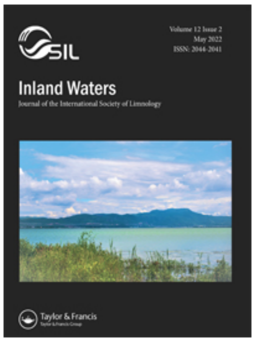

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Imperfect slope measurements drive overestimation in a geometric cone model of lake and reservoir depth

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ABSTRACT

Lake and reservoir (waterbody) depth is a critical characteristic that influences many important ecological processes. Unfortunately, depth measurements are labor-intensive to gather and are only available for a small fraction of waterbodies globally. Therefore, scientists have tried to predict depth from characteristics easily obtained for all waterbodies, such as surface area or the slope of the surrounding land. One approach for predicting waterbody depth simulates basins using a geometric cone model where the nearshore land slope and distance to the center of the waterbody are assumed to be representative proxies for in-lake slope and distance to the deepest point respectively. We tested these assumptions using bathymetry data from ~5000 lakes and reservoirs to examine whether differences in waterbody type or shape influenced depth prediction error. We found that nearshore land slope was not representative of in-lake slope, and using it for prediction increases error substantially relative to models using true in-lake slope for all waterbody types and shapes. Predictions were biased toward overprediction in concave waterbodies (i.e., bowl-shaped; up to 18% of the study population) and reservoir waterbodies (up to 30% of the study population). Despite this systematic overprediction, model errors were fewer (in absolute and relative terms, irrespective of any specific slope covariate) for concave than convex waterbodies, suggesting the geometric cone model is an adequate representation of depth for these waterbodies. But because convex waterbodies are far more common (>72% of our study population), minimizing overall depth prediction error remains a challenge.

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bathymetry; cone model; hypsography; lake depth; reservoir; slope

Introduction

Depth is an important factor controlling the physics, chemistry, and biota of lakes and reservoirs (hereafter, waterbodies). For example, deeper waterbodies generally have higher water clarity and less complete mixing compared to shallow waterbodies (Fee et al. 1996, Read et al. 2014). These differences are reflected in variation among waterbodies in terms of biological productivity (Qin et al. 2020) and rates of greenhouse gas production (Li et al. 2020). However, because measured depth data are only available for a small fraction of waterbodies (~15% of all waterbodies in the area encompassed by our study), our ability to understand and predict depth-dependent processes is limited. The importance of waterbody depth, coupled with its limited availability, has led to numerous attempts to predict depth using measures available for all waterbodies such as surface area or the nearshore slope of the surrounding land (Sobek et al. 2011, Heathcote et al. 2015, Oliver et al. 2016). Such efforts rely on a strategy of exploiting correlations between nearshore geomorphology of land and in-lake geometry, which at limited extents (e.g., within a single North American state or province) can be quite strong while at larger extents can be dependent on geographic location and waterbody type (Branstrator 2009, Oliver et al. 2016).

Given the limited prediction accuracy of prior depth prediction efforts (±6–7 m; Sobek et al. 2011, Heathcote et al. 2015, Oliver et al. 2016), a major focus has been on improving accuracy using strategies such as using more diverse covariates (Oliver et al. 2016), varying waterbody buffer sizes (Heathcote et al. 2015), or discrete classifications (e.g., fitting different models for waterbody size classes) among waterbodies (Sobek et al. 2011, Cael et al. 2017). Unfortunately, the predictive accuracy of these efforts has been limited (±6–7 m).

One intuitive approach for predicting waterbody depth involves using a geometric model that assumes waterbody basins correspond to an idealized shape such as a cone, bowl, or an elliptic sinusoid (Neumann

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1959, Hollister et al. 2011, Getirana et al. 2018, Yigzaw et al. 2018). All such geometric models for waterbody depth prediction involve implicit assumptions about the terms of geometric formulae. In the simplest case, where waterbody basins are treated as cones (equation 1, Fig. 1), 2 assumptions are required to make depth predictions for all waterbodies: (1) that nearshore land slope is a representative proxy for in-lake slope, and (2) that the distance to the center of the waterbody is a representative proxy for the distance to the deepest point of the waterbody (Fig. 1). This cone model imposes the following fixed (i.e., geometric) relationship between slope and horizontal distance:

$$\text{depth}_{\text{geometric}} = \tan(\text{slope}) \times \text{distance}, \qquad (1)$$

where the product of slope and horizontal distance yields an exact geometric depth estimate ($\text{depth}_{\text{geometric}}$). Cone models of waterbody basins have been used extensively to estimate hypsography in waterbodies with no knowledge of volume or mean depth (Read et al. 2014, Winslow et al. 2017).

