

Research Paper

An improved unified constitutive model for rock material and guidelines for its application in numerical modelling



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ABSTRACT

This paper presents the theoretical backgrounds, guidelines for selection of inputs, validation, and limitations/assumptions for a proposed improved unified constitutive model (IUCM). The IUCM is a collection of the most notable and widely accepted work in rock mechanics and is a unified constitutive model that can better and more accurately predict the stress-strain relationships of the rock mass or intact rock samples in continuum numerical models than conventional constitutive models.

The IUCM accounts for important and fundamental mechanisms, such as the transition from brittle to ductile response, confinement-dependent strain-softening, dilatational response, strength anisotropy, and stiffness softening. The IUCM was developed with the intention to provide a unified constitutive model that has the complexities required for application to a wide range of geotechnical applications and conditions, yet is simple enough to be used by most geotechnical practitioners.

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1. Introduction

Numerical modelling is increasingly used in the field of applied rock mechanics to predict the response of the rock mass to various engineering activities.

In recent years, increased computational power has helped facilitate the application of more sophisticated discontinuum codes for practical engineering design purposes. These codes can model the rock response to increasing stress levels at a more fundamental level by explicitly representing the discontinuities and the induced fracturing in the rock mass. As a result, these models can replicate complex failure mechanisms much more accurately than conventional continuum or semi-discontinuum models (semi-discontinuum models only include the major rock mass discontinuities explicitly and the remainder of the model is still represented through a continuum representation, in which its behaviour is controlled by a continuum constitutive model). Some recent applications of high-end discontinuum numerical methods were presented by Vyazmensky [71], Vakili and Hebblewhite [69], Hamdi et al. [29], Elmo et al. [26], Vakili et al. [70], Vakili et al. [68], Lisjak et al. [41] and Mahabadi et al. [44]. These studies showed that advanced discontinuum codes can reproduce complex failure mechanisms, such as time-dependent progressive failure, brittle damage, and caving mechanisms more realistically than semi-discontinuum or continuum models.

However, owing to computational limitations, most of the above studies were either conducted in 2D or small-scale 3D. Significantly greater computer power is required for construction and analysis of large-scale 3D discontinuum models. Furthermore, the input parameters required to construct these models are often not available, nor well understood. Consequently, fewer methods have been developed to derive representative inputs for discontinuum models.

In practice, continuum and semi-discontinuum models are quicker to construct and require less computational run time, which has resulted in their wider application amongst practitioners. Consequently, research over the last decade within the field of rock mechanics has largely focused on methods to define input parameters, refine failure criteria, and develop constitutive models for continuum medium.

The main problem associated with continuum models, however, is that they rely heavily on constitutive models. The role of a constitutive model is to implicitly represent the underlying failure mechanisms that are in place without explicitly including the micro-structures, block interactions, or the fracturing process. Therefore, a suitable constitutive model is one that can correctly represent the major controlling mechanisms that occur during the process of rock mass loading and failure.

The Mohr-Coulomb peak strength criterion and associated constitutive models have gained wide acceptance and application in the field of geotechnical engineering. Many analysis methods and software programs still use this criterion as part of their default

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Nomenclature

C	cohesion	$\sigma_{c \max}$	highest directional unconfined compressive strength of the intact rock (anisotropic rocks)
ϕ	friction angle	β	the orientation of the anisotropic plane with respect to the specimen loading axis
m	Hoek-Brown material constant	miMax	Hoek-Brown constant mi for the intact rock matrix (IUCM input)
mb	Hoek-Brown material constant	Ei	elastic Modulus of the intact rock (IUCM input)
a	Hoek-Brown material constant	E_M	elastic Modulus of the rock mass (IUCM input)
mi	Hoek-Brown material constant for intact rock	miMin	Hoek-Brown constant mi for the plane of anisotropy of the intact rock (IUCM input)
miMax	Hoek-Brown constant mi for the intact rock matrix	AnisoFac	anisotropy factor (IUCM input)
miMin	Hoek-Brown constant mi for the plane of anisotropy of the intact rock	AnisoDip	dip angle of the plane of anisotropy (IUCM input)
s	Hoek Brown material constant	AnisoDipD	dip direction angle of the plane of anisotropy (IUCM input)
UCS	unconfined compressive strength	DisFac	disturbance factor (IUCM input)
σ_c	unconfined compressive strength	CRes	residual cohesion of the rock mass (IUCM input)
σ_t	unconfined tensile Strength	FricRes	residual friction angle of the rock mass (IUCM input)
GSI	geological strength index (IUCM input)	TenRes	residual tensile strength of the rock mass (IUCM input)
σ_1	minor principal stress	CritRed	reduction factor for critical strain (IUCM input)
σ_2	intermediate principal stress	MR	modulus ratio
σ_3	minor principal stress	ν_M	rock mass Poisson's ratio
D	disturbance factor	ε_1	major principal strain
ψ	slope of the failure envelop in the principal stress space	ε_2	Intermediate principal strain
d	equivalent edge length of the model zone	ε_3	minor principal strain
ψ_{peak}	peak dilation angle	ΔV	change in volume
σ_{ci}	unconfined compressive strength of the intact rock	V_0	initial volume
σ_{ti}	unconfined tensile Strength of the intact rock		
Sigci	unconfined compressive strength of the intact rock (IUCM input)		
$\sigma_{c \min}$	lowest directional unconfined compressive strength of the intact rock (anisotropic rocks)		

constitutive model. This criterion is obtained from two key parameters—cohesion (c) and friction angle (ϕ), which became widely accepted parameters for describing soil and rock strength properties. In addition, many of the well-known concepts such as Factor of Safety are still based on Mohr-Coulomb parameters to describe the strength of soil or rock.

Despite its wide application, several research works and studies have questioned its usefulness and accuracy for many rock mechanics applications. This is particularly pronounced when a linear Mohr-Coulomb model is used in an unsuitable modelling software code or when other fundamental failure processes like strain-softening, dilation, confinement dependency, anisotropy, etc. are ignored.

According to Brown [13], the linear Mohr-Coulomb consisting of two independent cohesive and frictional components does not provide a realistic representation of the progressive failure and disintegration of rock under stress. Some recent studies such as Haji-abdolmajid et al. [28], Barton and Pandey [6], and Barton [7] also highlighted the limitations of this model and its application to predict damage in rock material.

At a fundamental level, when dealing with laboratory triaxial test results, Hoek and Brown [30] reviewed several sets of laboratory test results and found that unlike the traditional Mohr-Coulomb criterion, the peak failure envelope at different confinement levels follows a non-linear relationship in major and minor principal stress space. As a result, they proposed an empirical criterion, where material constants m and s and uniaxial compressive strength (UCS) of intact rock represents the curvature and position of the failure envelope. The material constant m represents the characteristics and size of the micro-grains that form the rock sample (and also reflect the ratio between UCS and tensile strength) and s represents the degree of rock jointing or blockiness of the sample. Subsequently Hoek et al. [35] provided some relationships to derive the rock mass properties using the Geological Strength Index (GSI) and the disturbance factor.

Because most modelling software codes (and also the majority of practitioners) still use the Mohr-Coulomb failure criterion, a line-fitting procedure was proposed to find equivalent cohesion and friction angle values based on the Hoek-Brown curves and the maximum confinement pressures. Nonetheless, there remains two fundamental problems associated with applying a linear model which often lead to a considerable mismatch between the Hoek-Brown and the Mohr-Coulomb predicted peak stress values. The first problem is that in many rock mechanics applications, such as mining, a significant variation exists in the level of confinement at different locations within the rock mass. Secondly, even for a particular excavation at a given depth there can be a large variation in confinement levels depending on the position with respect to excavation boundaries. This is mainly caused by the redistribution of stress around excavations and also new phases of confinement that can be induced as result of nearby excavations or yielded material. These two issues can be a lot more pronounced in high stress and high yield environments, or in rock material that exhibits a more curved peak failure envelope (rocks with high Hoek-Brown material constant m values).

It is globally accepted that the Hoek-Brown criterion (and a non-linear failure criterion in general) can forecast the “peak failure” state of rock samples with better accuracy than a linear criterion such as Mohr-Coulomb. However, when speaking of rock damage, “peak failure” is not the main or only controlling mechanism. Other contributing factors including residual strength, strain-softening, confinement-dependency, dilatational response, stiffness softening mechanism, and anisotropic behaviour also have significant influence on how damage initiates and propagates within the rock material. These factors are particularly important when stress elevates to levels that can initiate brittle intact rock failure.

This paper introduces an improved unified constitutive model (IUCM), which is based on widely accepted research cases of rock damage noted in the literature. It also outlines several examples

using well-documented case histories of rock damage, where extent and severity of damage is forecast more accurately than other conventional methods. The IUCM is believed to be applicable to a wide range of ground conditions, from intact brittle hard rock to heavily jointed and ductile rock, and also anisotropic rocks.

In developing this model, new theories and techniques were avoided where possible, so that the adopted processes are based on well-accepted and widely applied rock mechanics techniques and theories. A key conclusion of this paper is that if the key controlling mechanisms of the rock damage process that have been well-researched and understood over past years are included in a model that uses this unified approach, then the rock damage process can be predicted more accurately without the need to introduce any new theories or criteria.

Guidelines for determining the input parameters and interpreting the model results using this unified model are also presented in this paper.

2. Components of the proposed improved unified constitutive model (IUCM)

2.1. Peak failure criterion

The IUCM employs the Hoek-Brown criterion to determine the instantaneous Mohr-Coulomb parameters cohesion (c) and friction (ϕ) at each level of confining stress. There were a number of motives for continuing to employ the Mohr-Coulomb failure criterion in the IUCM including:

- The Mohr-Coulomb peak failure criterion has the advantage of having a simple mathematical representation, a clear physical meaning of the material parameters, and a general level of acceptance [40].
- The Mohr-Coulomb failure criterion is available in almost all geotechnical numerical modelling programs.
- Interpretation of results, particularly when using Factor of Safety values, becomes easier when dealing with well-known Mohr-Coulomb parameters.
- Mohr-Coulomb failure criterion has been applied and tested in various numerical modelling programs for many years. Therefore, it offers a highly robust numerical method and is associated with minimal solver-related errors or bugs.

As discussed earlier in this paper, despite all the above advantages, the Mohr-Coulomb criterion can lead to considerable errors in more extreme conditions such as higher stress (lower strength) or more non-linear rock failure behaviour (due to greater Hoek-Brown material constant m values, varied discontinuities patterns, etc.). This is demonstrated visually in Fig. 2.1. In extreme conditions (e.g. deep, large nearby excavations, lower rock strength, and higher m_i values), these errors can exceed 700%.

During development of the IUCM, various methods were employed to address issues associated with fitting linear envelopes to non-linear criteria. Sainsbury [60] demonstrated that a bi-linear fit can provide a more accurate strength estimate over the range of expected stresses. While Vakili et al. [70] showed that a multi-linear Mohr-Coulomb peak failure criterion was sufficient to reproduce the observed excavation damage.

However, subsequent works including back-analysis and validation studies during the development of the IUCM revealed that notable errors can still result, particularly in extreme conditions, when applying the simplified bi- or multi-linear criteria.

As a result, to minimise the potential errors, a complete global fit to the Hoek-Brown criterion [35] was adopted for the IUCM through calculation of instantaneous Mohr-Coulomb parameters for small stress increments.

