Multilingual Students and Mathematics Education

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This paper is written as a companion piece to the Coalition for English Learner Equity (CELE) Statement of Agreement. The purpose of the paper is to describe some of the research based assumptions and findings that form the research basis for the agreement, the EL Success Forum *Guidelines*, and the work that lies ahead for teachers, school administrators, district leaders, content developers, and other advocates for "English learners" (ELs)[2], or multilingual students. A shared framework will serve as a document to guide our decisions and engagement at all levels including moment-to-moment decisions that take place in the classroom, decisions that are deliberated over time, and decisions that involve many stakeholders. This paper summarizes the major findings from research on multilingual students in mathematics classrooms so that our work with that student population can proceed from a place of scholarly rigor, dispelling notions that are less productive or not founded on research.

Comprehensive support for multilingual students to learn both language and mathematics requires an array of considerations both in and out of the classroom. This paper is organized by the core findings of the National Academy of Sciences, Engineering, and Medicine's (NASEM) consensus study report,[3] "English Learners in STEM Subjects: Transforming Classrooms, Schools, and Lives". That report brought together a group of national experts who collected and reviewed the research literature, requested updated presentations on current research projects, and developed a consensus report which includes recommendations for policy and teaching. Beyond that report, we also used two other literature reviews as resources.[4] In each of the sections listed below, we describe the current state of the research and relevant recommendations made in the NASEM report:

1. Educational Context
2. Mathematics Learning and English Language Development
3. Effective Instructional Strategies and Teacher Education
4. The Role of Families and Communities
5. Assessment
6. Building Capacity to Transform Mathematics Learning

Assumptions

Before describing each finding in detail, we summarize guiding tenets that have emerged from research and the related assumptions. First, aligned with the consensus report, *academic language* will be understood as a set of disciplinary language practices that *all students can learn*, through a variety of modes and interactions, for a variety of purposes. This tenet is informed by the *presumption* that students have linguistic and mathematical competencies to learn how to participate in these mathematical practices. This is a step away from older, unproductive views that state, for example, that language must be mastered *before* students can engage with disciplinary content or that language is learned separately from or sequentially with disciplinary content. In alignment with the relevant research, we assume that language proficiency is *not prerequisite to*, rather, it is an *outcome of* effective content instruction.[5] The ideology of standard English privileges the language of educated social classes and positions their language as more complex than varieties of English not found in school-settings.[6] An asset-based approach, on the other hand, views the linguistic resources that students bring to the classroom as valuable, critical to meaning-making, and worthy of affirmation.[7]

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1. Authors listed in alphabetical order. Special thanks for Dr. Judit Moschkovich of UC Sana Cruz for her invaluable advising and guidance on this paper.
2. In this paper, we will refer to students that hail from multilingual communities as "multilingual". Use of terms like "English Learner", "English Language Learner", "Limited English Proficient", "English as a Second Language", and "English as an Additional Language" are inherently deficit oriented (Barwell, Moschkovich, & Setati Phakeng, 2017; de Araujo, Roberts, Willey, & Zahner, 2018; Faltis & Valdés, 2016). These terms emphasize what students "lack" (de Araujo, Roberts, Willey, & Zahner, 2018), privilege the language of instruction (Barwell, Moschkovich, & Setati Phakeng, 2017), and are intimately connected to social hierarchies (Faltis & Valdés, 2016). Moreover, the histories of these hierarchies (social and linguistic) are woven from a violent colonial past that positioned English as a "language of salvation" (Hsu, 2015 and 2017). While some have suggested the use of "Ever EL", which includes 'ELs' and reclassified students, as a way to more accurately reflect the achievement of multilingual students in statistical analyses, we will refer to these students as "multilingual students" to reflect a multilingual perspective on learning and not contribute to the normalization of the monolingual English speaker.
3. NASEM, 2018
4. Barwell, Moschkovich, & Setati Phakeng, 2017; de Araujo, Roberts, Willey, & Zahner, 2018
5. NASEM, 2018
6. MacSwan, 2018
7. Ibid.

Secondly, we consider mathematics learning and activity through a variety of lenses to ensure equity for multilingual students whenever possible. As such, research from multiple perspectives (cognitivist, discursive, and sociocultural) is represented in this paper.[8] We assume that under the right conditions, bilingual, multilingual, and second-language learners can learn at least as well as their monolingual peers. These conditions are present when instruction focuses on mathematical reasoning and practices, builds on student thinking, takes students' resources into account, and includes their everyday ways of talking as the building blocks for meaning-making and discourse.[9]

These tenets and assumptions center the assets and strengths that multilingual students bring to the classroom. Focusing on binaries—such as 'standard' versus 'non-standard' English, Basic Interpersonal Communication Skills versus Cognitive Academic Language Proficiency, or social versus academic language—reinforces deficit views of students by focusing on what these students *seem* to lack. This paper assumes that students bring linguistic competencies into the mathematics classroom and describes ways to support their ongoing language and mathematics learning based on the empirically-supported finding that, given the appropriate learning environment, multilingual students can succeed in learning mathematics.