The assumptions of the cone model (as well as other geometric models) can be tested by comparing proxy measures of waterbody geometry against corresponding "true" (i.e., in-lake) values derived from bathymetric maps and by evaluating how waterbody cross-section shapes differ from that of an idealized cone (Johansson et al. 2007). For instance, waterbody cross-section shapes have been shown to vary from narrow convex forms to outstretched concave forms (Håkanson 1977). Because tests of geometric model assumptions require bathymetric map data, which are only available for a small fraction of waterbodies (including ~15% of

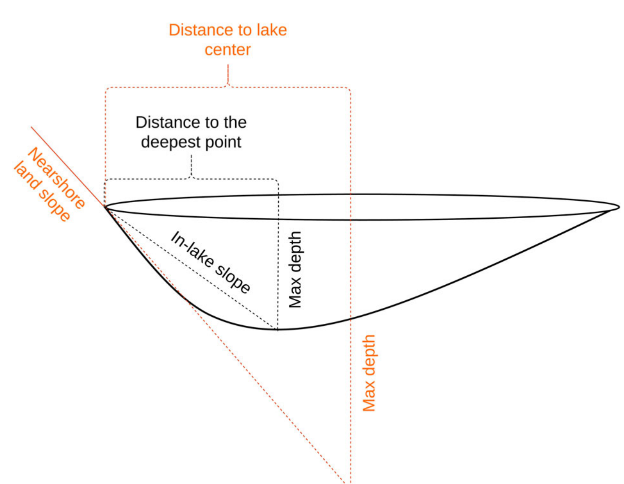

Figure 1. Relations between true (black) and proxy (orange) metrics of waterbody geometry. Geometric depth calculated via equation 1 requires a single distance and slope metric.

all waterbodies in our study footprint), existing evidence may not be applicable to all waterbodies. The few studies that have tested these assumptions have been limited to individual studies of large (>500 ha) waterbodies or on small numbers (<100) of waterbodies (Johansson et al. 2007). Studies focused specifically on reservoirs (as opposed to the more typical case where reservoirs and natural lakes are combined) have been even more restricted to large waterbodies >1000 ha (Lehner et al. 2011, Messager et al. 2016).

As a result of this limited testing, we lack knowledge on both the predictive performance of geometric models, the effect of proxies on depth prediction, and whether depth predictions are more sensitive to measurement errors in the horizontal dimension (i.e., distance to the deepest point of the waterbody) or measurement errors in the vertical dimension (i.e., in-lake slope). Additionally, it is unclear whether model prediction error is related to differences in waterbody type such those with different cross-section shapes (concave vs. convex) or those classified as reservoirs versus natural lakes. To address these knowledge gaps, we asked 3 research questions: (1) How representative is nearshore land slope of in-lake slope, and how representative is the distance to the center of a waterbody compared to the distance to the deepest point of a waterbody? (2) How does the use of proxies for waterbody geometry affect waterbody depth prediction error? (3) How does waterbody cross-section shape (i.e., concave vs. convex) and waterbody type (i.e., natural lake vs. reservoir) affect depth prediction error? To answer these questions, we extracted maximum depth (hereafter referred to as observed maximum depth), in-lake slope, cross-section shape (i.e., concave vs. convex), and distance to the deepest point, of ~5000 waterbodies from bathymetric map data. We supplemented these geometry measures with data classifying waterbodies as reservoirs or natural lakes. We used these data to compute geometric depth estimates (equation 1) and prediction offsets to these estimates using the random forest algorithm (equation 3). Such offsets are model quantities that minimize differences between observed and predicted depth as a function of covariates. In our case, covariates included a variety of waterbody, watershed, and hydrologic subbasin measures available for all waterbodies (Table 1).

By definition, the distance proxy (distance to the center of the waterbody) must always be greater or equal to the true distance value (distance to the deepest point of the waterbody). Therefore, we expected the use of this proxy would lead to overestimation of waterbody depth (Fig. 1). Furthermore, we expected to see greater

Table 1. Summary of waterbody characteristics for the present study and for waterbodies in the conterminous United States (in parentheses). Predictor variables for computing random forest offsets (equation 2) are printed in bold. Dashes (–) indicate an identical sample size among this study and that of the conterminous United States from the National Hydrography Dataset. The total number of waterbodies is reported as *n*.

Variable	Median	Q25	Q75	*n*
Max depth (m)	8.2 (7)	4.6 (3.7)	14 (12)	4820 (17 700)
Area (ha)	**55 (33)**	**21 (11)**	**140 (100)**	**4820 (17 700)**
Island area (ha)	**0 (0)**	**0 (0)**	**0.18 (0.076)**	**4820 (17 700)**
Perimeter (m)	**4400 (3500)**	**2500 (1800)**	**8100 (7300)**	**4820 (17 700)**
Shoreline development	**1.7 (1.7)**	**1.4 (1.4)**	**2.1 (2.2)**	**4820 (17 700)**
Elevation (m)	**300 (340)**	**180 (210)**	**400 (460)**	**4820 (17 700)**
Watershed-lake ratio	**7.8 (10)**	**3.8 (4.4)**	**17 (29)**	**4820 (17 700)**
Deepest point distance (m)	180 (–)	110 (–)	290 (–)	4820 (–)
Mean deepest point distance (m)	140 (–)	87 (–)	230 (–)	4820 (–)
Visual center distance (m)	**240 (–)**	**160 (–)**	**390 (–)**	**4820 (–)**
Inlake slope (m/m)	0.05 (–)	0.02 (–)	0.08 (–)	4820 (–)
Inlake slope online (m/m)	0.06 (–)	0.03 (–)	0.14 (–)	4800 (–)
Inlake slopes (m/m)	0.06 (–)	0.03 (–)	0.1 (–)	4820 (–)
Inlake slopes online (m/m)	0.07 (–)	0.03 (–)	0.15 (–)	4800 (–)
Mean inlake slope (m/m)	0.04 (–)	0.02 (–)	0.09 (–)	4820 (–)
Nearshore mean slope (m/m)	0.08 (–)	0.05 (–)	0.11 (–)	4820 (–)
Nearshore slope online (m/m)	0.08 (–)	0.04 (–)	0.13 (–)	4590 (–)
Nearshore slopes online (m/m)	0.08 (–)	0.04 (–)	0.13 (–)	4540 (–)