Various literature (e.g. [53]) demonstrated procedures to obtain instantaneous Mohr-Coulomb parameters from the Hoek-Brown failure envelope. The IUCM follows a simple computation procedure that uses the initialised pre-mining or induced stresses in a numerical model to obtain the instantaneous Mohr-Coulomb parameters. The initial inputs required are the unconfined compressive strength (UCS or σ_c) of intact rock, Hoek-Brown constant m_i , geological strength index (GSI), and disturbance factor (D). This computation procedure is outlined below:

1. Initialise the pre-mining stresses in the model.
2. Obtain the current minor and major principal stresses for each finite difference zone (or element in the finite element method).
3. Obtain minor principal stress increment ($\Delta\sigma_3$) by adding and subtracting 0.1% of the current σ_3 magnitude,

$$\sigma_3^1 = \sigma_3 - 0.001\sigma_3 \quad (1)$$

$$\sigma_3^2 = \sigma_3 + 0.001\sigma_3 \quad (2)$$

4. Calculate constants for the Hoek-Brown criterion based on equations provided by Hoek et al. [35],

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right) \quad (3)$$

$$s = \exp\left(\frac{GSI - 100}{9 - 3D}\right) \quad (4)$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{-\frac{GSI}{15}} + e^{-\frac{20}{3}}\right) \quad (5)$$

5. Obtain the major principal stress increment ($\Delta\sigma_1$) from the generalised Hoek-Brown failure criterion [35] and from the measured change in the minor principal stress ($\Delta\sigma_3$),

$$\sigma_1^1 = \sigma_3^1 - \sigma_c \left(m_b \frac{\sigma_3^1}{\sigma_c} + s\right)^a \quad (6)$$

$$\sigma_1^2 = \sigma_3^2 - \sigma_c \left(m_b \frac{\sigma_3^2}{\sigma_c} + s\right)^a \quad (7)$$

6. Obtain the slope (ψ) of the incremental stress envelope,

$$\tan \psi = \frac{\sigma_1^1 - \sigma_1^2}{\sigma_3^1 - \sigma_3^2} \quad (8)$$

7. Calculate instantaneous friction angle (ϕ) from ψ ,

$$\phi = \sin^{-1}\left(\frac{\tan \psi - 1}{\tan \psi + 1}\right) \quad (9)$$

8. Calculate instantaneous cohesion,

$$C = \frac{\sigma_1^1(1 - \sin \phi) - \sigma_3^1(1 + \sin \phi)}{2 \cos \phi} \quad (10)$$

9. Calculate uniaxial tensile strength,

$$\sigma_t = \frac{s\sigma_c}{m_b} \quad (11)$$

10. To avoid the tensile strength exceeding the value of σ_3 , which corresponds to the apex limit for the Mohr-Coulomb relation, compare the tensile strength value obtained from Eq. (11) and the maximum limit ($c/\tan \phi$), and use the lower value.

In the IUCM, the instantaneous peak Mohr-Coulomb parameters are updated in real-time as the model cycles and as new phases of confinement are formed owing to damage in nearby zones or new excavations (Fig. 2.2).

2.2. Post-peak failure criterion

Once the peak strength of a rock mass is reached, it fails and enters its post-peak state. If the loading condition continues, the rock mass will eventually reach its ultimate or residual strength.

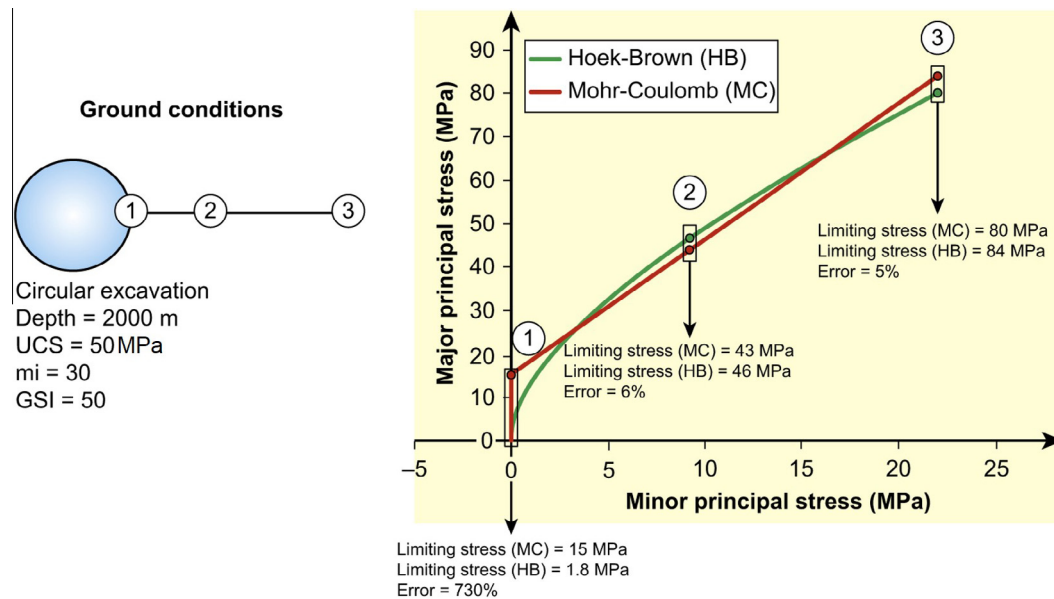


Fig. 2.1. An example showing the increased errors caused by application of Mohr-Coulomb criterion in extreme mining conditions.

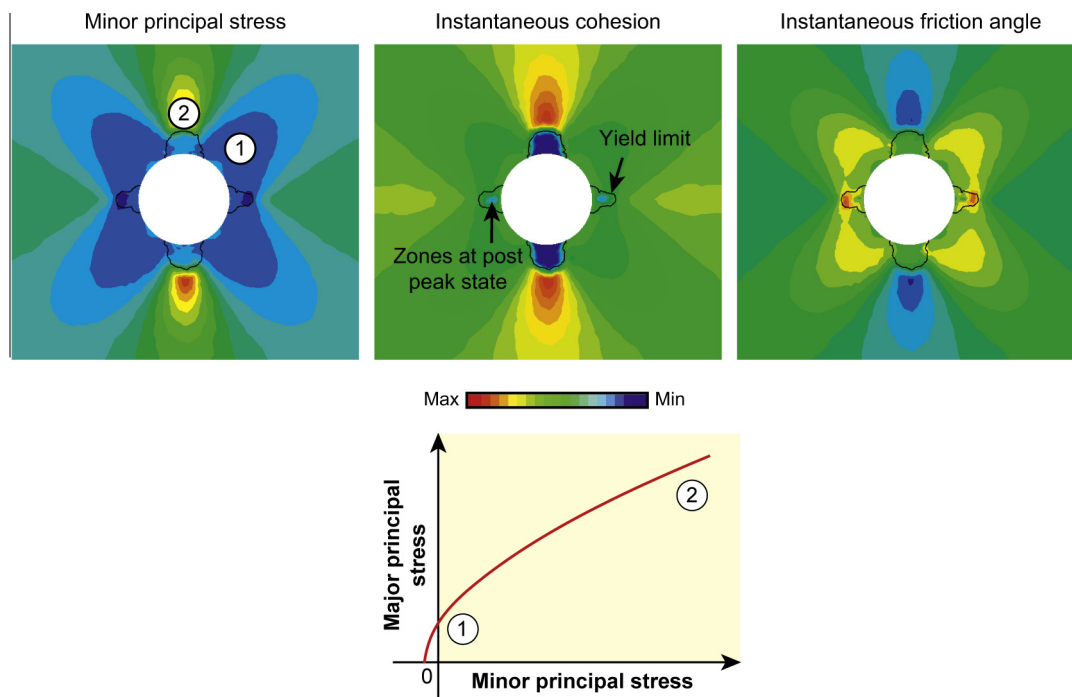


Fig. 2.2. Real-time update of instantaneous Mohr-Coulomb parameters as a result of new confinement levels.

Residual strength has an important role in the failure progression of the rock mass and the resulting stresses and strains. Cai et al. [14] demonstrated the influence of the residual strength on the model yielding zones around a tunnel. The importance of a post-peak failure criterion becomes more pronounced when the stress over strength ratio of a rock mass increases.

Owing to difficulties associated with obtaining the complete stress-strain curves from large-scale laboratory testing on jointed rock masses, a Synthetic Rock Mass (SRM) modelling technique, such as that proposed by Mas Ivars et al. [38] presents a rigorous method for determining the post-peak properties of the rock mass. However, this technique is time-consuming and requires structural data that is often not available at early stages of an engineering

project. Therefore, it is usually important to obtain quick estimates of the residual properties from the available data.

The most notable and widely accepted techniques for estimating the residual properties were presented by Cai et al. [14] and Lorig and Varona [42]. Cai et al. [14] provide a quantitative guideline that expresses the residual strength of the rock mass in terms of a reduced GSI value. Lorig and Varona [42] also provide a quantitative guideline that downgrades the peak strength of the rock mass using Hoek's disturbance factor. A relationship was provided to determine the residual properties based on Hoek's disturbance factor (D factor) as a function of GSI. These authors also recommended an alternative approach, where post-peak properties similar to those of completely broken rock (cohesion = 0, friction

angle = 35–55°) are assigned. This is based on the idea of cohesion-softening and friction-hardening (if a non-linear peak failure criterion such as Hoek-Brown is adopted).

During development of the IUCM, the above techniques were tested and compared against each other. This also involved the application of the SRM modelling technique (see [70]) that was used to obtain a post-peak failure criterion. The post-peak failure criterion obtained in the Vakili et al. [70] study correlated with the idea of cohesion-softening and friction-hardening proposed by several other authors.

Among all the previously recommended techniques, the author found the linear Mohr-Coulomb post-peak envelope employing the properties of a completely broken rock (cohesion = 0, friction angle = 35–55°) adequate for the IUCM. There were number of justifications and hypotheses for this selection including:

- The linear nature of the residual envelope in the IUCM, with respect to the peak Hoek-Brown envelop, replicates cohesion and friction-softening at low confinement levels and cohesion-softening and friction-hardening at high confinement levels (see Fig. 2.3). This feature of the model allows progressive failure to occur near the boundary of the excavation. At the same time, it limits the propagation of yield or plasticity zones away from the excavation boundaries, as observed in several case histories. This also correlates well with the previous studies that highlighted the importance of cohesion-loss and friction mobilisation when predicting depth-of-failure in underground excavations.
- A Mohr-Coulomb post-peak failure criterion implies that all rock masses will ultimately exhibit a soil-like response once they undergo enough deformation and crushing.

2.3. Transition from peak to residual failure state

The transition of a rock mass from its peak state to its residual state is controlled by its mechanical characteristics and the density of jointing. Hoek and Brown [32] demonstrated this phenomenon in a qualitative manner.

Under low confinement, a very good quality rock mass (high UCS, m_i , and GSI values) behaves in an elastic brittle manner, meaning that when the peak strength is exceeded, a sudden strength drop occurs. In an average quality rock mass, the reduction or transition of the strength from its peak occurs in a more gradual manner often referred to as strain-softening behaviour. In the case of a very poor quality rock mass, a perfectly plastic behaviour is expected, meaning that no reduction of strength occurs, and the strength remains equal to its peak value throughout its post-peak state.

The transition from peak to residual strength is often expressed in terms of critical plastic strain (as shown in Fig. 2.3). There is little known about the critical plastic strain required for different rock masses. Currently, the SRM technique is perhaps the only analytical way to determine the critical plastic strain. However, the relationship below was provided as part of the international caving study (reported by [42]) to estimate the critical plastic strain from the GSI of the rock mass:

$$\text{Critical plastic strain} = \frac{12.5 - 0.125\text{GSI}}{100d} \quad (12)$$

where d is the equivalent edge length of the zones (for the finite difference method) or mesh elements (for the finite element method). The equivalent edge length is the cube root of the zone volume.

In the IUCM, the above equation is used to replicate the strain-softening behaviour of the models. The critical plastic strain is calculated for each individual zone based on its edge length and the assigned GSI values. Because of the non-linear nature of the

peak failure envelope and the instantaneous cohesion and friction angle, the strain-softening behaviour is calculated separately for each zone based on its current confinement, cohesion, friction angle, and plastic shear strain.

In addition to the rock mass quality, confinement also has an important impact on the post-peak behaviour of the rock mass. It is well accepted that increased confinement leads to more ductile post-peak behaviour. This was shown by Wawersik and Fairhurst [73] and Rummel and Fairhurst [58] through laboratory triaxial testing on marble samples.

However, there is little known about the brittle-ductile transition limit of a rock mass. Mogi [51] and Seeber [62] presented some guidelines for selecting this limit for intact laboratory samples, which will not necessarily be valid for large-scale jointed rock mass. In addition, Cundall et al. [17] defined a new parameter called drop modulus in their new constitutive model, which can be adjusted based on different levels of confinement, to control the brittle-ductile transition limit. However, no guideline was provided for selecting the drop modulus for different rock mass conditions.

As shown in Fig. 2.3, the IUCM does not require a separate parameter to replicate the brittle-ductile transition. Instead, because of the non-linear shape of the peak failure envelope and the linear shape of the residual strength envelope, these two intersect each other at higher confinements. After this intersection point, both the peak and the residual failure criteria are represented by one envelope (the peak envelope), which replicates a perfectly plastic behaviour.

A brittle post-peak tensile behaviour is assumed in the IUCM, dropping the tensile strength to its residual value immediately after the yield point.

The complete computation procedure used in the IUCM for simulating the post-peak behaviour is outlined below:

1. Calculate the critical plastic strain for each zone using Eq. (12).
2. Obtain the current plastic shear and tensile strain for each model zone.
3. If the plastic tensile strain is greater than zero, assign the residual tensile strength value to the zone.
4. If the plastic shear strain is greater than zero, but smaller than the critical plastic strain limit, follow the calculation procedures below to obtain the updated instantaneous cohesion and friction angle values:

$$\text{maximum cohesion loss} = \text{peak cohesion} - \text{residual cohesion} \quad (13)$$

$$\text{current cohesion} = \text{peak cohesion}$$

$$- \left(\frac{\text{maximum cohesion loss} \times \text{current plastic shear strain}}{\text{critical strain}} \right) \quad (14)$$

$$\text{maximum friction loss or gain} = \text{peak friction} - \text{residual friction} \quad (15)$$

$$\text{current friction} = \text{peak friction}$$

$$- \left(\frac{\text{maximum friction loss or gain} \times \text{current plastic shear strain}}{\text{critical strain}} \right) \quad (16)$$

5. If plastic shear strain is greater than the critical plastic strain, assign the residual cohesion and friction angle to the zone.