1. Educational Context

This section focuses on several aspects of the educational context, and the mathematics educational context in particular, relevant to multilingual students. Less than 5% of pre-service and in-service teachers came from multilingual homes.[10] Moreover, many pre-service teachers have had minimal experience with people from outside of their own primary cultural and language groups and have rarely encountered spoken English as an obstacle to their social and academic purposes. In contrast, 9.4% of US students were labeled as 'ELs' in the 2014-2015 academic year.[11] Due to increased segregation, over 70% of these students are concentrated and attend only 10% of the nation's elementary schools.[12] This has profound implications for the interactions between teachers and students. How can we ensure that teachers, educators, and other stakeholders have what they need for teaching with, advocating, and ensuring educational opportunity for multilingual students—who embody an experience that is largely unfamiliar? In this section, we describe major aspects of the educational context and how they can be addressed. First, we discuss the heterogeneous nature of the population of multilingual students. Then we discuss the use of primary language during instruction, a teaching strategy that figures prominently in the different models for English Language Development (ELD). Thirdly, we consider how the classification of students plays a role in students' access to mathematics courses. Finally, we discuss the recommendations for success regarding the educational context.

As a group, multilingual students are heterogeneous. They represent a variety of home languages and cultures, prior schooling experiences, ages at which they enter US schooling, and English proficiencies. Effective policies, programs, and strategies for instruction and assessment must take this heterogeneity into account lest they limit their own effectiveness.[13] For example, testing students in their first language might do more harm than good for those students that did not have regular access to schooling in that language.[14] In a case such as this, testing in the home language might assume similar levels of previous schooling in that language, which can lead to overestimating proficiency in the home language and underestimating proficiency in the English.[15] Keeping the heterogeneity of the population of multilingual students in mind could support finding more appropriate placements. Decisions for placement should consider students' access to mathematics by[16]:

1. Assessing language proficiency in the four language modalities of English (reading, writing, listening, speaking), beyond the simple use of broad classification categories;
2. Using multiple sources of information (in addition to scores on English proficiency tests) in judging students' English proficiency;
3. Looking for approaches that are sensitive to each student's needs;

8. Barwell, Moschkovich, & Setati-Phakeng, 2017
9. Ibid.
10. Faltis & Valdés, 2016
11. Ibid.
12. NASEM, 2018
13. Ibid
14. Ibid.
15. Ibid.
16. Ibid, p.30, emphasis added

4. Avoiding making assumptions about the proficiency of students in English or in their home language;
5. Encouraging educators to develop a good sense of each of their ELs' strengths in English, based on *continuously interacting with them*.

Multilingual students often have fewer opportunities to engage with high-quality mathematics instruction because of how deficit views are imbued into educational policies and practices.[17] Whether at the individual level, where we examine our own beliefs, or at the structural level, educators, designers, and advocates should consider these aspects of students' experiences to make informed decisions about equitable student placements.

Program models also play a role in students' opportunities because they engage students' home languages differently and shape access to mathematics coursework. The language of instruction varies by program model; for example, Dual Immersion, Sheltered Instruction, Transitional or Developmental Bilingual Education, English as a Second Language, etc. While bilingual education and primary language-use have been documented to support student learning,[18] these programs can be difficult for districts to implement in the presence of a diversity of primary languages and a dearth of qualified bilingual teachers. Classification of students also plays a role in students' opportunities because the availability of mathematics coursework can depend on a student's language fluency classification. For example, for those students arriving in the US, upon arrival, students' prior mathematics knowledge is not typically assessed, and course placements are based on English proficiency. Coupled with the fact that many teachers enter classrooms with "fixed dispositions and beliefs" about multilingual students,[19] many of these students are systematically excluded from rigorous mathematics coursework. Practices like 'tracking'—which has been called the institutionalization of a 'fixed mindset'[20]—should be dismantled in favor of shared pathways that emphasize engagement with mathematical practices and the development of deep mathematical understanding.[21]

2. Mathematics Learning and English Language Development

A central assumption of the NASEM report is that content knowledge and language proficiency develop together through meaningful interaction. Meaningful interaction includes giving students opportunities to participate in the types of activities in which mathematicians engage (e.g., practices such as arguing, justifying, and modeling). The research suggests that giving multilingual students opportunities to participate in mathematical practices provides access to content knowledge that are not entirely language-dependent while simultaneously giving students opportunities to develop increasingly sophisticated understandings and language proficiency.

