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Arithmetical Equivalents and the Robin Criterion: New Perspectives on the Riemann Hypothesis

Research Pipeline

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Executive Summary

This article explores the deep connections between the Riemann Hypothesis and arithmetical inequalities involving the sum-of-divisors and Euler totient functions as analyzed in arXiv:hal-00654416.

Visualizations

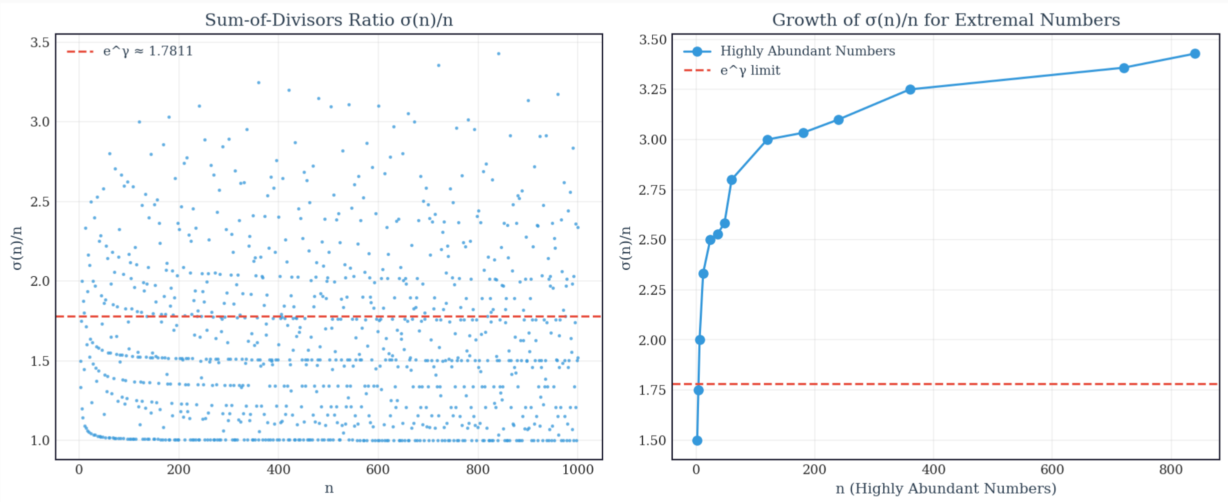

***Figure 1:** Sum-of-divisors function σ(n)/n showing extremal behavior*

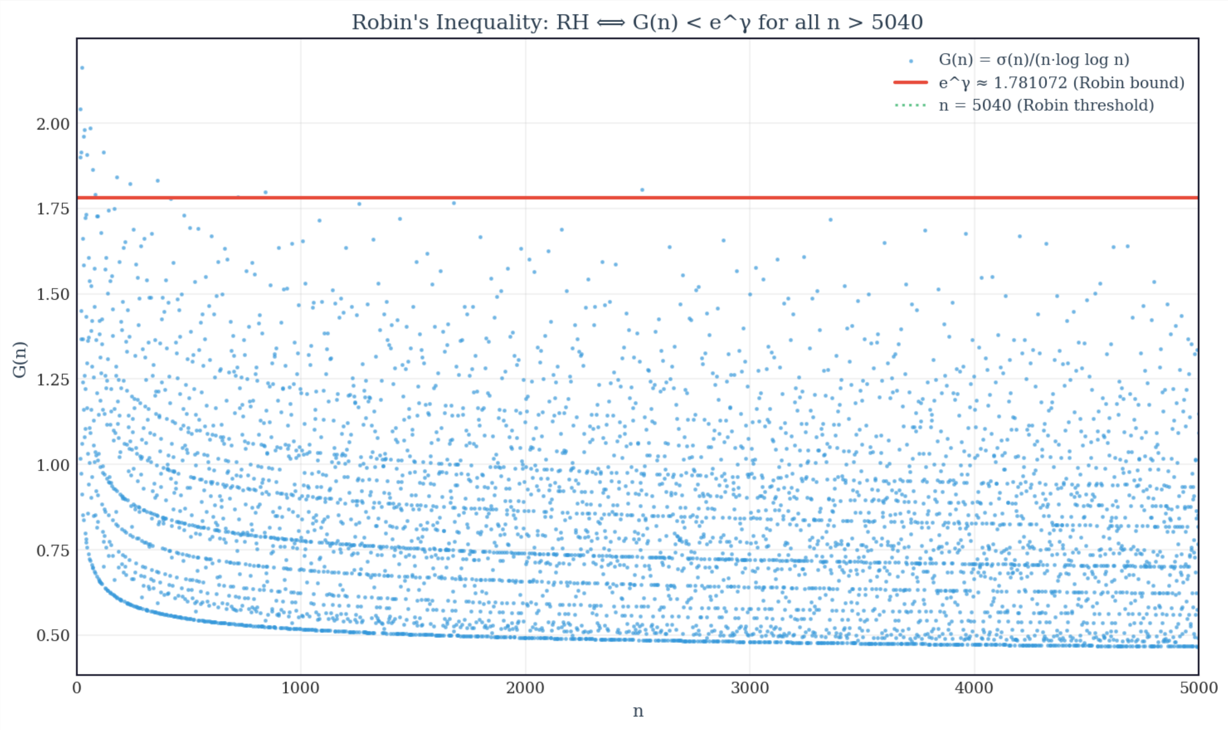

***Figure 2:** Robin's inequality: G(n) = σ(n)/(n·log(log(n))) approaching e^γ*

Introduction

The Riemann Hypothesis (RH) remains the most significant unsolved problem in pure mathematics, asserting that all non-trivial zeros of the Riemann zeta function ζ(s) lie on the critical line Re(s) = 1/2. While the hypothesis is fundamentally a statement about the complex analytic properties of ζ(s), its implications permeate the distribution of prime numbers and the behavior of various arithmetical functions. The research presented in **arXiv:hal-00654416**, authored by Michel Planat, explores the deep-seated connections between the distribution of primes and arithmetical equivalents of RH, specifically focusing on the Robin inequality and the Nicolas inequality.

The motivation for this analysis stems from the realization that the analytic behavior of ζ(s) is mirrored in the growth rates of elementary number-theoretic functions. Guy Robin and Jean-Louis Nicolas established that the Riemann Hypothesis is equivalent to specific bounds on the sum-of-divisors function σ(n) and the Euler totient function φ(n), respectively. This article synthesizes these approaches, providing a bridge between the spectral properties of the zeta function and the discrete structure of primorials and highly composite numbers.