The above computations are also updated in real-time as the model cycles and as new phases of confinement are formed owing to damage in nearby zones or new excavations.

Fig. 2.3 shows a conceptual representation of the post-peak rock mass behaviour as represented in the IUCM.

2.4. Elastic modulus-softening

When rock undergoes failure and continuous loading, voids are generated within the rock mass. The rock mass porosity subsequently increases. The greater the porosity of the rock mass, the

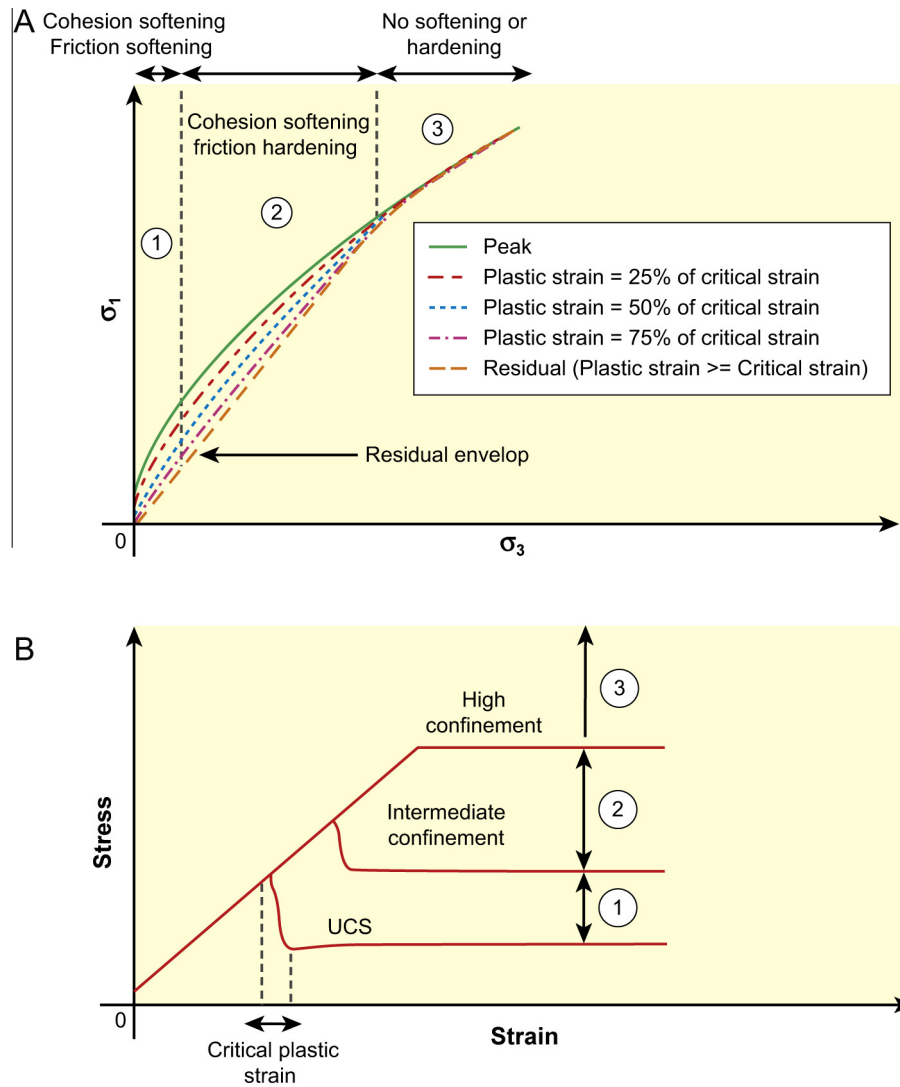


Fig. 2.3. Conceptual representation of the post peak rock mass behaviour as represented in the IUCM (a) in the principal stress space and (b) in the stress-strain space.

lower its elastic modulus. The drop in rock mass modulus can significantly affect the redistribution of stresses around a failed area and the subsequent phases of induced confinement. The impact of modulus-softening can be more pronounced in situations where significant rock mass yield or deformation is expected, for example in high-stress conditions, caving, or deep open pit mining.

Reyes-Montes et al. [56] and Sainsbury [60] reviewed and collated the previous literature in this area, and presented an empirical relationship between modulus drop and the level of porosity in a rock mass. The IUCM uses the relationship provided in those studies to update the elastic modulus values according to new porosity levels.

The complete computation procedure used in the IUCM for simulating elastic modulus-softening is outlined below:

1. Obtain the current volumetric strain increment for each model zone.
2. Check, from the volumetric strain sign, if the zone is in contraction or dilation mode.
3. If the zone is in dilation mode, calculate the void volume using the equation below:

$$Volume_{void} = \varepsilon_{volumetric} \frac{Deformed Volume}{1 + \varepsilon_{volumetric}} \quad (17)$$

4. Calculate porosity by dividing the void volume by the current deformed volume.
5. If porosity is greater than zero but smaller than 40% (0.4), calculate the modulus reduction factor using the equation below (adopted from Reyes-Montes et al. [56]):

$$Modulus\ reduction\ factor = 6.4Porosity^2 - 5Porosity + 1 \quad (18)$$

6. If the above reduction factor is smaller than 0.02, assign 0.02 instead.
7. Assign the updated elastic modulus by multiplying the initial elastic modulus by the reduction factor from Eq. (18).
8. If the porosity is greater than 40%, multiply the initial modulus by 0.02 (2%).
9. If the resulting modulus is smaller than 250 MPa, assign 250 MPa instead (adopted from Reyes-Montes et al. [56]).

2.5. Post-failure dilatancy

Dilatancy has a significant impact on the evolution and progression of damage in the post-failure state of a rock mass. However, dilatancy usually receives less attention during a geotechnical numerical analysis. This can be mainly attributed to the fact that many geotechnical engineering problems, in civil engineering in

particular, deal with the rock masses prior to their failure states, e.g. geotechnical designs in low stress environments. Alejano and Alonso [2] stated that dilatancy seems to receive much less attention because many problems in rock mechanics are solved by avoiding failure and also because of the inherent difficulties in estimating its value.

Zhao and Cai [75] demonstrated the importance of dilatancy for the prediction of excavation induced failure and displacement near the boundary of an excavation.

The conventional methods of estimating the dilation angle in numerical modelling analysis are often based on simplistic associated/non-associated flow rules ($\psi = \phi$, $\psi = 0^\circ$), or based on values recommended by Hoek and Brown [32] who suggested that values of the dilation angle, of $\phi/4$, $\phi/8$ and 0, may be appropriate for the case of good, average, and poor quality rock masses, respectively (ϕ is the equivalent Mohr–Coulomb friction angle at peak state).

Detournay [21], Zhao and Cai [74,75] and Alejano and Alonso [2] are perhaps the most notable and rigorous works completed on the subject of rock dilatancy. They demonstrated the importance of confinement and plastic shear strain dependency on the post-failure dilatancy of the rock mass and also presented improved methods for implementing dilatancy in numerical models that are more scientifically backed by field observations and physical experiments. Zhao and Cai [74] stated that “a constant dilation angle using either perfectly plastic or strain-softening model produces unrealistic dilation behaviour which cannot be supported by experimental data”.

In the IUCM, the equation below, proposed by Alejano and Alonso [2], is used to estimate the peak dilation angle:

$$\psi_{peak} = \frac{\phi}{1 + \log_{10} \frac{\sigma_{ci}}{\sigma_{3+0.1}}} \log_{10} \frac{\sigma_{ci}}{\sigma_{3+0.1}} \quad (19)$$

Following Alejano and Alonso [2], it is assumed in the IUCM that once the plastic deformation starts, the dilation angle begins to drop from a peak value. Both methods proposed by Zhao and Cai [74], [75] and Alejano and Alonso [2] provide a decay relationship for the reduction of the dilation angle, depending on the level of plastic deformation. However, both methods require constants that need to be obtained separately for different rock types.

In the IUCM, for simplicity and to reduce the number of required inputs, the decay of dilation with increasing plastic deformation is replicated through a fundamental understanding, as shown by Detournay [21], that the dilation angle is at its peak when the volumetric strain is negligible and tends to zero, once the maximum possible volumetric strain is reached in the model. For this purpose, Eq. (18) is also used here to reduce the dilation angle with increasing volumetric strain levels.

The complete computation procedure used in the IUCM for simulating the plastic decay of the dilation angle is outlined below:

1. Obtain the current plastic shear strain for each model zone.
2. If the plastic shear strain is greater than zero, assign the dilation angle using Eq. (19) and using the current instantaneous peak friction angle and the confinement stress for each zone. In addition, the rock mass unconfined compressive strength is used instead of σ_{ci} based on equations provided by Hoek et al. [35] (Eq. (6) and using $\sigma_3 = 0$).
3. If the volumetric strain is greater than zero (the zone is dilating) and is less than the maximum dilatational volumetric strain (66% based on a maximum porosity of 40%) obtain the reduction factor from Eq. (18) and multiply by the peak dilation angle obtained from Eq. (19) and assign as the zone's new dilation angle.
4. If the volumetric strain is greater than the maximum dilatational volumetric strain, assign a dilation angle of zero.

2.6. Anisotropic response

Strength anisotropy is perhaps one of the most important but commonly neglected mechanical properties that exists in some rock types. The significance of rock strength anisotropy in geotechnical design is often ignored or underestimated. This is partly because most geotechnical design methods (whether empirical, numerical, or analytical) are largely developed for isotropic rock mass conditions. Thus, there is a tendency to ignore its impact to simplify the design process or apply conventional design methods.

Experience suggests that in many cases, anisotropy can even override other geotechnical factors in controlling the failure mechanism. In high-stress conditions, anisotropy can significantly change the time-dependent failure mechanism and progression of damage into the rock mass.

Two forms of anisotropy are often present in rock material – intact rock anisotropy, and rock mass anisotropy. Intact rock anisotropy is a result of natural fabrics, such as schistosity, foliation, and bedding, constituting the rock. This can cause directional dependency even in a homogeneous intact anisotropic rock at very small scales.

Rock mass anisotropy, on the other hand, is large in scale and often results from the presence of well-defined and persistent joint sets in the rock mass. In the majority of cases, rock mass anisotropy occurs when the constituting rock types exhibit intact rock anisotropic behaviour; however, it is also possible for a rock mass to exhibit directional-dependent behaviour without any small-scale (intact) anisotropy. This is often the result of various in situ stress histories or other geological occurrences that have caused well-defined joint sets in the rock mass.

Vakili et al. [67] described the key components that were implemented in the IUCM to account for strength anisotropy. In the IUCM, the strength anisotropy feature is only applicable for rock types that have the ‘intact rock anisotropy’ characteristic and therefore exhibit directional dependency at both small and large scales. Typical rock type examples are gneiss, mylonite, migmatite, quartz schist, mica schist, hornblende schist, slate, shales, phyllite, and coal.

Anisotropy is often implemented in most commercially available numerical modelling codes as a ubiquitous joint model that is added to the constitutive model. The ubiquitous joint model introduces a weak plane with a given strength criterion embedded in a conventional isotropic constitutive model.

There are two limitations associated with the implementation of the strength anisotropy in the conventional constitutive models.

Firstly, the process of input parameter selection is not as well-defined as it is for isotropic rocks. For example, in isotropic rocks, there is a widely accepted approach using laboratory test results (UCS, UTS and Triaxial testing) and rock mass characterisation (e.g. GSI) data to derive the rock mass properties (e.g. Hoek–Brown parameters) for use in the numerical models. However, in anisotropic rocks, the process of deriving the rock mass properties does not follow any widely accepted process. These properties are often chosen using engineering judgment, or from back-analysis. The ubiquitous joint properties represent the overall macro-mechanical properties constituting the directional dependency of the continuum medium. This directional dependency is sourced from the intact rock fabrics (like schistosity, foliation, and bedding) and discontinuities (like foliation defects) that are parallel to these fabrics.