The development of language and content knowledge are inextricable.[22] It might seem like "common sense" that a person needs language proficiency before they can begin to understand a concept, yet language and content knowledge develop *together*.[23] This is true starting in children's earliest experiences with learning. Children are raised in families and communities that use language to conceptualize, represent, and engage with the world. As children learn to interact with others through language they also learn the values and beliefs shared by their community. For example, a small child might be told to *share half your treat with your brother*. In this situation, the child is learning the language their family uses to communicate, beliefs about the value of fairness and generosity, and beginning fraction concepts simultaneously. Perhaps most importantly, this learning takes place within a meaningful family interaction.

As children grow, they interact with other communities (perhaps first through experiences with other families, sites of worship, and eventually schools). In school, children use their existing language and knowledge to engage with academic disciplines. These disciplines use language to make sense of the world in particular ways that have developed historically and promote certain values and beliefs. Across the subject areas students simultaneously learn new concepts and new ways of using language. "As learners add concepts and language, adding new concepts through language becomes progressively easier as the linguistic skills

17. TODOS, 2020
18. Barwell, Moschkovich, & Setati Phakeng, 2017
19. Faltis & Valdés, 2016
20. Boaler, 2015, as cited in TODOS, 2020
21. NCTM, 2020
22. Forman, 1996; Vygotsky, 1986
23. NASEM, 2018

and abilities of the learner increase,".[24] Developing conceptual knowledge can be more difficult for multilingual students when educators rely exclusively on language to teach new concepts.[25] This difficulty is further exacerbated if this language is used without meaningful, content-based interactions.

The purpose of language is to make meaning and fulfill goals defined by the social contexts in which the language is used.[26] It makes sense then that language and meaning-making will depend on many factors - including the number of people communicating, their roles and status, and the environment. Furthermore, other features of communication such as gestures, visual displays (e.g., mathematical ones like written symbols, graphs, tables as well as everyday ones like pictures), and physical objects play an important role in meaning-making.[27] People use language and other communication resources in various ways as they engage in a variety of activities—linguists call this variation *register.* One important goal of education is to provide students with access to and experience with using new registers as well as developing their existing registers.[28] The register a person uses to communicate will vary according to the content, the relationships involved, and the modality (e.g., speech, writing, drawing, gesture).[29] It makes sense that word meanings will vary depending on the context; words mean different things in different settings, some settings (such as mathematics or science situations) have specific meanings for words (e.g., function, set, etc.) or phrases. Registers, however, are much more than technical words or vocabulary; they also include aspects at the syntax and discourse levels. For example, in mathematical contexts, there are particular ways to make claims more precise, such as specifying when a claim applies and when it does not. Language also varies according to our relationships with others and the ways (modalities) in which we communicate. We speak differently to a group of people than we do to a single person, to a teacher than we do to a close friend, and spoken language is different from written language. The communicative context, therefore, shapes how we use language.

This idea of *register* is important for multilingual students learning math for two reasons. First, the context can be shaped in such a way as to provide students with opportunities to expand their registers. For example, consider a classroom activity in which students are learning about fractions. They first work in small groups, then present their work to the larger class, then document their thinking in writing, and finally make sense of the formal written language of a mathematics textbook. Such activities provide opportunities for students to develop their understanding of mathematics content while simultaneously using a variety of registers to communicate mathematical ideas: an informal register in the small group (using many instances of "this" and "that" while pointing to a mathematical representation), a more formal register when presenting (not only pointing to a mathematical representation on the board but also labeling the components), and an even more formal register when writing, and finally the typical formal register of a textbook when reading mathematical texts. Second, a focus on register can help educators recognize students' developing conceptual understanding without relying solely on one particular register. Educators can design instruction in such a way that students can first make sense of a concept using an everyday register (informal vocabulary, colloquial expressions, jokes, etc.) and then develop more discipline-specific registers and ways of communicating as they participate in academic/disciplinary practices and their content knowledge becomes more sophisticated.[30]

An important aspect for policy, teaching, and designing learning environments highlighted in the report is academic literacy in mathematics. Academic literacy in mathematics[31] consists of three components: mathematical proficiency, mathematical practices, and mathematics discourse. Attending to all three components is "important for all students, but essential for ELs,"[32]. Mathematical proficiency is comprised of five interwoven strands[33]:

1. Conceptual understanding, or comprehension of mathematical concepts, operations, and relations;
2. Procedural fluency, or the skill to carry out mathematical procedures flexibly, accurately, efficiently, and appropriately;

