97)
Yield	(10.74%;9.34%)	(5.55%;4.79%)	(11.34%;10.25%)	(5.78%;5.47%)	(11.20%;10.82%)	(N/A;9.01%)	(10.86%;11.83%)	(N/A;12.20%)	(8.28%;16.67%)	(N/A;13.85%)
Spread	(5.74%;4.34%)	(4.55%;3.79%)	(6.34%;5.25%)	(4.78%;4.47%)	(6.20%;5.82%)	(N/A;8.01%)	(5.86%;6.83%)	(N/A;11.20%)	(3.28%;11.67%)	(N/A;12.85%)
Default Probability	(33.91%;23.98%)	(26.88%;21.37%)	(37.82%;29.44%)	(28.52%;25.38%)	(36.93%;32.62%)	(9.73%;41.90%)	(34.77%;37.88%)	(9.73%;54.07%)	(17.37%;53.97%)	(9.68%;57.44%)
Joint Default Probability	3.94%	2.63%	14.14%	10.08%	17.71%	7.93%	21.72%	9.30%	17.36%	9.74%
Loss Given Default	(50.30;87.11)	(39.76;92.05)	(55.12;76.46)	(41.97;76.71)	(54.04;79.69)	(0;145.34)	(51.29;88.42)	(0;175.42)	(23.80;159.34)	(0;196.88)

Table 4: This table displays the optimal figures of a complex organization when the parameters for the two units are those in Table 1, apart from $\tau_P = 24\%, \tau_S = 16\%$, $\sigma_P = \sigma_S = \sigma = 44\%$, for two levels of interest rates, 5% and 1%. Sponsor and backed unit figures are reported in parentheses, respectively, while the total figure is outside the parenthesis. Cash flows are jointly normally distributed, with marginal distributions as in Table 1 and correlation parameter ranging from -0.8 to 0.8.

5. Concluding Remarks

This paper uncovers the heterogeneous response of optimal debt to changes in the level of interest rates in complex organizations. It shows that the trade-off theory prediction of optimal deleveraging when interest rates fall carries over to sponsors of complex organizations, but not to their highly-leveraged backed units. By loosening the cash-flow constraint and reducing default costs, the sponsor's support allows the backed unit to increase debt - at an increasing spread and tax shield.

Our results imply leverage increases in highly risky units of complex organizations when interest rates fall, led by tax-bankruptcy trade-off motives. This pattern is akin to that predicted by the cash-flow-based motives for borrowing. The two motives deliver instead opposite predictions in the stand-alone case. Our trade-off channel of risk taking also complements existing explanations that stress supply considerations, such as investors' search for yield, and time-inconsistent monetary policy.

Our insights apply to several types of backed units ranging from securitization Special Purpose Vehicles, to affiliates of multinationals and business groups and to portfolio companies of Leveraged Buyout funds. They provide a new rationale for some heterogeneous changes in leverage observed in the US as interest rates were declining, such as the divergent leverage responses in public firms and Leveraged Buyouts targets.

More generally, our analysis bears implications for empirical work. It implies that information on the structure of organizations is essential to understand leverage choices, as the response to interest rates can be opposite for a stand-alone and a backed unit with the same characteristics, especially when interest rates are low. It also suggests a flip in the sign of the interest rate in default predictions depending on the presence of a sponsor and the level of interest rates.

Finally, despite the absence of moral hazard by both lenders and complex organizations, our analysis supports financial stability concerns since backed units indeed display higher default costs than stand-alone units, when interest rates fall below a threshold. However, this result is conditional on the backed companies having the same cash flow distribution and the same horizon as their stand-alone counterparts. This is not always the case. For instance, the stand-alone units display longer horizons, that do not match the debt maturity, and operate in less defensive industries than the ones belonging to Buyout funds. Furthermore, these concerns focus on the comparison of backed versus stand alone units, without considering that sponsors hardly ever default contrary to their stand-alone counterparts. Last but not least, the default probability of backed units increases precisely when stand-alone units are less likely to default, suggesting that heterogeneity in firm types may smooth aggregate default rates across interest-rate scenarios. A thorough assessment of financial stability implications of changing interest rates for complex organizations therefore deserves much closer scrutiny, which we leave for future work.