Secondly, in all commercially available constitutive models, the ubiquitous joint model is embedded in a linear or bi-linear constitutive model that employs the Mohr–Coulomb peak failure criterion. However, as shown by Donath [25], McLamore and Gray [50], Brady and Brown [12], and Saroglou and Tsiambaos [61], the behaviour of the anisotropy plane is also best described by a

Hoek-Brown failure envelope. Barton [9] also highlighted the importance of non-linear failure criterion for filled (or unfilled) discontinuities that can behave similar to intact rock fabrics like schistosity, foliation, and bedding.

In the IUCM, the above two limitations have been overcome through application of a Hoek-Brown failure envelope for the anisotropy plane as well as the main rock matrix. Two additional input parameters are required, including the anisotropy factor and the Hoek-Brown constant m_{iMin} (associated with the anisotropy plane). Both of these parameters can be obtained from laboratory testing.

In anisotropic rocks, the lowest strength value ($\sigma_{c\ min}$) occurs when the orientation of the anisotropic fabric element (bedding, foliation) with respect to the specimen loading axis (β angle) is between 30° and 45° . The highest strength ($\sigma_{c\ max}$) is achieved when this angle is either 0° or 90° . The anisotropy factor is the ratio of $\sigma_{c\ max}$ to $\sigma_{c\ min}$.

For the post-peak response of the anisotropy plane, a completely brittle transition to residual properties is considered. For the residual properties, only cohesion and tension are downgraded to their residual values and the friction angle remains similar to its peak value. The concept of keeping the friction angle replicates the response of filled rock joints under a direct shear test (see [9] for example). Fig. 2.4 shows a conceptual representation of how strength anisotropy is implemented in the IUCM. For the dilatancy of the anisotropy plane, currently, the IUCM uses the same algorithm used for the rock matrix.

The complete computation procedure used in the IUCM for the simulation of strength anisotropy is outlined below:

1. Obtain $\sigma_{c\ min}$ by dividing the rock matrix UCS ($\sigma_{c\ max}$) by the anisotropy factor.
2. Replacing $\sigma_{c\ min}$ and m_{iMin} with σ_c and m_i , follow steps 1–10 listed in Section 2.1 to obtain instantaneous ubiquitous joint properties (weakness plane cohesion, friction angle and tension).
3. Obtain the current ubiquitous joint's plastic shear strain for each model zone.

4. If the plastic shear strain is greater than zero, follow steps 1–4 in Section 2.5 to assign the ubiquitous joint's dilation angle using the current instantaneous peak ubiquitous joint's friction angle, $\sigma_{c\ min}$ and the confinement stress for each zone. Drop the ubiquitous joint's cohesion and tension to their residual values.

3. Guidelines for selection of the input parameters

One of the main emphases during the development of the IUCM was to establish a unified model that could capture the more complex failure mechanisms and still require minimum additional input properties, which could also be obtained through conventional data collection procedures.

Table 3.1 shows the input parameters that are used in the IUCM. Some input parameters in the IUCM can be left unassigned, and guidelines explained in this paper will be used to assign a default value instead.

The following sections describe the input parameters in more detail and provide guidelines for their selection.

3.1. Intact unconfined compressive strength ($\sigma_{c\ i}$) and elastic properties

In the IUCM, the $\sigma_{c\ i}$ property represents the unconfined compressive strength of the intact rock blocks bounded by rock mass discontinuities. In rock mass conditions with higher densities of discontinuities (lower GSI values), the compressive strength of the intact rock for the scale of laboratory samples can be used to assign the $\sigma_{c\ i}$ property. This is because for these rock masses the size of the intact blocks, bounded by the discontinuities, is close enough to the laboratory samples that the effect of increasing sample size on the strength of the intact rock can be neglected. However, in rock mass conditions with lower densities of discontinuities (i.e. GSI greater than 65), the effect of increasing sample size on the strength of the intact rock blocks should be considerably more pronounced.

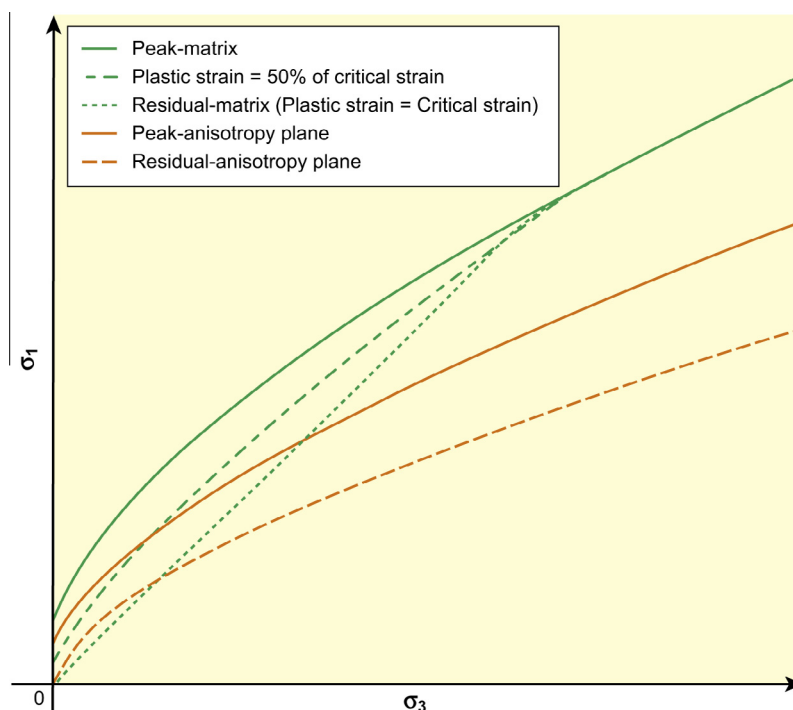


Fig. 2.4. Conceptual representation of the strength anisotropy criterion implemented in the IUCM.

Table 3.1

Input parameters used in the IUCM.

Property	Description	Required/Optional	Used for isotropic Rock	Used for anisotropic rock
Density	Density of the rock mass	Required	Yes	Yes
Sigci	Uniaxial Compressive Strength of the intact rock	Required	Yes	Yes
GSI	Geological Strength Index of the rock mass	Required	Yes	Yes
m_{iMax}	Hoek-Brown constant m_i for the intact rock matrix	Required	Yes	Yes
E_i	Elastic Modulus of intact rock	Required	Yes	Yes
m_{iMin}	Hoek-Brown constant m_i for the plane of anisotropy of the intact rock	Required for anisotropic rock	No	Yes
AnisoFac	Anisotropy Factor	Required for anisotropic rock	No	Yes
AnisoDip	Dip angle of the plane of anisotropy	Required for anisotropic rock	No	Yes
AnisoDipD	Dip direction angle of the plane of anisotropy	Required for anisotropic rock	No	Yes
DisFac	Disturbance factor	Optional	Yes	Yes
C_{Res}	Residual Cohesion of the rock mass	Optional	Yes	Yes
Fri_{Res}	Residual Friction Angle of the rock mass	Optional	Yes	Yes
Ten_{Res}	Residual Tensile Strength of the rock mass	Optional	Yes	Yes
$Crit_{Red}$	Reduction factor for critical strain	Optional	Yes	Yes

Based on previous back-analysis and calibration studies completed for underground excavations using the IUCM, the following rule of thumb is recommended when assigning the Sigci property where calibration data is not available:

- For rock masses with GSI values less than 65, use the unconfined laboratory tests to assign the UCS property.
- For rock masses with GSI values greater than 65, reduce the strength obtained from the unconfined laboratory tests according to an approximate relationship provided by Hoek and Brown [30] and using the approximate block sizes associated with each GSI value as recommended by Cai et al. [15].

For laboratory unconfined compressive strength tests, special care should be taken when dealing with anisotropic rocks, where the Sigci should represent the rock matrix strength. In these conditions, the average peak strength values associated with β angles (the orientation of the anisotropic fabric element to the specimen loading axis) close to 0° or 90° should be chosen for the Sigci, and not the average of all data.

As shown in Table 3.1, in the IUCM, the elastic modulus of the intact rock is the only elastic property that can be provided as input. If E_i is available, the Hoek and Diederichs [33] equation shown below is used to assign the elastic modulus in the models.

$$E_M = E_i \left(0.02 + \frac{1 - D/2}{1 + e^{((60+15D-GSI)/11)}} \right) \quad (20)$$

If not available from testing, E_i can be estimated from the equation below that uses the modulus ratio (MR), introduced by Deere [20] and updated by Hoek and Diederichs [33].

$$E_i = (MR)\sigma_{ci} \quad (21)$$

Brown [13] also suggested that in addition to demonstrating a better fit to known data that Eq. (20) provides, this method has the great advantage that because of the shape of the sigmoidal curve used, it avoids the unrealistically high elastic modulus values given by other methods.

For calculation of Poisson's ratio, the following equation proposed by Hoek et al. [36] is used in the IUCM to assign the Poisson's ratio in the models:

$$\nu_M = 0.32 - 0.0015GSI \quad (22)$$

3.2. Hoek-Brown material constant (m_i)

The Hoek-Brown constant m_i for the intact rock matrix (m_{iMax} in the IUCM) is an important parameter that can largely control how damage initiates and propagates in the numerical model.

More importantly, this constant can control the confinement dependency of the modelled rock under high stress conditions.

Hoek and Brown [32], Brown [13], and Hoek et al. [35] recommended the use of a series of well-conducted triaxial compression tests to provide the most reliable means of establishing the value of m_i .

In the absence of triaxial test data, practitioners often use the recommended m_i values for various rock types proposed by Hoek and Brown [30,32]. However, as recently demonstrated by Richards [57], there is often a poor correlation between m_i values obtained from well-controlled triaxial tests and those provided in the guideline charts.

Carter et al. [16], Diederichs et al. [23], Brown [13], and Richards [57] suggested that the ratio of uniaxial compressive to true uniaxial tensile strength (σ_{ci}/σ_{ti}) provides a more reliable estimate of m_i than published guidelines. As a result, in the absence of triaxial test results, this ratio should be used to establish the m_i value, and the published guidelines should only be used as a last resort when none of the above methods are available.

3.3. Geological Strength Index (GSI)

The Geological Strength Index (GSI) is a classification system based upon an assessment of the lithology, structure, and condition of discontinuity surfaces in the rock mass [46]. The main application of the GSI classification is the estimation of rock mass properties through reduction of the material constants σ_{ci} and m_i .

Hoek and Brown [32], Marinos and Hoek [45], Marinos et al. [46], and Hoek et al. [37] provided guidelines for selection of the GSI for various geological conditions. Estimation of the GSI values from direct visual observations of the rock conditions in the field should always take the highest precedence.

However, in most practical mining or civil conditions, direct exposure mapping is either not available or limited to very isolated locations throughout the rock mass. Information from geotechnical drill holes are often the main source of data available for the purpose of rock mass classification and establishing a GSI value.

The method proposed by Cai et al. [15] should preferably be used to derive the GSI value from the geotechnical drill hole data. In cases where RQD is the only data available for representation of joint spacing, the method presented by Hoek et al. [37] can be used for quantification of the GSI. This method uses the ratings applied in the most widely accepted rock mass classification systems, Rock Mass Rating (RMR) and Tunnelling Quality Index (Q), developed by Bieniawski [11] and Barton et al. [8] respectively. This technique has been used during the majority of recent validation studies carried out for the IUCM.

In cases where the large-scale rock mass stability is under study, e.g. mine scale assessments, large open pit stability assessments, caving assessments, etc., if the variability in the GSI values throughout the rock mass is not sufficiently represented in the models (for example through a geotechnical block model), the GSI value will usually need to be downgraded towards the lower percentile values. This is to account for the effect of rock mass variability that is further discussed in the following section.

3.4. Disturbance factor (*D*)

Disturbance factor (*D*) was originally proposed as a means to account for various factors that are often not considered in the Hoek-Brown failure criterion, and can reduce the overall strength of the rock mass. The main drivers for proposing such a parameter were the various back-analysis and case histories reviewed and referenced by Hoek et al. [35], where application of the Hoek-Brown criterion for undisturbed in situ rock masses resulted in rock mass properties that were too optimistic.

Hoek et al. [35] associated the requirement for a disturbance factor largely to blast damage, and stress relaxation due to removal of the overburden. However, review of other works, including Jefferies et al. [39] and Cundall et al. [19], reveals that other factors, such as scale and rock mass heterogeneity effects can also largely reduce the overall strength of the rock mass.