overestimation error in reservoirs compared to natural lakes because many reservoirs are known to be drowned river valleys where the deepest point is close to the edge at the end of the reservoir (i.e., next to the dam) rather than in the center of the reservoir (Lanza and Silvey 1985). In a similar fashion, we expected to see overestimation error associated with using a nearshore land slope proxy in waterbodies with differing cross-section shapes, such that the depth of bowl-shaped (i.e., concave) waterbodies would be overpredicted, whereas the depth of V-shaped (i.e., convex) waterbodies would be underpredicted (Supplemental Fig. S1). Finally, we expected that depth predictions themselves would be strongly related to waterbody area and hydrologic subbasin variables because these measures have been influential in previous studies (Oliver et al. 2016).

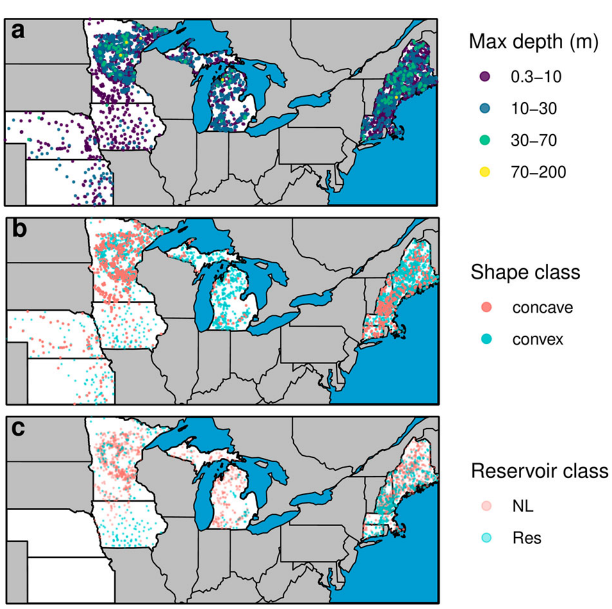

Figure 2. Study waterbodies showing (a) waterbody maximum depth measurements, (b) cross-section shape class, and (c) reservoir classification. The distribution of waterbody depths from panel a is reported in Supplemental Fig. S10.

By testing these expectations, we could establish whether barriers to increased depth prediction accuracy lie in lack of correspondence between true and proxy measures of waterbody geometry or in specific characteristics among waterbodies (such as waterbody cross-section shape or reservoir status). This information could help direct future research efforts to focus on particular dimensions of waterbody geometry (i.e., horizontal vs. vertical) or to stratify model predictions based on specific waterbody types and cross-section shapes. Ultimately, achieving increased depth prediction accuracy would allow more precise estimates of depth-dependent biotic and chemical processes across broad spatial extents.

Methods

Data description

We compiled bathymetry data on ~5000 waterbodies in the Northeastern and Midwestern United States from 9 official state databases (Fig. 2). These data represent ~15% of all waterbodies in the United States included in our study and are a diverse cross section in terms of their characteristics and surface areas, and they span a wide geographic extent including glaciated and non-glaciated regions (Table 1). Thus, they can be considered representative of the entire population of lakes in our study.

The original data were in a variety of formats, including pre-interpolated rasters (Minnesota), contour lines (Nebraska, Michigan, Massachusetts, Kansas, Iowa), contour polygons (New Hampshire, Connecticut), or point depth soundings (Maine). For the Minnesota data, we simply clipped the raster for each waterbody to its outline. For data from the remaining states, we processed each waterbody by converting its original representation to a point layer (if necessary), rasterizing these points, and creating an interpolated bathymetry "surface" using a simple moving window average in the *raster* R package (Hijmans 2019). The size of the moving window was adjusted iteratively to ensure that each bathymetry raster contained no missing data.

All waterbody bathymetry was specifically calculated relative to high resolution (1:24 000 scale) National Hydrography Dataset (USGS 2019) waterbodies such that source data and bathymetry surface outputs were clipped to the area of each waterbody polygon. We restricted the waterbodies in our study to those with an area of at least 4 ha and a maximum depth of at least 0.3 m to ensure that waterbodies had enough contours (or points, or polygons) to generate adequately smooth interpolations to calculate in-lake geometry metrics.