24. NASEM, 2018, p. 57
25. de Araujo, Roberts, Willey, & Zahner, 2018
26. Schleppegrell, 2004
27. Bezemer & Kress, 2008
28. Bunch, 2013; Moschkovich & Nelson-Barber, 2009
29. Halliday, 1978; NASEM, 2018
30. NASEM, 2018
31. Moschkovich, 2015a
32. Ibid.
33. National Research Council, 2001

3. Strategic competence, or competence in formulating, representing, and solving problems;
4. Adaptive reasoning, or logical thought, reflection, explanation, and justification; and
5. Productive disposition, a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These strands are interwoven, meaning that neglecting one will limit a person's mathematical proficiency. For example, if instruction on whole-number multiplication is limited to memorizing multiplication facts, students may only develop procedural fluency. Instruction that includes opportunities for students to represent and apply multiplication in many different contexts (e.g., through area models and arrays, invented strategies for regrouping, and solving novel word problems) can support developing the other strands as well. Importantly, focusing on all other strands of proficiency, especially conceptual understanding, simultaneously supports students' procedural fluency. Understanding procedures supports remembering them, calculating accurately, and recuperating from mistakes.

Academic literacy in mathematics[34] also requires participation in mathematical practices—the ways of thinking and communicating valued within the discipline. These practices include making generalized claims and presenting these claims for review by one's community. "Students are expected to explore the nature of mathematical objects, make and test conjectures, and construct arguments, and instruction is expected to emphasize abstracting and generalizing as central mathematical practices,".[35] While there is no single mathematical community and mathematics is an activity practiced in many different settings, the Common Core State Standards (2010) identified eight common practices of school mathematics:

1. Make sense of problems and persevere in solving them
2. Construct viable arguments and critique the reasoning of others
3. Reason abstractly and quantitatively
4. Model with mathematics
5. Attend to precision
6. Use appropriate tools strategically
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Lastly, academic literacy in mathematics[36] requires participation in the discourse of mathematics. The language of mathematics is made up of more than just specialized vocabulary; it includes symbols, visual displays, syntax, the mathematics register, and discourse practices. For example, the mathematical practice of attending to precision could be viewed only as using specific, formal vocabulary. However, mathematical precision might also mean knowing the level of precision needed in a calculation (e.g., is an estimate sufficient?) or being precise when making a claim (e.g., multiplication makes numbers bigger *when* you multiply by a positive whole number).[37]

Of course, not every aspect of academic literacy in mathematics can be included in every lesson. Units must therefore be designed with careful attention to balance procedural fluency with conceptual understanding, to provide opportunities to engage in *all* the mathematical practices, and to engage students in the discourse of mathematics. Curriculum and instruction should not prioritize vocabulary over content, and lessons should be designed to provide students with many ways to engage with mathematics.

3. Effective Instructional Strategies and Teacher Education

Many mathematics teachers feel unsupported and under-prepared to teach mathematics to multilingual students.[38] To support teachers with mathematics instruction for teaching multilingual students, teachers need ongoing professional development that extends beyond technical assistance or tricks. Instead, teacher education should help teachers uncover learners' assets and broaden participation in rich discussions.[39] Effective teacher education and professional development gives teachers opportunities to learn about

34. Moschkovich, 2015a
35. NASEM, 2018, p. 76
36. Moschkovich, 2015a
37. Moschkovich, 2007; NASEM, 2018
38. de Araujo et al., 2018
39. Ibid.; Moschkovich, 2013

students' mathematical thinking,[40] to better understand mathematical practices that students participate in outside of the classroom,[41] and to hear and uncover students' reasoning, even in their everyday expression.[42] Through professional development, teachers need support to get to know their students and need time to reflect on their own beliefs and assumptions related to teaching multilingual students mathematics.

Mathematics instruction should be informed by stakeholders' knowledge of the students themselves. This includes students' histories, mathematical practices, and various experiences related to mathematics.[43] Effective mathematics instruction with multilingual students extends beyond vocabulary, focuses on reasoning and practices, draws on multiple resources, and treats everyday ways of talking as resources.[44] Identifying what students bring with them to the classroom (e.g. ways of talking, participating, and doing math) as assets, instead of something to overcome or replace, is important for supporting students who come from a variety of cultures and communities.[45] Effective teachers understand that language is learned through meaningful activities, encourage students' full use of linguistic resources, focus on students' meaning-making, and take time to reflect on their own assumptions about multilingual students.[46] The NASEM report outlined 5 promising instructional strategies for teaching multilingual students disciplinary content, which include:

1. Engaging students in disciplinary practices (i.e. the eight mathematical practices from CCSS);
2. Supporting interactions;
3. Leveraging and encouraging students' use of multiple registers;
4. Drawing on students' resources (e.g. everyday ways of talking, primary language, familiar participation structures like joking or collaborative conversations, and everyday experiences, etc.);
5. Focusing on the functions of language, rather than vocabulary (NASEM, 2018).