Some key points that need to be noted when downgrading the rock mass properties from their original values obtained from laboratory testing and rock mass characterisation. These are outlined below:

- Many of the previous back-analysis studies were based on application of simplified constitutive models (e.g. employing Mohr-Coulomb, perfectly plastic, constant elastic modulus, etc.) and non-suitable numerical methods. It is uncertain how much the inaccuracies experienced during those studies were associated with the chosen rock mass properties, and how much of it was actually related to modelling effects (due to inaccurate constitutive models or modelling methods).
- The stress relaxation effect or disturbance caused by mining induced stresses comprises a large weight (approximately 70%) in the original guideline provided by Hoek et al. [35] for selection of the *D* factor. It is important to note that this effect is mainly controlled by the strain and elastic modulus softening nature of the rock mass that has already been accounted for in more advanced constitutive models like the IUCM.
- According to guidelines provided by Hoek et al. [35], the blasting effect has a much smaller weight than the stress relaxation effect in selecting the *D* factor. Saïang [59] conducted a detailed study on the blast-induced damage zone (BIDZ) around underground tunnels and concluded that the BIDZ in most practical cases, on average, ranges from 0.3 m to 0.5 m away from the excavation boundary. It is also important to note that the BIDZ is more likely to influence the more competent rock masses that have fewer discontinuities. This is because blasting energy in rock masses with more discontinuities (say $GSI < 65$) can be released through in situ fractures, minimising the induced damage to intact rock bridges. Most other key recent literature including Hoek and Marinos [34] and Brown [13] also suggest that the blasting effect should only be limited to the immediate boundary of the excavation and cannot be applied to the entire rock mass.
- The strength anisotropy can also have a substantial impact on strength properties of both intact rock and rock mass specimens. In anisotropic rocks, the compressive strength of the rock mass or the intact rock can vary by a factor of 6 or more depending upon the direction of loading. Despite this considerable

effect, many practitioners still use isotropic constitutive models and often choose the average strength value from laboratory test results. This will often lead to overestimation of stability conditions in certain geometrical conditions, and underestimation in other conditions. Therefore an arbitrary parameter like the *D* factor is necessary in certain conditions to reproduce the observed failure in anisotropic rocks. The strength anisotropy is explicitly included in the IUCM.

- The scale effect is also another important factor that impacts both the intact rock strength and the rock mass strength. Hoek and Brown [32] recommend that the Hoek-Brown criterion should only be applied in conditions where the intact block size is relatively small compared to the size of the rock mass being analysed. This implies that the Hoek-Brown criterion only accounts partially for the size effect. In situations where the size effect is not appropriately reflected, a reduction factor, such as the *D* factor, would be required to downgrade the strength parameters accordingly.
- Rock mass variability is also another factor that can largely influence the overall rock mass strength, particularly in larger scale problems like open pit mining or caving operations. The work of Jefferies et al. [39] demonstrated that the stability of the rock slopes is often controlled by the weakest portions of the heterogeneous rock mass. Therefore, the conventional methods, which apply the median or average rock mass conditions, can overestimate the overall strength of the rock mass. This is, perhaps, another factor contributing to the need for a *D* factor in the traditional ways of analysis.

Considering the above discussions, and based on previous applications of the IUCM in several numerical back-analysis studies, the use of a *D* factor is not recommended when using the IUCM in numerical modelling studies. This is mainly because some of the above factors (e.g. strain-softening, anisotropy, etc.) are explicitly accounted for in the IUCM, and the remaining factors (e.g. scale effects, rock mass variability, etc.) are accounted for during the input parameter selection process (e.g. downgrading of UCS or GSI values).

3.5. Post-peak material properties

In the IUCM, a linear Mohr-Coulomb criterion is used for the residual state of the rock mass. There is limited information available on residual properties of rock masses.

In the IUCM, zero residual cohesion and tensile values, and a friction angle between 35° and 55° are recommended. These recommended values are based on:

- Work carried out by Barton and Kjærnsli [5] and Aghaei Araei et al. [3].
- Recommendations provided by Lorig and Varona [42], Cai et al. [14] and others.
- Several numerical back-analysis studies completed during the development of the IUCM (e.g. [70]).

In all studies carried out for validation of the IUCM (see Section 4), zero cohesion, zero tension and a friction angle of 45° were used. Therefore, these values are currently assigned in the IUCM as default values.

3.6. Additional input parameters for anisotropic rocks

The IUCM requires four additional input parameters when analysing anisotropic rocks. These are listed in Table 3.1. A detailed discussion on how to assign the anisotropic input parameters, when using the IUCM, is provided by Vakili et al. [67].

It should be noted that anisotropic behaviour is often overlooked during data collection or in laboratory tests. In fact, much of the well-accepted literature on rock mechanics often recommends the use of isotropic behaviour with an average or lower bound of the rock strength properties, instead of explicitly accounting for the anisotropic behaviour. Vakili et al. [67] demonstrated that these conventional approaches can lead to significant errors, particularly in more challenging ground conditions.

The guidelines provided by Vakili et al. [67] should be followed when selecting and preparing core samples for laboratory testing and also when carrying out the laboratory tests and interpreting the results.

Ideally, the m_{iMin} parameter should be obtained from triaxial tests conducted on samples with β angles (the orientation of the anisotropic fabric element with respect to the specimen loading axis) of between 30° and 45° (direction with lowest strength). However, in the absence of this information, the ratio of tensile strength to UCS (both at their lowest value based on the β angle) can be used to estimate the m_{iMin} . As a rule of thumb, if none of the above methods could be used, a m_{iMin} equal to half of the m_{iMax} can be assigned.

The anisotropy factor (ratio of $\sigma_{c max}$ to $\sigma_{c min}$) should be obtained from the results of laboratory UCS tests on samples with varying β angles. This ratio can be estimated from the U shape graphs of UCS versus β angles. In the absence of this information, Table 3.2 can be used to assign an indicative anisotropy factor based on typical foliation intensity present in the intact rock specimens.

$Aniso_{Dip}$ and $Aniso_{DipD}$ parameters are the dip and dip direction of the plane of weakness (anisotropy). Unless folding is explicitly

represented in the model, the average values of dip and dip direction from mapping or core logging should be used.

4. Verification and qualitative validation of the IUCM

Development of the IUCM took place over a period of eight years. The individual components of the model were gradually added to the model as a result of new information obtained from several mining-related case histories and numerical back-analyses. In its current form, the model has been successfully validated in several mining operations to back-analyse various geotechnical case histories. Owing to confidentiality matters, many of these case histories cannot be published in the public domain, and also, a more detailed inclusion of all case histories is outside the scope of this paper and will be included in future publications. Future publications will include more detailed and quantitative validation of the IUCM.

Vakili et al. [67], Mahabadi et al. [44], and Watson et al. [72] outline case histories in which the IUCM was successfully employed.

A number of case examples are briefly described in this section, where the accuracy and predictability of the IUCM were validated against known rock mass responses. The IUCM is currently implemented in Itasca's FLAC3D as a FISH function and a C++ DLL plugin and also as a FISH function in Itasca's 3DEC. Analyses presented in this paper were all completed using the FLAC3D code.

In the validation studies presented in this paper, dilatational volumetric strain model output is mainly used to represent the damage severity. Volumetric strain is the unit change in volume due to a deformation ($\Delta V/V_0$) and is calculated by the sum of the major, minor, and intermediate principal strain components ($\varepsilon_1 + \varepsilon_2 + \varepsilon_3$). A negative volumetric strain implies a contraction of the rock mass and a positive value indicates dilation. Contraction occurs at high confinement levels, while dilation happens in lower confinement zones, such as near the boundary of an excavation. The rock mass damage is most often controlled by a dilatational volumetric strain induced at low confinement levels. However, in cases of high confinement levels, rock can still become highly damaged and pulverised, while undergoing contraction (negative volumetric strain). Therefore, it is always important that in continuum models, other parameters like velocity and displacement are monitored for interpretation of results. Vakili et al. [67] provided a visual representation of various volumetric strains with respect to degree of disintegration as simulated in a fully discontinuum model through modelling of uniaxial loading of a rock sample, as shown in Fig. 4.1.

Table 3.2

Classification of anisotropy intensity in various rocks (modified from Tsidzi [65,66], 1990; Singh et al. [63]; Ramamurthy et al. [54]; and Palmström [52]).

Anisotropy classification	Example rock types	Anisotropy factor
Isotropic	Quartzite, hornfels, granulite	1–1.1
Low anisotropy	Quartzofeltspathic gneiss, mylonite, migmatite, shales	1.1–2.0
Medium anisotropy	Schistose gneiss, quartz schist	2.0–4.0
High anisotropy	Mica schist, hornblende schist	4.0–6.0
Very high anisotropy	Slate, phyllite	>6.0

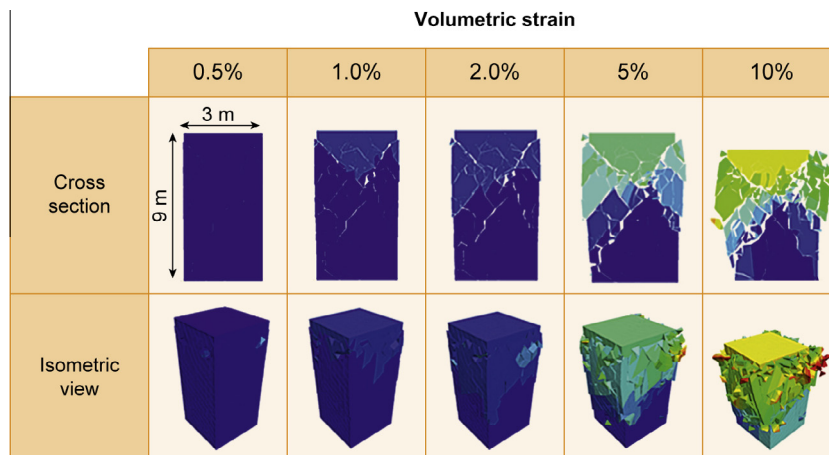


Fig. 4.1. Visual representation of degree of rock disintegration at various levels of volumetric strain (after Vakili et al. [67]).

4.1. Simulated triaxial testing

Simulation of triaxial testing is often used to validate the accuracy of a constitutive model. A series of simulated tests were completed using a $6\text{ m} \times 6\text{ m} \times 6\text{ m}$ cubic sample to confirm that different components of the IUCM function appropriately.

Fig. 4.2 shows the results of a simulated triaxial test on an intact brittle hard rock sample. Results are presented in principal stress space, a stress-strain curve, and a volumetric strain versus axial strain curve. In addition, visual plots of displacement and volumetric strain are presented to demonstrate the condition of the sample after failure. The model outputs correlate well with the expected response of a brittle intact rock sample, where axial type failure occurs at low confinement pressures. At intermediate confinement pressures, shear type failure is evident from the model results. At very high confining pressures, the sample behaves in a ductile manner and shows a uniform volumetric strain throughout the sample implying a pulverised condition. These results correlate

well with the findings of Bewick et al. [10] who conducted several triaxial tests on various rock types.

Fig. 4.3 shows the results of a simulated triaxial test on a jointed rock mass with a smaller $m_{i\text{Max}}$ value. The model outputs show similar trends at intermediate and high confinement levels, where shear type failure occurs at intermediate confinement pressures and ductile plastic flow at high confinement pressures. However, at low confinement pressures, the model outputs depict multiple shearing throughout the sample with formation of several shear planes. This behaviour correlates well with observations of Tiwari and Rao [64] who conducted a number of triaxial and true triaxial tests on physical models of a rock mass comprising continuous joints.

Fig. 4.4 shows the results of a simulated triaxial test on an anisotropic and jointed rock mass. The model outputs are in agreement with analytical relationships provided by Hoek and Brown [31] and laboratory tests carried out by Donath [25] and McLamore and Gary [50].