We used our generated bathymetry surfaces to find the location of the deepest point in the waterbody and resolved ties by choosing the deepest point closest to the center of the waterbody. We used the location of this deepest point to calculate "distance to the deepest point" as the minimum distance to the waterbody shoreline. To account for waterbodies where the centroid does not intersect waterbody bathymetry because it is located within an embedded island or peninsula, we calculated the center of the waterbody not as its centroid, but rather by finding the point farthest from the waterbody shoreline (i.e., its visual distance to waterbody center). For these calculations, we used the *polylabelr* R package (Larsson 2019), which interfaces with the Mapbox pole, an algorithm to find the polygon pole of inaccessibility (Agafonkin 2019). We calculated (maximum) in-lake slope as depth at the deepest point divided by the shortest distance to the deepest point from the shoreline. We calculated (mean) nearshore land slope for each waterbody by computing the slope within a 100 m buffer using data from a high-resolution digital elevation model ($\sim$15 m $\times$ 15 m grain) accessed using the *elevatr* R package (Hollister and Shah 2017). We explored alternative buffer sizes ranging between 50 and 1000 m following Sobek et al. (2011), and although 100 m provided the lowest model error, we ultimately found little appreciable effect of varying buffer sizes on model performance. Slope computations proceeded by passing a 3 $\times$ 3 moving window over the 100 m buffer to calculate the slope at each point using Horn's algorithm via the *terrain* function in the *raster* R package (Hijmans 2019). Reported nearshore land slope values are the mean of all points in the buffer. In addition to the aforementioned techniques of calculating in-lake (and nearshore) slopes and distances, we tried 7 alternate techniques (described in Supplemental Fig. S2 and Supplemental Table S1), including measures such as median slope (results not shown).

We categorized waterbodies based on their cross-section shape and reservoir class (e.g., natural lake, reservoir). For cross-section shape, we categorized waterbodies as either convex or concave following the method of Hakånson (1977) by computing normalized waterbody depth–area relationships (i.e., hypsographic curves) and assigning class membership based on whether the midpoint of a waterbody's curve falls above or below that of a simple straight-sided cone (Supplemental Fig. S3).

We further categorized waterbodies using the output of a deep convolutional neural network model trained on satellite images labeled according to visual evidence of a water control structure significantly impacting flow (Polus et al. 2021). This model had an overall validation accuracy of 81% and produced a probability for each waterbody as to whether it is a reservoir or a natural lake. For our purposes, we set a conservative classification probability threshold of 0.75 to determine whether a waterbody would be considered a reservoir. For example, if the Polus et al. (2021) dataset classified a particular waterbody as a reservoir with a probability of 0.74, we categorized it as a natural lake, but if the probability was $\geq$0.75, we categorized it as a reservoir. Note that our reservoir classification defines reservoirs as any permanent waterbody with a water control structure likely to significantly impact flow or pool water. It makes no distinction between different dam types, heights, or uses/purposes because the Polus et al. (2021) dataset is based only on visual interpretation of waterbody images (via deep convolutional neural network models). However, the Polus et al. (2021) dataset is unique in that it provides data using a standardized approach at broad spatial extents for waterbodies >4 ha.

Covariates used in random forest modeling (Table 1, equation 3; detailed later) for waterbody elevation, area, island area, perimeter, shoreline development, watershed to waterbody area ratio, and hydrologic subbasin (i.e., HUC4s) were obtained from the LAGOS-US LOCUS database. One such measure, that of shoreline development, is a measure of waterbody perimeter shape defined as:

$$\text{shoreline}_{\text{devel}} = \frac{\text{perimeter}}{2 \times \sqrt{\pi \times \text{water area} \times 10000}}, \quad (2)$$

where sinuous waterbodies have larger values of shoreline development and circular waterbodies have smaller values of shoreline development. Watershed to waterbody area ratio is an approximation of water residence time and is defined as watershed area divided by waterbody area (Timms 2009).

Proxy evaluation

We conducted a qualitative assessment of whether proxy measures of waterbody geometry (e.g., nearshore land slope, distance to the center of the waterbody) are representative of their true values (e.g., in-lake slope, distance to the deepest point of the waterbody) by visual inspection (i.e., plotting each proxy measure against its corresponding true value) and by computing coefficients of determination (R^2). We further tested proxy measures by examining their effect on waterbody depth prediction error. Our approach involved several steps. First, we computed a geometric estimate of waterbody depth using only geometry information ($\text{depth}_{\text{geometric}}$, equation 1). Second, we fit a random forest model to predict observed (i.e., true) depth as a function of geometric depth along with several covariates available for all waterbodies (Table 1). The purpose of this random forest "offset" modeling was to more rigorously test our expectations regarding prediction error among different formulations of $\text{depth}_{\text{geometric}}$ and among different waterbody types. Each of these steps was executed iteratively for each combination of true and proxy values of slope and distance (Table 2). We conducted additional sensitivity analysis to examine possible interactions between different proxy measures of waterbody geometry and different subsets of the entire dataset where model data were restricted (i.e., filtered) to include only reservoirs, only natural lakes, only convex waterbodies, or only concave waterbodies (Table 2).

Table 2. Model fit and predictive accuracy metrics (RMSE = root mean square error, R^2 = coefficient of determination, MAPE = mean absolute percent error) for all combinations of true (in-lake slope, distance to the deepest point of the waterbody) and proxy (nearshore land slope, distance to waterbody center) metrics.