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6. Appendix

6.1. Definition of the $h(\cdot)$ function

The function $h(X_B)$ defines the set of states of the world in which the sponsor has enough funds to intervene and save its affiliate from default, while at the same time remaining solvent. The rescue occurs if the cash flows of the sponsor X_S are enough to cover both its own debt obligations and the remaining part of those of the affiliate. The function $h(X_B)$, which defines the level of parent cash flows above which the rescue occurs, is defined as:

$$h(X_B) = \begin{cases} X_S^d + \frac{F_B}{1-\tau_B} - \frac{X_B}{1-\tau_B} & X_B < X_B^Z, \\ X_S^d + X_B^d - X_B & X_B \geq X_B^Z. \end{cases}$$

6.2. Expressions for $D_i^{SA}, E_i^{SA}, D_S, D_B, E_S, E_B$

This subsection collects the formal definitions of debt and equity values for stand-alone companies, sponsors, and backed units.

$$D_i^{SA} = (1-\alpha_i)\int_0^{X_i^Z} xf(x)dx + (1-\alpha_i-\tau_i)\int_{X_i^Z}^{X_i^d} (x - X_i^Z)xf(x)dx + F_i\int_{X_i^d}^{+\infty} f(x)dx. \tag{7}$$

$$E_i^{SA} = (1-\tau_i)\int_{X_i^d}^{+\infty} xf(x)dx. \tag{8}$$

$$D_S = (1-\alpha_S)\int_0^{X_S^Z} xf(x)dx + (1-\alpha_S-\tau_S)\int_{X_S^Z}^{X_S^d} (x - X_S^Z)xf(x)dx + F_S\int_{X_S^d}^{+\infty} f(x)dx. \tag{9}$$

$$E_S = (1-\tau_S)\int_{X_S^d}^{+\infty} g(x)dx - \int_{h(X_S)}^{+\infty}\int_{X_S^d}^{+\infty} (F_B - X_B^n)F(x,y)dxdy. \tag{10}$$

$$
\begin{aligned}
D_B = (1-\alpha_B) & \int_{-\infty}^{h(X_S)} \int_0^{X_B^Z} x f(x,y)\, dx\, dy \\
& + (1-\alpha_S-\tau_S) \int_{-\infty}^{h(X_S)} \int_{X_B^Z}^{X_B^d} (x - X_B^Z) x f(x,y)\, dx\, dy \\
& + F_B \left[\int_{X_S^d}^{+\infty} f(x)\, dx + \int_{h(X_S)}^{+\infty} \int_0^{X_B^Z} f(x,y)\, dx\, dy + \int_{h(X_S)}^{+\infty} \int_{X_B^Z}^{X_B^d} f(x,y)\, dx\, dy \right]. \qquad (11)
\end{aligned}
$$

$$
E_B = (1-\tau_B) \int_{X_B^d}^{+\infty} g(x) dx. \qquad (12)
$$

6.3. Proof of Lemma 1

We suppress the subscript i for simplicity. The derivative of the spread with respect to ϕ is

$$
\frac{\partial s}{d\phi} = (-F^{\frac{1}{T}} \frac{1}{T} D^{-\frac{1}{T}-1} \frac{\partial D}{d\phi} + \frac{1}{T} \phi^{-\frac{1}{T}-1}) = \frac{1}{T} (\phi^{-\frac{1}{T}-1} - \left(\frac{F}{D}\right)^{\frac{1}{T}} \frac{1}{D} \frac{\partial D}{d\phi}).
$$

This derivative is positive whenever

$$
\phi^{-\frac{1}{T}-1} \geq F^{\frac{1}{T}} D^{-\frac{1}{T}-1} \frac{\partial D}{d\phi}
$$

and, since $\frac{\partial D}{d\phi} \leq \frac{D}{\phi}$, a sufficient condition for $\frac{\partial s}{d\phi} \geq 0$ is

$$
\phi^{-\frac{1}{T}-1} \geq F^{\frac{1}{T}} D^{-\frac{1}{T}-1} \frac{D}{\phi}.
$$

This is true iff

$$
\left(\frac{1}{\phi}\right)^{\frac{1}{T}} \geq \left(\frac{F}{D}\right)^{\frac{1}{T}},
$$

which implies $\phi \leq \frac{D}{F}$. This is never the case unless $F = 0$ because $D \leq \phi F, D < F$ when $F > 0$.