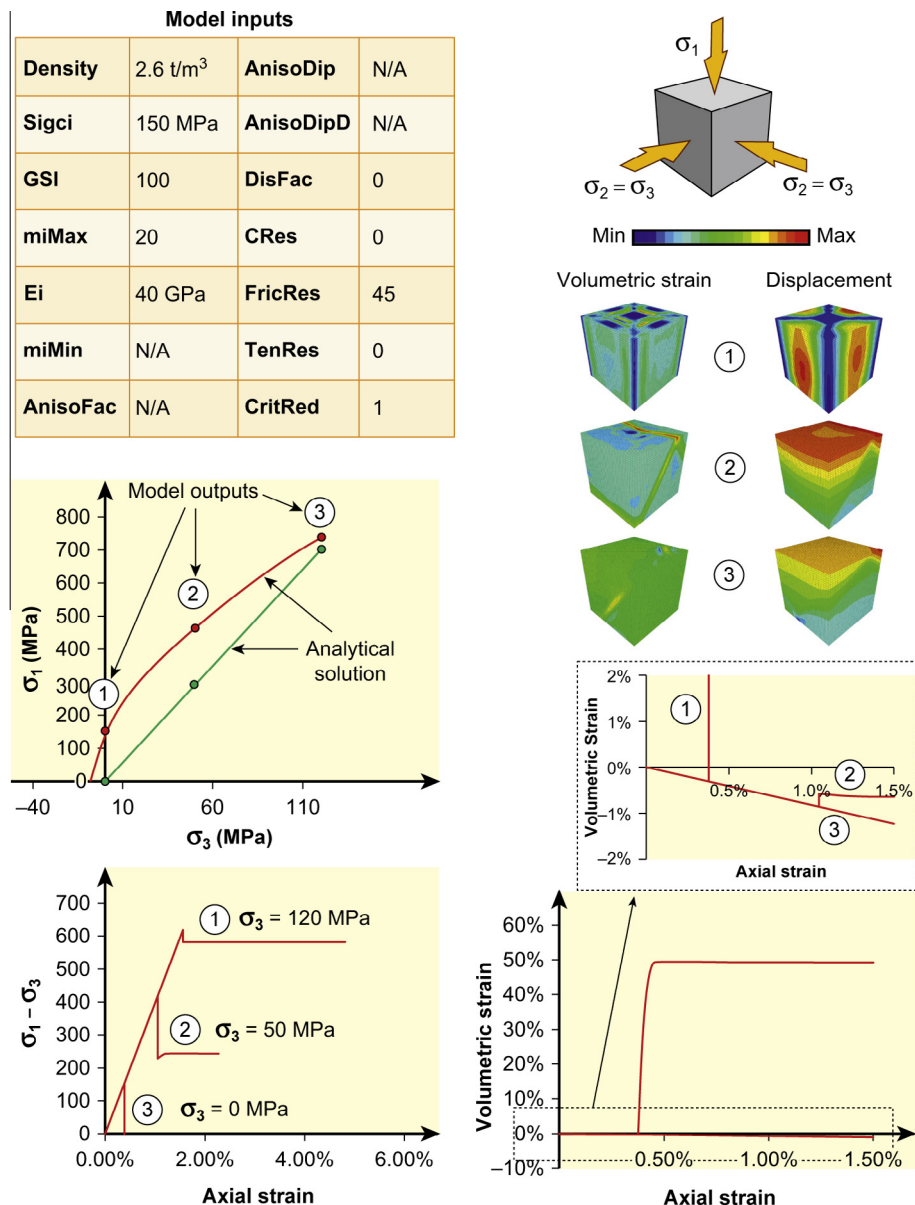


Fig. 4.2. Simulated triaxial test on an intact brittle hard rock sample.

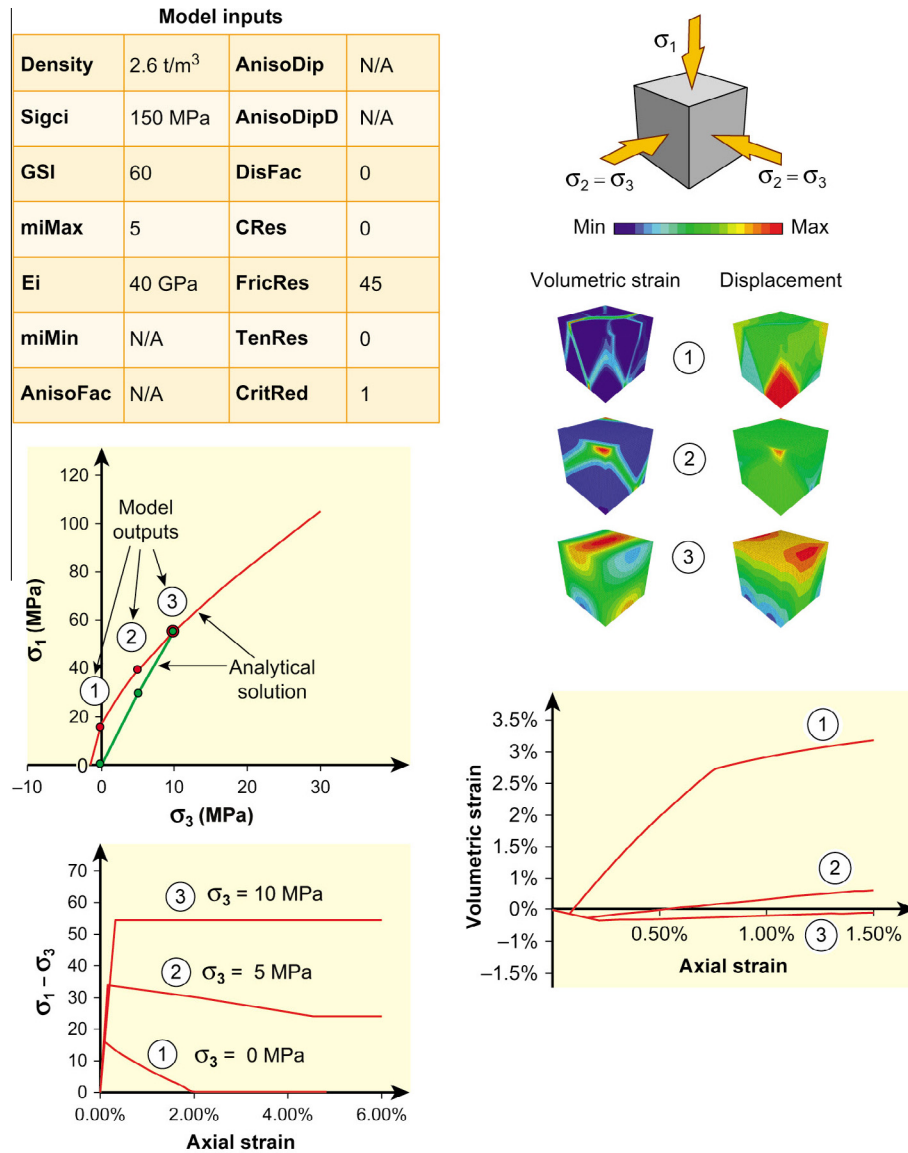


Fig. 4.3. Simulated triaxial test on a jointed rock mass with smaller m_{iMax} value.

4.2. Back-analysis of observed damage in a massive brittle rock mass

The Mine-by-Experiment project completed by Atomic Energy of Canada Ltd. (AECL) has been the subject of several research efforts attempting to better understand the failure mechanism of massive brittle rock masses. A test tunnel 46 m long and 3.5 m in diameter, excavated as part of this project, was used most frequently in the various studies. Read [55] and Martin [48] documented more details about the project. Works conducted by Martin [48], Martin et al. [49], Diederichs [22], Diederichs et al. [24], and Hajiabdomajid et al. [28] demonstrated the limitations and errors associated with the application of conventional analysis techniques and constitutive models in predicting the complex mechanisms involved in the brittle intact rock failure. The conclusion of those studies was that conventional techniques cannot reproduce the failure mechanism that was observed in real life.

Diederichs [22] proposed an empirical criterion, commonly known as the S-shape criterion, based on the concept of an empirically driven damage threshold (30–50% of the intact UCS) and a spalling limit (σ_1/σ_3 varying from 10 to 20). Hajiabdomajid et al. [28] introduced another modelling approach, called cohesion

weakening and frictional strengthening (CWFS), which was based on the concept of cohesion loss and delayed frictional mobilisation. Both of these criteria were shown to capture the notch formation around the AECL's Mine-by-Experiment tunnel. However, for more ductile rock mass conditions, such as those present in moderately to highly jointed rock masses or soft rock, the proposed criteria are generally not suitable to predict the correct failure mechanism, and are therefore unsatisfactory to be used as a comprehensive failure criterion/constitutive model.

The studies that led to development of the proposed criteria mainly used elastic boundary elements or implicit finite element codes to back-analyse the AECL case history. It is well understood that the elastic codes are not able to replicate plasticity and the subsequent changes to excavation induced stresses. Implicit finite element codes also have the limitation of not being able to follow the development of a failure, or in other words, the progressive damage process of a system.

It was well documented that the brittle damage and the resulting V-shaped notches evolved gradually in a progressive and time-dependent manner. In this type of failure, the initial yield within the rock mass gradually initiates new phases of confinement, and

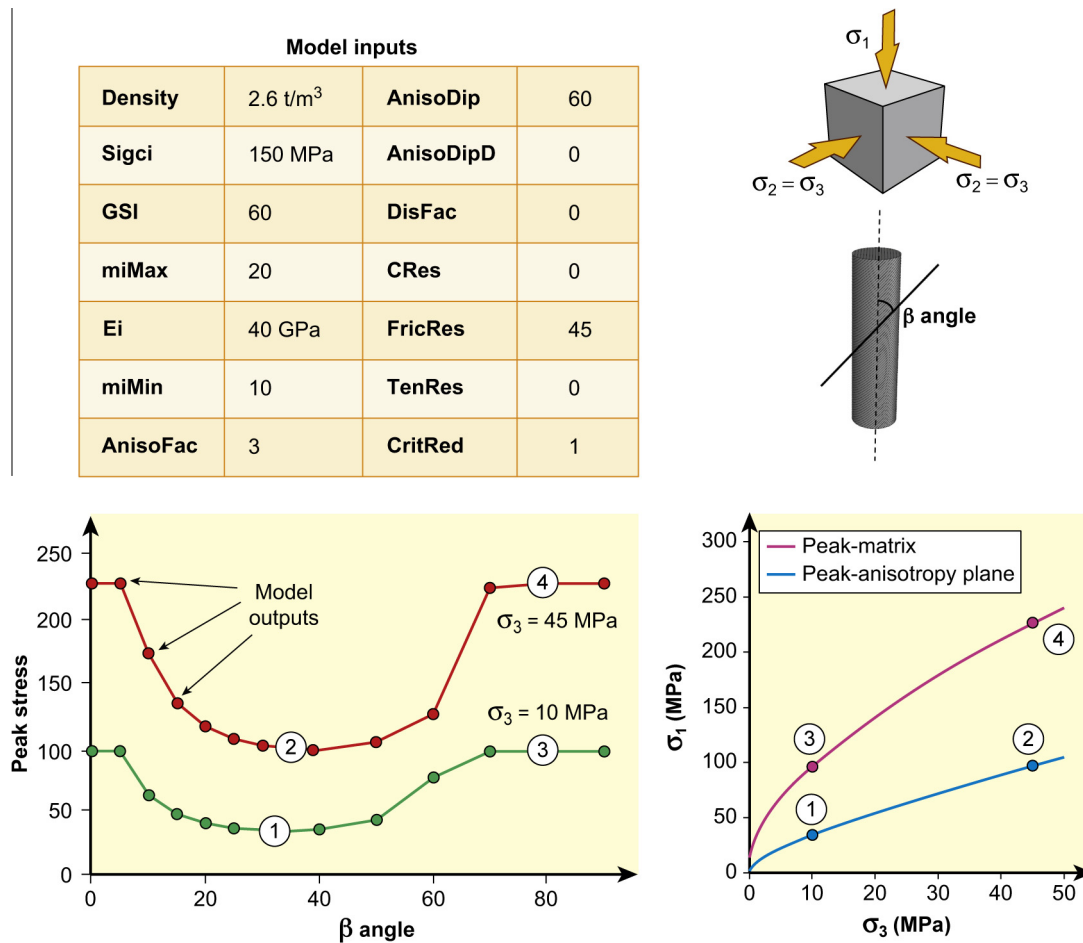


Fig. 4.4. Simulated triaxial test on an anisotropic and jointed rock mass.

therefore, new phases of yield and failure can also be initiated. This response generally cannot be replicated in the commercially available numerical modelling codes used in the above studies.

It is also important to know that a reliable constitutive model should be able to replicate multiple aspects of rock damage, from depth and extent of fracturing (yield) to displacement, velocity, and strains. Displacement in particular is a key factor when using the numerical models for ground support design. Extent of the fracturing zone or yield would not necessarily represent the actual ground fall-off or over-break, and depending on new phases of stresses, a yielded material can undergo further strain (damage) or it may remain unchanged and with no additional crushing. As a result, strain is generally a more realistic indicator of damage severity as was shown in this paper. The work conducted by Hajiabdolmajid et al. [28] showed that using the newly developed constitutive model, the location and geometry of the failed notch correlated well with the yielded elements in the model; however, very small plastic strains were reported from the model. This highlights the fact that the proposed constitutive model is not able to sufficiently follow the material damage progression in its post-failure state. Although this might not be critical in simple examples like the AECL case, where a single excavation with simple geometry exists, but in more complex problems this issue could potentially lead to more errors. For example, when dealing with multiple nearby excavations with complex geometries, in caving conditions, or in yielding pillars, the global failure mechanism is heavily reliant on the progressive damage process.

The Mine-by-Experiment tunnel case was used as a case example to validate the IUCM for predicting the failure mechanism in a

massive brittle rock mass. As shown in Fig. 4.5, the model replicates the observed failure mechanism reasonably well. All key input parameters adopted in the model are within the ranges provided by Read [55] and Martin [48]. Note that UCS was downgraded by approximately 50% from its original laboratory test result to account for the scale-effect explained earlier in this paper. Further, the m_{iMax} value was chosen according to the long-term m_i values reported by Read [55] and Martin [48].