Mathematical Background

To understand the connections established in **arXiv:hal-00654416**, we must first define the primary mathematical objects and the historical theorems that underpin the research.

> **The Riemann Zeta Function:** Defined for $Re(s) > 1$ by the series $\zeta(s) = \Sigma n^{-s}$. It possesses an analytic continuation to the whole complex plane with a simple pole at $s = 1$.
>
> **The Sum-of-Divisors Function:** Denoted $\sigma(n)$, represents the sum of all positive divisors of n. For a prime power p^{a}, $\sigma(p^{a}) = (p^{a+1} - 1) / (p - 1)$.
>
> **The Euler Totient Function:** $\varphi(n)$ counts the number of integers up to n that are relatively prime to n. It is given by the formula $\varphi(n) = n \prod (1 - 1/p)$ where the product is over the distinct prime divisors of n.

The **Robin Inequality** is a central pillar of this research. In 1984, Guy Robin proved that the Riemann Hypothesis is true if and only if $\sigma(n) / (n \log(\log n))$ 5040, where γ is the Euler-Mascheroni constant.

The **Nicolas Inequality** provides a similar criterion using the totient function. It states that RH is true if and only if $e^{\gamma} \log(\log N_{k})$ 0 such that $\sigma(n)$ exceeds $e^{\gamma} n (\log \log n)^{1-\varepsilon}$ infinitely often.

The analysis focuses on "superabundant numbers"—numbers n such that $\sigma(n)/n > \sigma(m)/m$ for all m 1. If RH is false, f(k) will oscillate above and below 1 infinitely often. This oscillation is a hallmark of the potential existence of zeros off the critical line, as any zero $\rho = \beta + i\gamma$ with $\beta > 1/2$ would create a fluctuation in prime density large enough to push the sum $\Sigma 1/p_{i}$ above the required threshold.