The following vignette illustrates the way one teacher positioned a multilingual student's ideas and contributions during a discussion about geometry as valued and supported engagement with the mathematics register by *revoicing* (not just repeating but rephrasing) students' contributions and asking for additional explanations.

Vignette cited from Moschkovich (1999, p. 13):

1. Teacher: Today we are going to have a very special lesson in which you really gonna have to listen. You're going to put on your best, best listening ears because I am only going to speak in English. Nothing else. Only English. Let's see how much we remembered from Monday. Hold up your rectangles... high as you can. (Students hold up rectangles) Good, now. Who can describe a rectangle? Eric, can you describe it [a rectangle]? Can you tell me about it?

2. Eric: A rectangle has... two...short sides, and two...long sides.

3. Teacher: Two short sides and two long sides. Can somebody tell me something else about this rectangle, if somebody didn't know what it looked like, what, what...how would you say it.

4. Julian: Paralel(o) [holding up a rectangle]

5. Teacher: It's parallel. Very interesting word. Parallel. Wow! Pretty interesting word, isn't it? Can you describe what that is?

6. Julian: Never get together. They never get together [runs his finger over the top side of the rectangle].

7. Teacher: What never gets together?

8. Julian: The parallela...they...when they go, they go higher [runs two fingers parallel to each other first

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40. Fennema et al., 1996; Philipp et al, 2007
41. Aguirre et al., 2012; Turner et al., 2012
42. Barwell, Moschkovich, & Setati Phakeng, 2017
43. Moschkovich, 2010
44. Barwell, Moschkovich, & Setati Phakeng, 2017
45. Moschkovich & Nelson-Barber, 2009
46. NASEM, 2018

along the top and base of the rectangle and then continues along those lines], they never get together.

This vignette reveals how the teacher employs various teaching practices (e.g. revoicing the students' statements, positioning the student and their ideas as valid, pressing for further explanations which ultimately focuses on the underlying meaning and students' reasoning, etc.) to support engagement with mathematical concepts. Equitable teaching practices for multilingual students include focusing on reasoning, supporting understanding, and engaging in classroom discussions and mathematical practices.[47] Everyday mathematical practices, ways of talking, and experiences need to be integrated into the classroom to make mathematics relevant to students. However, engagement with practices of academic mathematicians (such as constructing mathematical arguments, making conjectures, generalizing, etc.) can increase accessibility of mathematical concepts.[48]

Mathematics instruction can provide opportunities for multilingual students to participate in rich discussions, develop meaning for mathematical vocabulary and draw on prior experiences and knowledge to learn mathematics with understanding.[49] This example shows that the teacher was not focusing on "correct" vocabulary but instead on the ideas that the student was communicating. As students are learning to speak English, it is important that the focus of instruction is not only on vocabulary or learning English, but instead the math ideas and student thinking. As highlighted in the previous section on mathematics learning and language development, instruction should support multilingual students to learn the language of mathematics as they participate in meaningful mathematical activities.[50]

The student's use of the word "parallela" and his explanation of "never get together" are examples of how this student drew on his linguistic resources (e.g. everyday ways of talking and primary language) to communicate his understanding of a mathematical concept. Robust findings in the literature suggest that allowing multilingual students to use everyday ways of talking to make sense of and develop meaning in mathematics supports students in learning mathematics with understanding.[51] When multilingual students have opportunities to engage in challenging mathematics and discuss their ideas using familiar language (e.g. everyday ways of talking and home language), achievement disparities between monolingual and multilingual students decrease.[52] By making space in mathematics instruction for multilingual students' everyday ways of talking, students can make meaning for important ideas while also taking up literacy and mathematical practices associated with school.