This experiment not only confirms the applicability of the IUCM for brittle and massive rocks but also shows that as long as all important features of rock mass failure, such as confinement-dependent softening, correct dilatational response, and suitable transition from peak to residual are accounted for in the constitutive model, there will be no need to introduce any new failure criterion.

4.3. Back-analysis of observed damage in an anisotropic, highly jointed, and ductile rock mass

The complex buckling type failure mechanism associated with a vertical shaft was the subject of another case study, which was rigorously investigated throughout the development of the IUCM. The subject shaft was excavated using a raise-boring technique at an approximate underground depth of 1500 m. The rock mass at the site was highly foliated and anisotropic with a high degree of jointing.

As reported by Vakili et al. [70] and Vakili et al. [67], shortly after excavation, significant buckling was observed in the east and west walls of the shaft with around 1 m of initial breakout.

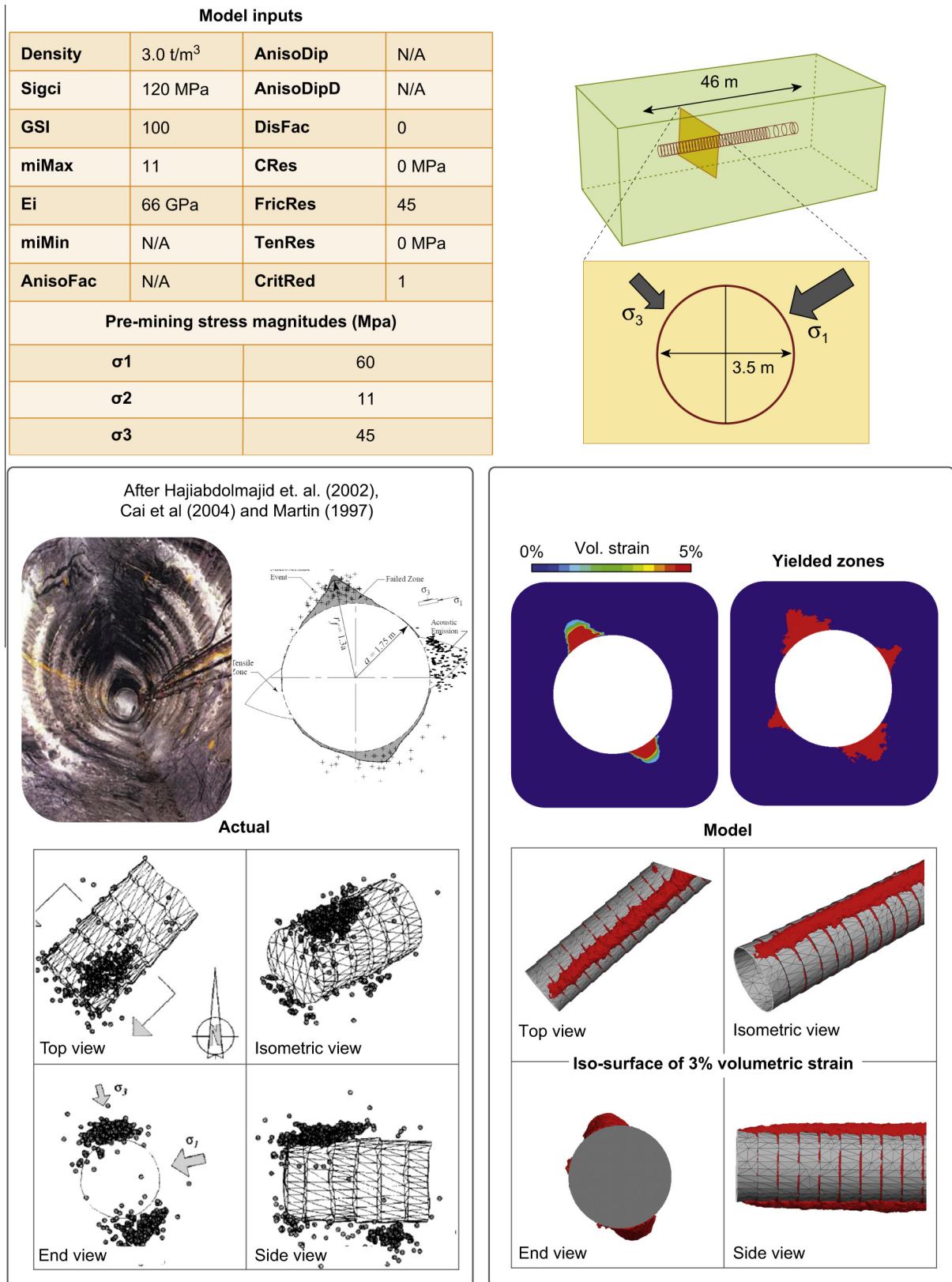


Fig. 4.5. Validation of the IUCM for modelling of a massive brittle rock mass using the Mine-by-Experiment case history.

Ultimately, after approximately 6 months of shaft excavation, the east and west walls experienced up to 2.5 m of breakout with a final ellipse-like shape.

Vakili et al. [70] and Vakili et al. [68] showed that only discontinuum models were able to reproduce the complex failure mechanism observed at the site; continuum models using the

commercially available constitutive models were not able to reproduce the observation with a sufficient level of accuracy.

Fig. 4.6 shows the comparison between actual observations and model results using the IUCM. The input parameters used were mostly derived from geotechnical logging data, structural mapping and laboratory testing with some minor adjustments to obtain a better match with the observations.

The IUCM, using similar input parameters, was also used at this site to back-analyse the response of the rock mass in a development drive that was carefully monitored using extensometers. As shown by Vakili et al. [67], a very good correlation was also

observed between model and actual observations in terms of location of failure, mechanism of failure, depth of failure, and monitored displacement.

4.4. Back-analysis of observed damage in a deep underground open stope

The rock mass damage observed in the hanging wall of an underground open stope excavation was back-analysed using the IUCM (see [67]). The subject excavation was found to be a good case for validation of the IUCM for the following reasons:

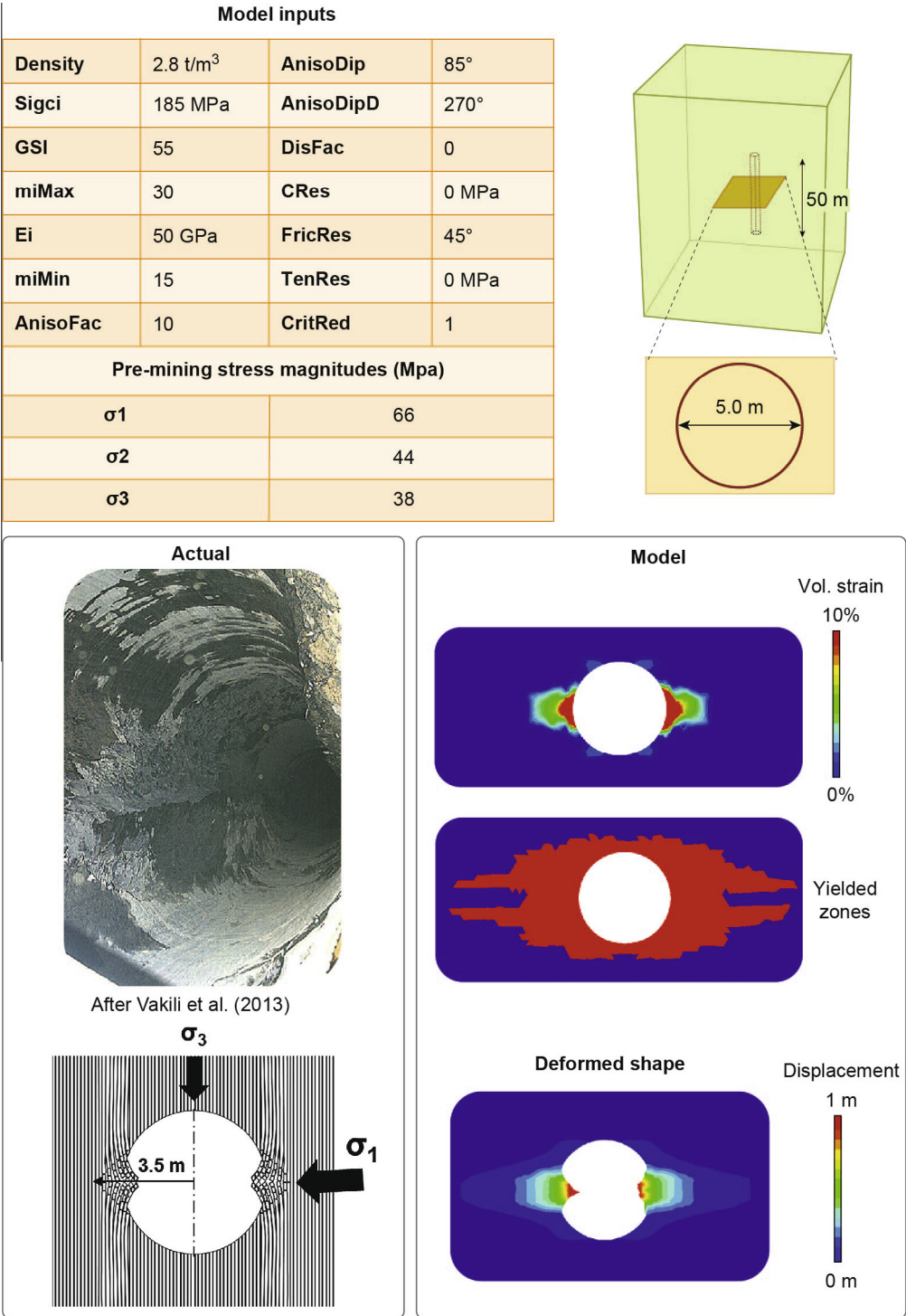


Fig. 4.6. Validation of the IUCM for modelling observed damage in an anisotropic, highly jointed and ductile rock mass.

- The subject slope was excavated in a relatively isolated location from other mining excavations, ensuring that no mining induced stresses affected the observed failure mechanism.
- The observed failure was mainly stress-induced and no major influence from the local structures was evident.
- The ground condition under study was high stress and experiencing both massive brittle hard rock and strength anisotropy associated with the foliated fabric in the rock mass. The combination of all these factors led to significant complexity associated with this rock mass, which makes it a good case for validation of the IUCM.

The mining depth was approximately 1500 m and the slope was excavated by blasting in three separate firings. No major failure was observed until the final firing took place. Significant hanging wall overbreak (to a depth of up to 17 m) was recorded.

As demonstrated by Vakili et al. [67], the isotropic strain-softening Mohr-Coulomb constitutive model, which is most commonly used by practitioners, was not able to replicate the complex failure mechanism observed for this slope.