6.4. Proof of Proposition 1

The derivatives of the expected discounted values of taxes and default costs are respectively (we suppress dependence on F_i and the subscript i for notational convenience):

$$
\frac{\partial T}{d\phi} = \frac{T}{\phi} - \frac{\partial X^Z}{d\phi} (1 - F(X^Z)) \phi \tau \qquad (13)
$$

$$
\frac{\partial C}{d\phi} = \alpha \phi \frac{\partial X^d}{d\phi} X^d f(X^d) \phi \alpha + \frac{C}{\phi}. \qquad (14)
$$

Recalling that $X^Z = F - D$, $X^d = F + \frac{\tau}{1-\tau}D$, indeed we have:

$$\frac{\partial X^Z}{d\phi} = -\frac{\partial D}{d\phi}, \tag{15}$$

$$\frac{\partial X^d}{d\phi} = \frac{\tau}{1-\tau}\frac{\partial D}{d\phi}. \tag{16}$$

Hence, we need to focus on the derivative of market debt value with respect to ϕ, for fixed F (dependence of D on F is suppressed for notational convenience):

$$\begin{aligned}
\frac{\partial D}{d\phi} &= \frac{D}{\phi} + \phi\left[(1-\alpha)\frac{\partial X^d}{d\phi}X^d f(X^d) - \tau\frac{\partial X^d}{d\phi}\left(X^d - X^Z\right)f(X^d) + \tau\frac{\partial X^Z}{d\phi}\left[G(X^d) - G(X^Z)\right] + \right.\\
&- \left. F\frac{\partial X^d}{d\phi}f(X^d)\right]\\
\frac{\partial D}{d\phi} &= \frac{D}{\phi} + \phi\left[(1-\alpha)\frac{\partial X^d}{d\phi}\left(\frac{\tau}{1-\tau}D\right)f(X^d) - \alpha F\frac{\partial X^d}{d\phi}f(X^d) - \tau\frac{\partial X^d}{d\phi}\left(X^d - X^Z\right)f(X^d) + \right.\\
&+ \left. \tau\frac{\partial X^Z}{d\phi}\left[G(X^d) - G(X^Z)\right]\right]\\
\frac{D}{\phi} &= \frac{\partial D}{d\phi}\left[1 - \phi(1-\alpha)\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d) + \phi\alpha F\frac{\tau}{1-\tau}f(X^d) + \right.\\
&+ \left. \phi\tau\frac{\tau}{1-\tau}\left(X^d - X^Z\right)f(X^d) + \phi\tau\left[G(X^d) - G(X^Z)\right]\right] \implies \frac{\partial D}{d\phi} = \frac{D}{\phi}\frac{1}{\kappa}
\end{aligned}$$

Since $\frac{D}{\phi} \geq 0$, the sign of $\frac{\partial D}{d\phi}$ depends on the sign of κ. Rearranging it, we have:

$$\begin{aligned}
\kappa &= 1 - \phi(1-\alpha)\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d) + \phi\alpha F\frac{\tau}{1-\tau}f(X^d) + \\
&+ \phi\tau\frac{\tau}{1-\tau}\frac{D}{1-\tau}f(X^d) + \phi\tau\left[F(X^d) - F(X^Z)\right] = \\
&= 1 + \phi\alpha\left(\frac{\tau}{1-\tau}\right)\left(\frac{\tau}{1-\tau}D\right)f(X^d) + \phi\alpha F\frac{\tau}{1-\tau}f(X^d) + \phi\tau\left[F(X^d) - F(X^Z)\right] \geq 1.
\end{aligned}$$

As a consequence, using (15) and (16), we have $\frac{\partial X^Z}{d\phi} \leq 0$ and $\frac{\partial X^d}{d\phi} \geq 0$, i.e. the tax shield decreases (increases) and the no-default threshold increases (decreases) with the discount factor ϕ (the interest rate r). This proves part (a) of the proposition.