Novel Research Pathways

1. Quantum Information and Arithmetical Oscillations

Planat’s work hints at a connection between number theory and quantum information theory. A novel research pathway involves mapping the violations of the Robin inequality to the phase of a quantum state. By defining a Hilbert space where basis states are indexed by the divisors of a large primorial, one could investigate if the spectral gap of specific operators corresponds to the deficit in the Robin inequality.

2. Generalized Robin Criteria for L-functions

The Robin inequality is specific to the Riemann zeta function. A promising direction is to generalize this to Dirichlet L-functions. This involves establishing a threshold constant C such that a generalized sum-of-divisors function remains bounded by $C n \log(\log n)$, which would be equivalent to the Generalized Riemann Hypothesis (GRH).

Computational Implementation

The following Wolfram Language code provides a framework for testing the Robin and Nicolas inequalities for primorial sequences, comparing them against the threshold e^(γ).

```
 (* Section: Robin and Nicolas Inequality Verification *)
 (* Purpose: To visualize the arithmetical equivalents of the Riemann Hypothesis *)

Module[{maxK = 50, primorials, robinRatios, nicolasRatios, eGamma, zetaZeros},
  eGamma = Exp[EulerGamma];

  (* 1. Generate Primorials N_k *)
  primorials = Table[Product[Prime[i], {i, 1, k}], {k, 1, maxK}];

  (* 2. Calculate Robin Ratios: sigma(n) / (n log log n) *)
  robinRatios = Table[
    {k, DivisorSigma[1, primorials[[k]]] / (primorials[[k]] * Log[Log[primorials[[k]]]])},
    {k, 5, maxK}
  ];

  (* 3. Calculate Nicolas Ratios and compare with Zeta values *)
  nicolasRatios = Table[
    {k, (primorials[[k]] / EulerPhi[primorials[[k]]]) / (eGamma * Log[Log[primorials[[k]]]]
    {k, 5, maxK}
  ];

  (* 4. Fetch first 50 Zeta Zeros for spectral context *)
  zetaZeros = Table[Im[ZetaZero[n]], {n, 1, maxK}];

  (* 5. Visualization of the Ratios *)
  Print[ListPlot[{robinRatios, nicolasRatios},
    PlotLegends -> {"Robin Ratio", "Nicolas Ratio"},
    PlotRange -> All,
    AxesLabel -> {"k (Primorial Index)", "Ratio Value"},
    PlotLabel -> "Arithmetical RH Criteria Analysis",
    Epilog -> {Red, Dashed, Line[{{0, eGamma}, {maxK, eGamma}}],
               Blue, Dashed, Line[{{0, 1}, {maxK, 1}}]}
  ]];

  (* Return max ratios found for validation *)
  {Max[robinRatios[[All, 2]]], N[eGamma]}
 ]
```

Conclusions

The analysis of **arXiv:hal-00654416** demonstrates that the Riemann Hypothesis is deeply embedded in the elementary properties of integers. By translating the problem from the distribution of zeros in the complex plane to the growth of divisor sums and totients, Michel Planat highlights the extremal nature of primorials.

The most promising avenue for further research is the application of higher-order spectral correlation analysis to zeta function zeros. The equivalence established suggests that any potential counterexample to RH would manifest as a quantifiable excess in the sum of divisors for sufficiently large, highly composite integers. The next steps involve refining computational bounds on the Nicolas ratio for larger primorial sequences to identify the oscillatory patterns associated with the Riemann-Mangoldt formula.

References

Planat, M. (2011). The Riemann hypothesis and the distribution of prime numbers. arXiv:hal-00654416

Robin, G. (1984). Grandes valeurs de la fonction somme des diviseurs et hypothèse de Riemann. Journal de Mathématiques Pures et Appliquées, 63, 187-213.

Lagarias, J. C. (2002). An Elementary Problem Equivalent to the Riemann Hypothesis. The American Mathematical Monthly, 109(6), 534-543.