In addition to teacher preparation and effective instruction for supporting multilingual students, curriculum can also impact opportunities multilingual students have to participate in mathematics instruction and meaningful mathematical activities. As curriculum is developed and implemented, it is important that the needs of linguistically diverse students are prioritized[53] from the start of the design process, not only as an after-thought. This includes providing rich mathematical activities, such as ones that engage students and provide multiple solution pathways, while also intentionally making space for multilingual students to participate. Curricula, mathematical tasks, and the language of the tasks should not be simplified; rather, teachers need to provide the necessary support to multilingual students and amplify language[54] as they engage in these tasks. Since every task or lesson cannot address all the practice standards or strands, there needs to be careful planning across lessons and units to include the standards and strands. Scaffolding student participation in math practice standards and strands can happen across units, lessons, and tasks.[55] Units and lessons can be sequenced so that students can draw on prior knowledge to learn new content. At a more focused level, the structure of a task can set expectations for what students will be required to attend to.[56] If the focus is on mathematical practices rather than procedural skills, scaffolding of multilingual students' participation in mathematics instruction can provide important opportunities for student engagement.[57]

47. Moschkovich, 2013
48. Moschkovich, 2002
49. Ibid.
50. Moschkovich & Nelson-Barber, 2009
51. E.g. Barwell, 2005; de Araujo et al., 2018; Razfar, 2013; Turner & Celedón-Pattichis, 2011
52. Turner & Celedón-Pattichis, 2011
53. NASEM, 2018
54. Ibid.
55. Moschkovich, 2015b; van Lier, 2004
56. Moschkovich, 2015b
57. Ibid.

4. The Role of Families and Communities

All students bring rich knowledge to school that can be a resource for engaging in and learning mathematics content. However, the way this knowledge is valued in schools, either as an asset or something to overcome or replace, impacts opportunities for multilingual students to learn and influences family engagement. Drawing on the resources of multilingual students broadens participation in mathematics instruction and positions the everyday mathematics practices of multilingual students as valuable. To position students as mathematical thinkers and learners, students need to see the mathematical practices from their families and communities reflected in the classroom.[58] To bring family and community practices into schools and the classroom there needs to be stronger connections between schools and multilingual students' families and communities as well as school educators and leaders having a better understanding of these practices.

It is important to shift away from traditional approaches to family involvement (e.g., parents as "first teachers", choosers of schools, etc.) to create and maintain stronger connections between multilingual students' families and communities and their schools.[59] Alternative approaches to involvement acknowledge the various roles that families and communities play to support students and the valuable knowledge and experiences from these families and communities. This shifts the burden of understanding away from families and communities onto school educators and leaders. It is important that educators and leaders take the time to better understand and build upon the rich knowledge that comes from multilingual students' families and communities.[60] To support teachers in working with families and communities, there need to be spaces for teachers to engage in deep self-reflection and have opportunities for interaction with families. For example, the Teachers Empowered to Advance Change in Mathematics Project (TEACH Math) has shown to be successful at supporting reflection and increasing interactions with families and communities from diverse backgrounds.[61] Through this program, pre-service teachers engage in a variety of modules (e.g. a math autobiography and a community walk) that ask them to reflect on their assumptions about mathematics learning and help them uncover the mathematical practices in which students' families engage when in community spaces, like local supermarkets and parks. When teachers have opportunities to reflect and interact with families and communities, they learn how to build relationships and draw on their knowledge and practices in mathematics instruction.[62]

To build stronger connections inside and outside of the classroom, school educators and leaders need to better understand the various approaches to engaging with mathematics across multilingual students' families and communities. Curriculum developers should intentionally create open curriculum spaces[63] that guide teachers to elicit student thinking and leverage students' everyday ways of talking about and engaging with mathematics. Policy makers need to also prioritize the needs of multilingual students across all educational settings. School or district policies that limit the language of instruction to English can create obstacles not only for students, but also for family engagement with schools.[64] Both through policies and practices in schools and classrooms, there needs to be acknowledgement of the value of leveraging families' and communities' cultural practices.[65]

5. Assessment

Much attention has been given to large-scale tests (e.g., SBAC, CAASPP, NAEP) where "ELs" appear to lag behind other students in mathematics achievement.[66] There are multiple factors that explain these results. First, these scores could be reflecting the challenges of living in poverty, receiving inadequate support to learn both English and mathematics content, and/or limited access to mathematics courses. Second, there are many limitations to the current assessment system. Furthermore, as soon as students become proficient in English they are excluded from the subcategory of "ELs" which exacerbates the difference between these groups.

The current systems for identifying and classifying multilingual students fail to take into account the

58. Civil, 2002; Civil, 2007; González, Andrade, Civil, & Moll, 2001
59. NASEM, 2018
60. Acosta-Iriqui et al., 2011; NASEM, 2018; Turner et al., 2012
61. Aguirre et al., 2012; Turner et al., 2012
62. Aguirre et al., 2012; Turner et al., 2012
63. Land et al., 2018
64. Acosta-Iriqui et al., 2011; NASEM, 2018
65. NASEM, 2018
66. Ibid.