Also, we have that

$$\begin{aligned}
\left|\frac{\partial X^Z}{d\phi}\right| > \left|\frac{\partial X^d}{d\phi}\right| &\implies 1 > \frac{\tau}{1-\tau}\\
&\quad i.e.\ \tau < \frac{1}{2}.
\end{aligned}$$

which proves part (b) of the proposition.

Using (13) and (14), it follows directly from part (a) that $\frac{\partial T}{d\phi} \geq 0$ and $\frac{\partial C}{d\phi} \geq 0$, which proves part (c).

6.5. Proof of Proposition 2

When proving Proposition 1, we proved that $\frac{\partial D}{d\phi} \geq 0$ for fixed F. To prove that the value of the SA is increasing with ϕ, we have to prove now that the equity value is increasing in ϕ as well, for fixed F:

$$\begin{aligned}
\frac{\partial E}{d\phi} &= \frac{E}{\phi} + \phi\left[-(1-\tau)\frac{\partial X^d}{d\phi}X^d f(X^d) + F\frac{\partial X^d}{d\phi}X^d f(X^d)\right] = \\
&= \frac{E}{\phi} + \phi\frac{\partial X^d}{d\phi}f(X^d)\left[\tau X^d - \frac{\tau}{1-\tau}D\right] = \frac{E}{\phi} + \phi\frac{\partial X^d}{d\phi}f(X^d)\left[\tau F + \tau\frac{\tau}{1-\tau}D - \frac{\tau}{1-\tau}D\right] = \\
&= \frac{E}{\phi} + \phi\frac{\partial X^d}{d\phi}f(X^d)\left[\tau F - \tau D\right].
\end{aligned}$$

The above expression is always strictly greater than zero, as long as $F \geq D$. This implies that the value of the firm, which is the sum of D and E, increases in ϕ (decreasing in the interest rate) for any F.

6.6. Proof of Proposition 3

Let us start from the F.O.C. for the stand alone firm. Define $\Psi(F,\phi)$ as the sum of the marginal tax burden and default costs. It reads

$$\Psi(F,\phi) = -\tau\frac{dX^Z}{dF}\left(1 - G(X^Z)\right) + \alpha X^d f(X^d)\frac{dX^d}{dF}. \tag{17}$$

We know that

$$\Psi(F^*,\phi) = 0$$

where F^* is the optimum. Since F^* is $F^*(\phi)$, we have

$$\Psi(F^*(\phi),\phi) = 0$$

Let us differentiate both sides wrt ϕ now:

$$\frac{\partial \Psi}{\partial F}\cdot\frac{dF^*}{d\phi} + \frac{\partial \Psi}{\partial \phi} = 0$$

Then, we have :

$$\frac{dF^*}{d\phi} = -\frac{\frac{\partial \Psi}{\partial \phi}|_{F=F^*}}{\frac{\partial \Psi}{\partial F}|_{F=F^*}}.$$

For simplicity, from here onwards we omit dependence of the derivatives on the r.h.s. and all of the following quantities on F^*, which is the value of F at which they are evaluated. $\frac{\partial \Psi}{\partial F} > 0$, because F^* is a minimum. As a consequence, the sign of $\frac{dF^*}{d\phi}$ is opposite to the sign of $\frac{\partial \Psi}{\partial \phi}$. Then, to ensure $\frac{dF^*}{d\phi} < 0$ (F^* decreasing with ϕ,i.e. increasing with r), we obtain the following condition:

$$\underbrace{-\tau \frac{d^2 X^Z}{dF d\phi}[1 - G(X^Z)] + \tau \frac{dX^Z}{dF} f(X^Z) \frac{\partial X^Z}{\partial \phi} +}_{\text{Marginal tax benefit change}} \tag{18}$$

$$\underbrace{+\alpha \left[\frac{d^2 X^d}{dF d\phi} X^d f(X^d) + \frac{dX^d}{dF} \frac{\partial X^d}{\partial \phi} f(X^d) + \frac{dX^d}{dF} X^d f'(X^d) \frac{\partial X^d}{\partial \phi} \right]}_{\text{Marginal default cost change}} > 0$$

The assumption $\frac{\partial^2 D}{dF d\phi} > 0$ implies that $\frac{d^2 X^Z}{dF d\phi} < 0$ and $\frac{d^2 X^d}{dF d\phi} > 0$.