Filter	Slope	Distance	RMSE	R^2	MAPE
All	True	True			
	True	Proxy	4.8 m	0.73	27%
	Proxy	True	7.3 m	0.31	64%
	Proxy	Proxy	7.1 m	0.36	61%
Reservoir	True	True			
	True	Proxy	5.3 m	0.66	36%
	Proxy	True	7.0 m	0.40	61%
	Proxy	Proxy	7.0 m	0.41	60%
Natural lake	True	True			
	True	Proxy	4.1 m	0.74	22%
	Proxy	True	6.7 m	0.26	68%
	Proxy	Proxy	6.6 m	0.29	64%
Convex	True	True			
	True	Proxy	4.7 m	0.74	30%
	Proxy	True	7.2 m	0.34	59%
	Proxy	Proxy	6.9 m	0.39	58%
Concave	True	True			
	True	Proxy	1.6 m	0.78	20%
	Proxy	True	3.1 m	0.14	46%
	Proxy	Proxy	3.0 m	0.17	45%

Model description

Geometric model

We used a geometric model of waterbodies in which basins are treated as cones with a fixed relationship between slope and distance (equation 1), in part because, unlike other idealized shapes, the cone model does not require knowledge of waterbody volume or mean depth. Note that equation 1 is a geometric formula and has no intercept or coefficients, and it produces an exact depth value given true values of slope and distance. To use this model to predict the depth of all waterbodies, an assumption was made that proxy slope and distance measures, which are available for all waterbodies, are representative of true slope and distance (Fig. 1).

Random forest models

Prior studies using geometric models to predict waterbody depth included a statistical or machine learning model "layer" or "offset" to boost predictive accuracy (Hollister et al. 2011, Yigzaw et al. 2018), which involves fitting a statistical or machine learning model to the residuals of an initial geometric model. For our purposes, such offset modeling enabled us to test our expectations that prediction error would be different among different formulations of $\text{depth}_{\text{geometric}}$ and among different waterbody types. It also facilitated direct comparison against prior models of waterbody depth, including those that are non-geometric. We generated an offset to geometric depth (sensu Hollister et al. 2011) using the random forest algorithm and the *ranger* R package (Wright and Ziegler 2017) to predict observed maximum depth as a function of covariates including geometric maximum depth (from equation 1) along with the waterbody elevation, area, perimeter, and ratio/index measures (Table 2):

$$\text{depth}_{\text{observed}} \sim \text{depth}_{\text{geometric}} + \text{covariates}. \quad (3)$$

We evaluated the relative importance of individual covariates by comparing performance between (1) models in which a given covariate was left untouched versus (2) models in which a given covariate was permuted randomly (Prasad et al. 2006, Wright and Ziegler 2017). Neither cross-section shape nor reservoir class was used as a covariate in any random forest models. Random forest training and test data were stratified on shape and reservoir class to match those of the overall waterbody population. We used the random forest algorithm because it makes no assumptions about the distribution of model residuals, allows non-linearity, and is insensitive to interactions (i.e., multicollinearity) among covariates (Prasad et al. 2006).

Model comparisons

We tested model sensitivity to slope and distance proxies by generating multiple "geometric maximum depth" estimates from 3 different model runs using each of the possible metric combinations for equation 1 (true slope–proxy distance; proxy slope–true distance; proxy slope–proxy distance). Before entering into equation 1, we standardized proxy distances to have the same numeric range as their true counterpart to prevent waterbodies with extremely long proxy distances from having an outsized impact on model evaluation metrics. In addition to comparing among model runs using different metric combinations, we compared among sets of model runs in which slope and distance measures were calculated using different sets of calculation techniques (Supplemental Table S1).

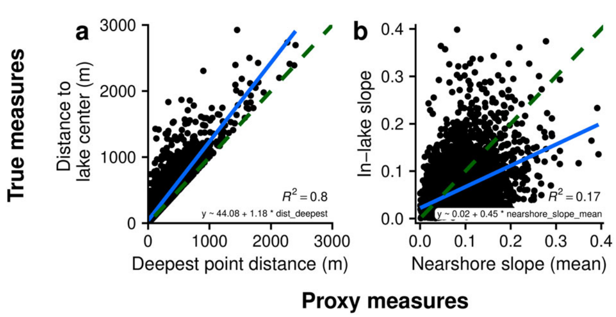

Figure 3. Comparison among proxy and true values of waterbody geometry for (a) distance to deepest point vs. distance to waterbody center and (b) nearshore land slope vs. in-lake slope. A best-fit line and coefficient of determination are shown to illustrate representativeness.

Model evaluations

We evaluated model fit and prediction error using root mean square error (RMSE), mean absolute percent error (MAPE), and coefficient of determination (R^2) metrics on a holdout set (i.e., a data subset not used for model training) containing 25% of all waterbodies. We evaluated the residuals of each model relative to waterbody cross-section shape and reservoir classes to determine whether depth is consistently over or under predicted for some waterbody types relative to others.

Results

Waterbodies belonging to each cross-section shape and reservoir class were not evenly distributed across our study area (Fig. 2). For example, concave waterbodies were nearly absent from Michigan, whereas Maine had more waterbodies categorized as neither concave nor convex (~3%) than other states. Waterbodies in the southern portions of our study area tended to be classified as reservoirs, whereas waterbodies in the northern portions of our study area were a more even mix between reservoirs and natural lakes (Fig. 2). Approximately 18%, 80%, and 2% of waterbodies were classified as having a concave, convex, or neither shape, respectively, whereas ~30% and 70% of waterbodies were classified as a reservoir or a natural lake, respectively.