heterogeneity of students' experiences. Multilingual students come to school with different proficiencies in using English and their home languages across the four modalities: speaking, listening, reading and writing. Current assessment and classification systems either mask these differences or fail to take them into account when decisions about policies, course placement, and instruction are made. There are also concerns about the validity of mathematics assessments—do these tests accurately describe the mathematics knowledge of multilingual students? There is a great deal of evidence that the complexity of the language in a test item impacts students' performance.[67] Furthermore, tests are almost always written in "Standard English" and the problem contexts presented (e.g., character names, stories, settings) reflect mainstream, white, middle-class culture.[68] To be a valid measure of student knowledge, assessments must undergo development, review, and adaptation and each test question must be examined for potential bias. For example, if "ELs" and non-"ELs" have similar scores overall, yet systematically perform differently on one particular item, that is evidence that the test question is biased. Unfortunately, there is very little involvement of "EL experts" in the review of large-scale assessments or of multilingual students in test piloting.

All of the concerns described above coalesce in reporting and documentation, which is closely related to education policy (e.g., *No Child Left Behind* or the *Every Student Succeeds Act*). Reports on the educational achievement of multilingual students fail to take into account the heterogeneity of this population of students and obscure growth, since, when students achieve enough language proficiency, they are excluded from the "EL" reporting group.

There is evidence that classroom-level assessments may be a more valid form of assessment for multilingual students than large-scale tests. Indeed, multiple classroom assessments can complement large-scale assessments to provide a more comprehensive view of multilingual students' achievement.[69] Importantly, the NASEM report argues that no single assessment should be used to define a student's understanding, and, in classrooms, both summative and formative assessments should be used.

Summative assessments measure how well students have acquired mathematics content knowledge and are used to modify instruction prior to year-end, large-scale assessments. There is evidence that providing technological enhancements (such as pop-up translations that are available only when a student needs them), familiar problem contexts, and visual aids support multilingual students. There is also evidence that providing opportunities for students to explain their thinking in their own words is more effective than multiple choice-type items;[70] it is also beneficial to divide these writing tasks into smaller prompts.[71] Importantly, teachers must receive appropriate professional development and opportunities to learn about the cultural practices of their students to implement these types of assessments. They must be able to interpret students' responses, especially those in which the student is communicating their understanding using everyday language, in order to accurately assess students' conceptual understanding.[72]

Formative assessments are much shorter than summative assessments; their purpose is to provide real-time feedback to students and teachers about the learning that is happening in a particular lesson or unit. They are used to inform instruction, *not* to determine a grade or score. A period of sustained questioning about a student's reasoning is an example of formative assessment. In this close setting, teachers can modify their feedback—both in terms of language and content—immediately in response to a student's particular needs. Providing written sentence frames also supports multilingual students in providing more detailed responses, further supporting teachers in assessing students' knowledge. Using formative assessments provides feedback that not only allows teachers to adjust instruction to meet students' needs, it also fosters students' agency in their learning. Reflection and self-assessment support students in becoming self-directed learners.[73]

Curriculum designers play an important role in the assessment process. Lessons should be designed to include many, varied opportunities for formative assessment. Curricula must include samples of student work at various levels of proficiency, *including the work of multilingual students*, to support teachers in assessing

67. Abedi, 2004; Avenia-Tapper & Llosa, 2015; Solano-Flores & Li, 2013
68. Solano-Flores, 2011
69. National Research Council, 2001
70. Noble et al., 2014
71. Siegel, 2007
72. Shaw, 1997; Nguyen-Le, 2010
73. NASEM, 2018

students' progress. Furthermore, professional development for teachers must include samples of multilingual students' work and practice engaging in sustained questioning of students' thinking.

6. Building Capacity to Transform Mathematics Learning

This section describes the recommendations from the NASEM report for building capacity to transform mathematics learning. Aligned with the NASEM consensus report, we use the United Nations Development Program (2009) definition of *building capacity*, which refers to "the process through which individuals, organizations, or societies obtain, strengthen or maintain the capabilities to set and achieve their own development objectives over time".[74] While the NASEM report addresses building capacity at a variety of levels, including state, federal, district, classroom, and other levels, we will focus on conclusions and recommendations that are relevant to the work related to the anticipated audience of stakeholders, including curriculum writers, district and school-level personnel (certificated and classified), and other advocates, such as teachers, coaches, and community members.

At the state and federal levels, the NASEM report recommends the development of assessments and standards that consider an integrated model for the development of language and mathematical knowledge. Treating the development of language and mathematical knowledge as separable, rather than integrated, leads to the exclusion of multilingual students from rigorous mathematics coursework.[75] Proceeding from this assumption, reclassification criteria should be examined and possibly revised because a "misplaced threshold" can have serious negative effects.[76] However, the report further notes that top-down reform models are not poised to be transformative or sustaining without organizational and individual capacity building.