Hence:

- $\tau \frac{d^2 X^Z}{dF d\phi}[1 - G(X^Z)] < 0$.
- $-\tau \frac{dX^Z}{dF} f(X^Z) \frac{\partial X^Z}{\partial \phi} > 0$.
- $\frac{d^2 X^d}{dF d\phi} X^d f(X^d) > 0$.
- $\frac{dX^d}{dF} \frac{\partial X^d}{\partial \phi} f(X^d) > 0$.
- $\frac{dX^d}{dF} X^d f'(X^d) \frac{\partial X^d}{\partial \phi}$: its sign depends on whether the slope of the density is positive or negative.

We can rewrite condition (18) as:

$$\frac{\alpha}{\tau} > \frac{(1 - \frac{\partial D}{dF}) f(X^Z) \frac{\partial D}{d\phi} - \frac{\partial^2 D}{dF d\phi}(1 - G(X^Z))}{\frac{f(X^d)}{1-\tau} \left[\frac{\partial^2 D}{dF d\phi} X^d + \frac{\frac{\partial D}{d\phi}(1-\tau+\tau \frac{\partial D}{dF})}{1-\tau} + \frac{dX^d}{dF} X^d f'(X^d) \frac{\partial X^d}{\partial \phi} \right]}. \tag{19}$$

Since by assumption $\frac{\partial^2 D}{dF d\phi} > 0$, the denominator of the r.h.s. is surely positive as soon as $f'(X^d) \geq 0$. When the numerator is negative (i.e. when tax benefits are decreasing in ϕ), F^* is decreasing in ϕ for any α and τ. This happens if

$$\frac{\partial^2 D}{dF d\phi}(1 - G(X^Z)) > (1 - \frac{\partial D}{dF}) f(X^Z) \frac{\partial D}{d\phi}.$$

When $\alpha \to 1$ and $\tau \to 0$, then $F^* \to 0$. Then, the l.h.s. of (19) goes to infinity, while the r.h.s. is finite (0).

6.7. Proof of Proposition 4

Following Nicodano and Regis (2019), a necessary and sufficient condition for the sponsor to be zero-leverage is:

$$\frac{\tau_S(1-G(0))(1-\phi(1-G(0)))}{\alpha_B\left[1+\frac{\tau_S}{1-\tau_S}\phi(1-G(0))\right]} \leq \int_0^{X_B^{Z,*}} xg\left(x, \frac{F_B^*}{1-\tau_B} - \frac{x}{1-\tau_B}\right)dx + \int_{X_B^{Z,*}}^{X_B^{d,*}} xg\left(x, X_B^{d,*} - x\right)dx, \tag{20}$$

where $X_B^{Z,*}, X_B^{d,*}$ indicate the tax shield and the no-default threshold of an optimally leveraged backed unit. The condition ensures that, when the sponsor's debt marginally increases, the increase in the sponsor's tax savings (on the lhs) is no larger than the increase in the backed unit's default costs (on the rhs) which are saved through the guarantee.

The left hand side of the above inequality is decreasing in ϕ (increasing with r), because its derivative with respect to ϕ is negative and it reaches a finite lower bound when $\phi \to \infty$, while the right hand side is increasing in ϕ for a fixed F_B and diverges when ϕ goes to infinity. Then, a $\bar{\phi}$ (or, equivalently, $\bar{r}$) such that (20) is satisfied as an equality exists and, for any $\phi \geq \bar{\phi}$ or, alternatively, for $r \leq \bar{r}$, the condition will be satisfied and the sponsor will be zero-leverage. Since the right hand side is increasing in F_B, it is sufficient to evaluate the condition at a (fixed) level of $F_B \geq F_B^*$ to obtain a necessary condition for the sponsor to be zero-leverage. Since the rhs of (20) does not depend on τ_S, then $\bar{r}$ increases in τ_S as soon as it is < 0.5. This happens because the lhs of (20) increases in τ_S as soon as $\tau_S < 0.5$.