Although proxy distance to waterbody center was often larger in magnitude than the true distance to the deepest point of waterbodies (rather than being identical), they were strongly related ($R^2 = 0.8$). Note that the coefficient of determination for this relationship is not strictly correct given that distance to waterbody center is an upper bound on distance to the deepest point of a waterbody. In contrast to distance metrics, proxy nearshore land slope and true in-lake slope were more weakly related ($R^2 = 0.17$). For slope measures, most waterbodies had higher magnitude (i.e., steeper) nearshore land slope compared to true in-lake slope (Fig. 3). Taken together, these results suggest that proxy distance to the center of waterbodies is representative of true distance to the deepest point of waterbodies, whereas proxy nearshore land slope is not representative of true in-lake slope. The strong relationship between distance to the center of waterbodies and distance to the deepest point means that converting between the 2 measures in subsequent analyses is possible (best-fit equations in Fig. 3).

In addition to overall differences between slope and distance measures, we found differences in these relationships among waterbody shape classes. For example, in-lake slope and distance to the deepest point of the waterbody metrics were consistently larger in magnitude for convex waterbodies than concave waterbodies (Supplemental Fig. S4). We found evidence that this difference was at least partly explained by the fact that convex waterbodies are deeper than concave waterbodies (Supplemental Fig. S5). Unlike concave and convex waterbodies, no clear differences were found among slope and distance metrics for natural lakes versus reservoirs.

Offset model fit and prediction error differed depending on the technique used to calculate in-lake and nearshore geometry metrics (Supplemental Table S1). We found that the best model fit and lowest model error occurred when in-lake slope was calculated as the average point-wise slope of all points at maximum waterbody depth rather than at a single point of maximum depth. However, given the small difference in the fit of models using either of these techniques and the significant cost in terms of computational load and complexity, we limit our discussion hereafter to the simpler case involving only a single deepest point.

The use of proxy nearshore land slope had a larger effect on model fit and prediction error than the use of proxy distance to waterbody center (Table 2). More specifically, the true slope (in-lake slope)–proxy distance (distance to the center of the waterbody) model had a better fit ($R^2 = 0.73$) and lower prediction error (RMSE = 4.8 m, MAPE = 27%) compared to the proxy slope–true distance model ($R^2 = 0.31$, RMSE = 7.3 m, MAPE = 64%). The fit of the proxy slope–proxy distance model ($R^2 = 0.36$, RMSE = 7.1 m, MAPE = 61%) was similar to the proxy slope–true distance model. Predicted depth values for this model were generally underestimates relative to measured depth values (Supplemental Fig. S6).

Furthermore, analysis of model residuals showed overestimation of waterbody depth for concave waterbodies when models included a proxy slope measure (Fig. 4). We observed similar but smaller overestimation depending on whether a waterbody was classified as a reservoir rather than a natural lake (Fig. 4). We found that models restricted to consider only concave lakes had lower error (both in absolute and relative terms) than models on other data subsets (e.g., convex lakes, reservoirs, natural lakes; Table 2). Conversely, we observed no notable geographic patterns in model residuals (Supplemental Fig. S7).

The most important covariates in these models were those relating to spatial location, waterbody area, and perimeter (Fig. 5). Conversely, watershed metrics and waterbody elevation made little contribution to random forest model fit. The spatial location (i.e., HUC4, hydrologic subbasin) covariate was notably less important in the true slope model than the 2 proxy slope models. To evaluate the contribution of our "offset" models relative to the "base" geometric model, we can look at model importance calculations for the geometric max depth input to the random forest model (Fig. 5). These calculations indicate that omitting a geometric max depth term results in a 130%, 60%, or 50% increase in mean square error depending on the formulation of geometric max depth.

Discussion

Our tests of the geometric cone model of waterbody depth models show that specific proxy measures of waterbody geometry are not representative of true geometry measures across a broad array of waterbodies. Models using nonrepresentative proxies showed increased error and systematic overestimation of depth in concave and reservoir waterbodies. Although our analysis was limited to waterbodies with available bathymetry data, their characteristics did not differ from those of the overall waterbody population (apart from the fact that our study waterbodies were somewhat larger in area than the overall waterbody population; Supplemental Figs S8 and S9, Table 1). Although some hidden bias not explored in our analyses could exist, this lack of difference suggests that our results are likely broadly applicable to nearly all waterbodies in the study area.

Representativeness of proxy measures of waterbody geometry

In comparing waterbody geometry measures, our analysis suggests that proxy distance to waterbody center is representative of true distance to the deepest point of the waterbodies, but that proxy nearshore land slope is not representative of true in-lake slope. A simple indication of this nonrepresentativeness is that proxy nearshore land slope was often (>74% of cases) steeper than true in-lake slope. This finding is consistent with Heathcote et al. (2015), whose results suggest that in-lake slopes are shallower than the surrounding land. Furthermore, the fact that in-lake slopes were shallower than the surrounding land even after controlling for differences in area (Supplemental Fig. S10) is consistent with the idea of topographic scaling (i.e., scale invariance) explored in previous work and detailed by Cael et al. (2017). The underlying reason for these shallow in-lake slopes may be related to slope-induced turbidity currents, which distribute sediment from shallow high-energy areas of waterbodies to deep low-energy areas (Håkanson 1981, Johansson et al. 2007). The strength of such sediment focusing is likely greater in