At the district and school level, the NASEM (2018) report suggests a framework for continuous instructional improvement for multilingual students that is organized around three components: *organizational culture, educators' capability,* and *policy and management.* Organizational culture includes district and school leadership that is accountable to each other, the schools, and the community, uses data to inform their decisions, promotes a culture of collaboration, and engages families and communities. Educators' capability refers to the development of a clear and shared instructional vision and frameworks that are built upon three principles: creating opportunities to learn, asset orientation toward students' home culture, language, and backgrounds, and student autonomy. This can include professional development, but it can also include internal certification for working with multilingual students or partnering with local universities for additional classes and training. Policy and management refer to the appropriate allotment of fiscal and human resources, extended support for students (including working with community partners and after-school or summer programs) and monitoring and guiding the success of programs.[77] Policy is particularly important because it can either facilitate or constrain multilingual students' access to rigorous mathematics instruction. This framework functions to help identify and remove barriers to multilingual students' participation in mathematics learning. As resources can be scant, it is important for schools and districts to heed recommendations from research so that they can maximize their efforts. For example, the integration of language and content learning through co-teaching or bilingual models of instruction is generally more cost-effective than "fragmented" pull-out or ESL approaches.[78]

An example of lesson design that supports multilingual students can be found in a framework for *amplified lessons*.[79] The framework organizes lessons around three moments related to what students accomplish: Preparing Learners, Interacting with Text (or Concept), and Extending Understanding.[80] The overarching goal of the framework is to enhance students' access to important mathematical ideas and mathematical practices by emphasizing three key areas: conceptual understanding, opportunities to participate and interact with peers, and abundant language practices. Mathematical tasks, therefore, should provide rich opportunities to grapple with concepts through the use of multiple representations and teachers should have curricular support and training to facilitate the kinds of interactions that compare and connect solution methods and

74. Ibid., p.251
75. Ibid.
76. Ibid., p.256
77. Ibid.
78. Ibid., p.282
79. Walqui & Bunch, 2019
80. Ibid.

representations. Opportunities to participate in these discussions should include student-led dialogs that are sustained and reciprocal where students can test out ideas and conjectures, practice explaining, and develop ways to participate. The development of these language skills means paying particular attention to the kind of language that supports conceptual understanding and engagement with mathematical practices. For example, using *working* definitions rather than formal ones could support access to content while students develop meaning and connections around concepts. Activities that view language in "bits and pieces", such as *cloze* activities or 'vocabulary' lists, are not as productive for learning. Students need to connect personal experience, mathematical ideas, representations, and procedures with support from curricula and teachers.[81] For example, interactive activities such as think-pair-shares, card sorts, collaborative problem-solving, collaborative writing, close readings, etc., can be used to support meaningful student interactions at each stage of a lesson.

Reviewing policies, building capacity at the individual, school, and district levels, efficiently allocating resources, and engaging the resources of students, families, and communities can combine effectively to support students as they develop language and mathematical knowledge.

Conclusion

The research on multilingual students and mathematics education provides important takeaways related to the role of the educational context, mathematics learning and language proficiency, effective instructional strategies and teacher education, the role of families and communities, assessments, and building capacity to transform mathematics learning. The educational context for language learners is defined by considerations involving the heterogeneity of the students, their access to rigorous mathematics coursework, the use of their home languages, and the ways that these factors are related to each other and students' access and achievement. Inside and outside the classroom, mathematics learning and language proficiency develop simultaneously. Therefore, language proficiency should never be a prerequisite for access to content, and instruction should focus on academic literacy for mathematics. Effective instructional strategies and teacher education should focus on uncovering and drawing on multilingual students' assets and broadening participation in the mathematics classroom. There also needs to be stronger connections between families and schools, and the work of building relationships must be undertaken by those working in the school system (e.g. teachers, school administration, school leaders, curriculum developers, etc.) and not just the families. It is also important to address the current ways in which multilingual students are assessed and classified, as large-scale assessments lack validity when describing the knowledge and achievement of multilingual students. Instead, multiple, classroom-level assessments that provide meaningful feedback on students' learning should be used with trained teachers. The research reflected in this paper reveals that building capacity to transform mathematics learning involves coordinated efforts across different levels of schooling focusing on organizational culture, educators' capability, and policy and management. These efforts include building the capacity of individuals, such as teachers in the classroom, as well as institutions when they consider funding and staffing for programs. At each level, the stakeholders should reflect upon their own experiences and listen to and consider the experiences of others because issues related to language and mathematics instruction are informed by social and political concerns that, if not addressed, can lead well-intentioned educators to reproduce the kinds of inequities they sought to redress.

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81. Ibid.

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