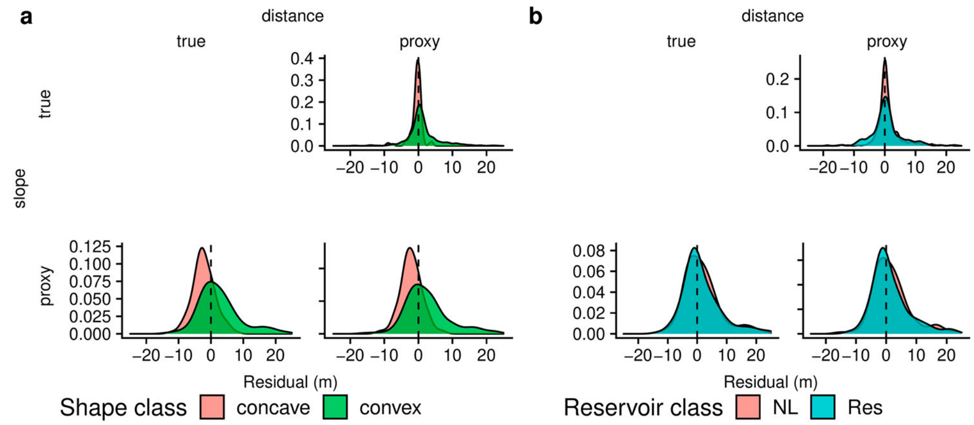

Figure 4. Depth model residuals (residual = observed − predicted) in meters by (a) cross-section shape and (b) reservoir class, indicating overprediction of concave and reservoir waterbodies. Dashed line is 1:1 relationship.

'younger' waterbodies with steeper slopes, leading to a smoothing of their bathymetry over time (Blais and Kalff 1995).

One surprising finding with respect to the relationship between true and proxy geometry measures when examined by waterbody class was the lack of a greater difference between proxy and true distances in reservoirs than for natural lakes. This finding is contrary to the idea that most reservoirs are drowned river valleys where the deepest point is close to the edge at the end of the reservoir (i.e., next to the dam) rather than in the center of the reservoir (Lanza and Silvey 1985). One possible explanation is that our reservoir classification data used a more general definition of a reservoir (i.e., any permanent waterbody that has a water control structure likely to significantly impact flow or pool water) than that of conventional classifications tied to specific dam types or dam heights. Another possible explanation is that conventional reservoir classifications are conceptually biased toward more southern areas with few natural lakes (Fig. 2). Southern Iowa, for instance, is typically considered to have few to no natural lakes. In the present study, all of the apparent natural lakes in southern Iowa were in fact oxbow lakes adjacent to the Missouri River.

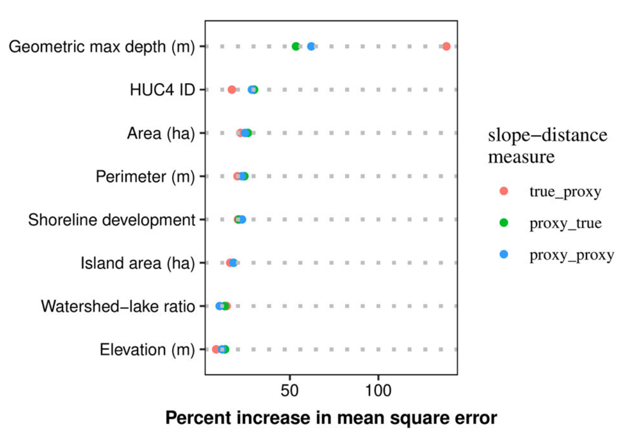

Figure 5. Importance plot for random forest variables showing increase in mean square error. Higher values indicate greater importance to model predictions. See equation 1 for a definition of geometric max depth. HUC4 ID is a "dummy" variable of geographic (hydrologic subbasin) location. The key indicates different combinations of true (in-lake slope, distance to the deepest point of the waterbody) and proxy (nearshore land slope, distance to waterbody center) metrics.

We found other differences among waterbody geometry measures according to waterbody cross-section shape. One finding was that convex waterbodies had longer distances than concave waterbodies to waterbody centers relative to corresponding distances to the deepest point. In addition, convex waterbodies often had steeper in-lake slopes relative to nearshore land slopes than concave waterbodies. Finally, and notably, convex waterbodies were deeper than concave waterbodies despite having similar distributions of waterbody surface area (Supplemental Fig. S5). The underlying cause of these differences is unknown, but one possibility is that geometry is tied to the circumstances of waterbody formation whereby the formation of concave waterbodies was a result of more intense glacial scouring compared to the formation of convex waterbodies (Gorham 1958). While our findings provide some evidence to support this idea, namely that a geographic hotspot of concave waterbodies is associated with the glaciated prairie pothole region (Hayashi and van der Kamp 2000), it is not supported by the overall geographic distribution of waterbody cross-section shapes. Instead of a concentrated area of concave waterbodies in formerly glaciated regions, a fairly even mix of concave and convex waterbodies is distributed among the northern (i.e., glaciated) and southern (non-glaciated) portions of our study area (Fig. 2).

Effects of proxy measures of waterbody geometry depth prediction error

Models using only proxy variables (Table 2) had prediction errors (RMSE = 7.1 m) of a similar magnitude to those in prior studies (RMSE = 6–7.3 m) predicting waterbody depth at broad geographic extents (Hollister et al. 2011, Messager et al. 2016, Oliver et al. 2016). When only a single proxy measure was used, model sensitivity differed depending on whether it was a horizontal distance measure or a vertical slope measure. In the case of a true slope and proxy distance combination, models were more accurate (±4.8 m, 27%) than even the most accurate of prior studies (Hollister et al. 2011, Messager et al. 2016, Oliver et al. 2016). Conversely, models using a proxy slope and true distance combination had prediction error rates (±7.3 m, 64%) of a similar magnitude as that of the baseline proxy-proxy model (±7.1 m, 61%). The greater sensitivity of depth predictions to proxy slope measures relative to proxy distance measures may be explained by the fact that proxy slope measures were a more imperfect representation of true in-lake slopes relative to proxy versus true distances. We found no evidence that the sensitivity of depth predictions to slope was dependent on variations in how these measures were calculated (Supplemental Table S1). In a general sense, the sensitivity of depth predictions to slope help explain the relatively poor predictive performance of prior nongeometric waterbody depth models given that they rely heavily on waterbody area as a predictor (Sobek et al. 2011, Messager et al. 2016, Oliver et al. 2016), and both horizontal distance measures and vertical slope measures seem to be decoupled from waterbody area (Supplemental Fig. S5).

Effects of waterbody shape and waterbody type on depth prediction error

As expected, we found that the maximum depth of concave waterbodies was systematically overpredicted by a simple geometric model using proxy nearshore land slope (Supplemental Fig. S1). However, contrary to our expectation, we did not observe underprediction of depth in convex waterbodies, likely because geometric depth itself was always greater than observed maximum depth because proxy distance is constrained to be greater than true distance. Models restricted to only concave waterbodies had low error (both in absolute and relative terms, irrespective of any particular slope metric), suggesting that despite evidence of overprediction, the cone model is an adequate representation of depth for these waterbodies. However, because only 18% of lakes in our study population are concave, minimizing overall depth prediction errors remains a challenge.

Future research

The only model parameterization that was more accurate than the most accurate of prior studies (which fit models to waterbody depth) requires data on in-lake slope (true slope, proxy distance), which are not available for all waterbodies, so using this method for general prediction is not practical. However, we propose that the error rate of this model (±4.8 m, 27%) be used as an out-of-sample prediction benchmark for future studies that should attempt to match but not expect to exceed it.

Because this most accurate model requires bathymetry data, producing depth predictions for all waterbodies with error rates below ~5 m or 30% may not be possible with current data and models. To achieve high prediction accuracy using data available for all waterbodies, future studies could explore alternative modeling approaches such as ordinal modeling, which would capture whether a waterbody crosses some important depth threshold that has ecological relevance but would not seek to predict a specific depth value. Another emerging approach is to use data such as topobathymetric products that integrate both topographic and bathymetric data in a seamless fashion rather than treating them as separate entities. Although topobathymetry would allow more robust tests of the representativeness of geometric model inputs, topobathymetric products are rare and have mostly been limited to nearshore marine environments, and as such are not yet widely available for inland waters (Danielson et al. 2016). Other potential explanatory data include information on waterbody origin such as glaciation status. Unfortunately, such waterbody ontogeny data are currently available only for select regions and limited to the largest waterbodies (Sharma and Byrne 2011).

Finally, our findings indicate that geometry measures differ according to waterbody cross-section shape, making them an attractive target for inclusion in depth prediction models. Unfortunately, identifying the cross-section shape of a waterbody requires bathymetry data, which are unavailable for most waterbodies. However, given the conceptual links between cross-section shape, glaciation, and sedimentation (Johansson et al. 2007), it may be advantageous for future studies to compile data on sedimentation to determine if these data can be used to predict cross-section shape and boost depth prediction accuracy. We note that such data do not currently exist for large numbers of waterbodies.

Conclusion

To our knowledge, the present study is the largest and most comprehensive test to date of the geometric cone model of waterbody depth. Using bathymetry data on ∼5000 waterbodies, we showed that proxy slope measures are a poor representative of true in-lake slope, leading to overestimates of depth in concave and reservoir waterbodies. Despite these apparent biases, overall prediction accuracy was equivalent to that of prior depth prediction studies (±6–7 m). In addition, although depth estimates may be biased for concave waterbodies, models restricted to only consider these waterbodies had fewer errors than other data subsets (both in absolute and relative terms), suggesting that the cone model is an adequate representation of depth for concave waterbodies.

Only our models using a true measure of in-lake slope had greater accuracy than those of prior studies (±4.8 m, 27%). Given that these in-lake slope models require data only available for waterbodies with bathymetry data, their use for general depth prediction is limited but can provide a best-case benchmark for future studies that use commonly available data. Lack of improvements in prediction accuracy (short of including data unavailable for most waterbodies) suggest that improved prediction may require new types of data or novel analysis techniques.

Data availability

All data used in the study are available in Stachelek 2021a (with Figshare DOI). All code for data processing, model fitting, and model evaluation is available at Stachelek 2021b (with Zenodo DOI).

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Author contributions

JS conceived the study, built models, analyzed data, and wrote the paper. PJH and PAS contributed to the conception of the manuscript, provided interpretation of results, and edited the paper. This work benefited from participation in the Global Lake Ecological Observatory Network (GLEON). We thank K.S. Cheruvelil for a friendly review of an earlier draft.

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