As shown in Fig. 4.7, a close correlation between model and actual observations existed when using the IUCM constitutive model. The majority of the applied input parameters were within

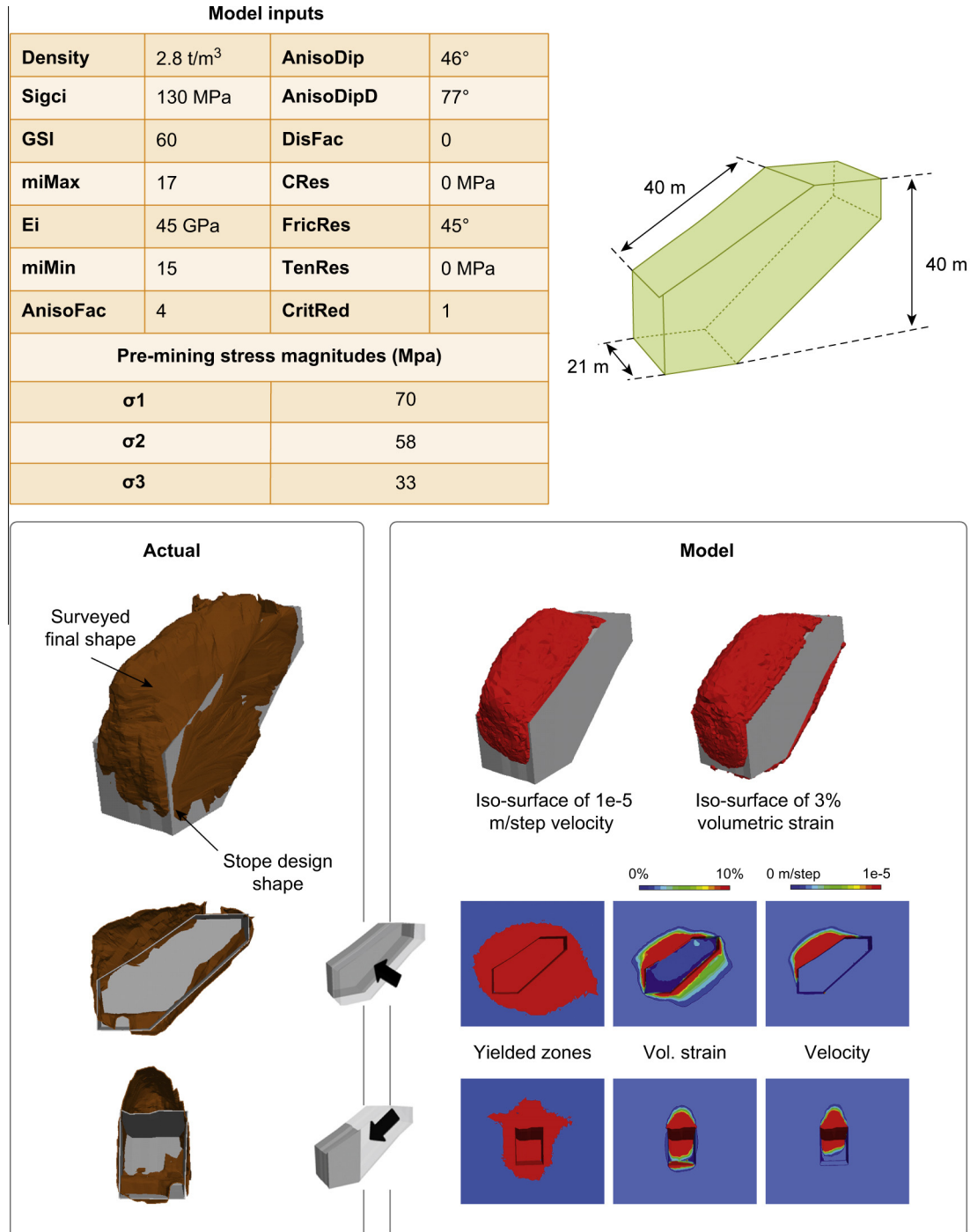


Fig. 4.7. Validation of the IUCM for modelling of observed damage in a deep underground open slope.

the ranges obtained from laboratory testing, underground mapping, and geotechnical logging. Minor adjustments were made to obtain a better match with the observations. These adjustments and more detailed descriptions about this case are outlined by Vakili et al. [67].

4.5. Pillar stability assessment of massive brittle rock mass

Pillar failure is perhaps one of the most complicated mechanisms to simulate. The effect of confinement on post-peak response and particularly the transition from brittle to ductile behaviour and also the effect of stiffness can play a significant role on the stability of a pillar and the ability to model it.

Several authors have investigated the application of numerical modelling for assessment of pillar stability, and highlighted the difficulties associated with representing the complex failure mechanism through a continuum constitutive model. The most extensive database of hard-rock pillar failures was compiled by Lunder and Pakalnis [43] who analysed 178 case histories. This database was used in several studies to compare and validate the suitability of the modelling techniques for simulating the previously observed case histories.

Martin and Maybee [47] demonstrated that application of the Hoek-Brown parameters in the continuum Phase2 code was not able to produce a good correlation with the Pillar Stability Graphs developed by Lunder and Pakalnis. Therefore, they proposed the use of modified Hoek-Brown parameters in accordance with the concept of damage threshold and spalling limits described earlier in this paper. Esterhuizen [27] used a similar method and showed that a closer match to the Lunder and Pakalnis graph could be achieved when the proposed criterion was used.

Fig. 4.8 shows the results of numerical analyses completed using the IUCM, and the same model geometries and model resolutions applied by Esterhuizen [27]. A close match was obtained between model results and the Pillar Stability Graph developed by Lunder and Pakalnis. The calibrated input parameters were all within the ranges provided by Martin and Maybee [47] and Esterhuizen [27] as typical rock mass properties for Canadian hard rock mines. The only parameter that needed adjustment was the critical strain reduction factor ($Crit_{Red}$), which was changed to 6, suggesting that a more brittle response was required from that recommended by the Lorig and Varona [42] equation.

A comparison between the model output and actual typical pillar appearance at various loading stages is also depicted in Fig. 4.8. This comparison shows that the model can also replicate the gradual deterioration of the pillar before reaching its residual state.

This case study again shows that the conventional Hoek-Brown parameters are suitable for a wide range of rock masses, as long as they are accompanied by suitable post-peak criteria and the appropriate numerical code is applied.

5. Current limitations and assumptions of the IUCM

The IUCM was developed with the intention of providing a unified constitutive model that has the complexities required for a wide range of geotechnical applications, but is also simple enough to be used by most geotechnical practitioners. In addition, the aim was to establish a constitutive model that could be used as a standard model and avoid having to apply non-consistent constitutive models. Having a consistent constitutive model and a consistent procedure for selection of input parameters can help when reviewing numerical modelling studies conducted by third parties, as well as establishing consistency when interpreting modelling results and enabling regeneration of modelling results when modelling is carried out by others. Finally, it also highlights areas of current

limitation, and focuses future research towards improving the IUCM.

It is important to identify the known limitations of the IUCM. There are certainly other limitations that might not be well-known at this stage, and future work and research are required to identify them. Some of these limitations will be addressed as the development of the IUCM progresses. The current known limitations and assumptions of the IUCM are listed below:

- The IUCM currently does not account for anisotropy of the elastic modulus and only accounts for anisotropic behaviour of the rock mass in the plasticity calculations. If future validation works suggest a considerable impact on model outputs, this feature will be added to the model.
- In the IUCM, the strength anisotropy feature is only applicable for rock types that have the 'intact rock anisotropy' characteristic, and therefore exhibit directional dependency at both small and large scales. The IUCM might not provide an accurate estimate of the rock mass response in layered rock where bending of individual layers is expected. This situation often occurs in conditions where intact rock assumes isotropic behaviour, but the rock mass contains a single well defined joint set that controls the shearing behaviour of the rock mass. In this case, the shearing occurs in a direction not parallel to the layering and it is largely controlled by the spacing of the dominant joint set. This limitation of the ubiquitous joint models is further discussed by Adhikary [1], and Cundall and Fairhurst [18]. This effect becomes more pronounced as the spacing of the dominant joint set increases. In these situations, explicit inclusion of the dominant joint set in the model is recommended.
- The equations for dilation angle calculations proposed by Alejano and Alonso [2] were derived based on laboratory testing on intact rock samples. In the IUCM, it is assumed that a similar equation is applicable for the rock mass dilation angle given that the intact UCS and the friction angle values used in this equation are downgraded to their rock mass values accordingly.
- The dilation angle logic for the plane of anisotropy follows a similar logic to that of the rock matrix using the equation proposed by Alejano and Alonso [2]. Considering the nature of the fabrics constituting anisotropic rocks, it might be more appropriate to use the dilation angle equations developed specifically for filled/healed discontinuities, such as those proposed by Barton and Choubey [4].
- The majority of the back-analyses and validation studies completed during the development of the IUCM used mining case histories dealing with small to medium underground excavations including development drives/tunnels, vertical shafts, pillars, and open stopes. Therefore, use of the IUCM for other applications such as open pit slope stability, subsidence assessment, and caveability assessment should be accompanied by the relevant model calibration and back analysis studies before results can be used for design purposes.
- In the IUCM, the effect of small-scale rock mass discontinuities is accounted for implicitly through the Hoek-Brown criterion and use of the GSI value. However, the IUCM does not account for intermediate to large-scale discontinuities within the rock mass. These discontinuities need to be modelled explicitly if their characteristics are known.
- The majority of applications, validations, and back-analysis studies carried out using the IUCM were for rock masses with GSI values ranging between 40 and 100. Use of the IUCM for rock masses with GSI values less than 40, and intact UCS below 15 MPa, requires careful consideration and model calibration. In general, the IUCM is developed as a constitutive model for rock masses and should not be applied for soil-like materials.

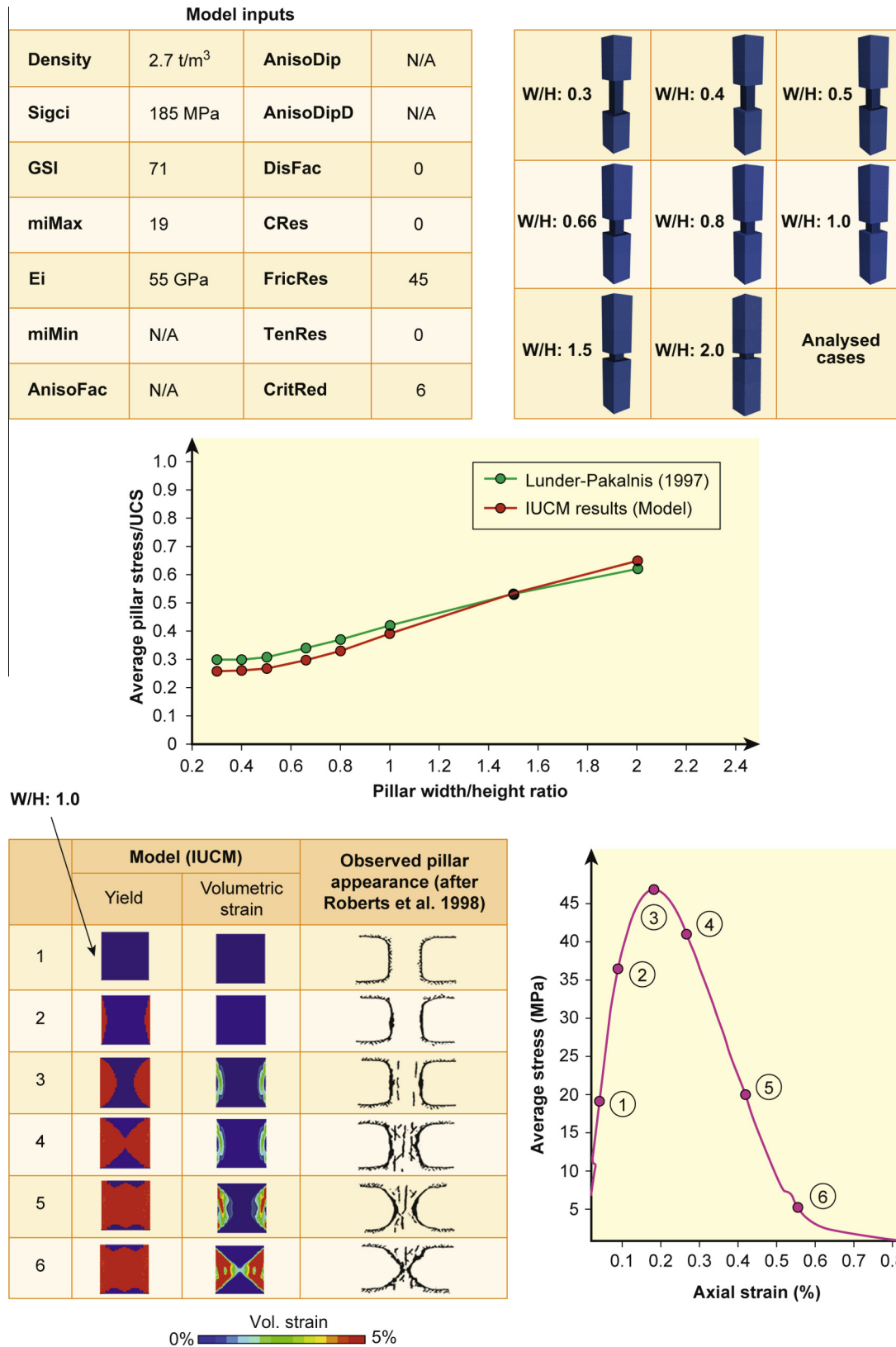


Fig. 4.8. Validation of the IUCM for modelling mining pillars in massive brittle rock masses.

- The IUCM currently relies on a number of empirical correlations (for example, Eqs. (18) and (19)) to establish some important parameters. Some of these parameters are still difficult to accept as being universally applicable. However there are

presently no better alternatives for these equations and validation work completed so far suggest they provide a reasonable estimates for some key parameters that need to be taken into account.

6. Summary

This paper has outlined the theoretical backgrounds, guidelines for selection of the inputs, validation, and limitations/assumptions for an improved unified constitutive model (IUCM).

The IUCM gathers the most notable and widely accepted previous research work in the area of rock mechanics and integrates them into a unified constitutive model that can better and more accurately predict the stress-strain relationships in a continuum model. Several back-analyses and validation studies completed during development of this model confirmed its applicability for a wide range of ground conditions, from intact brittle hard rock to heavily jointed and ductile rock, and also anisotropic rock.

In developing this model, the use of any new theory or technique was avoided where possible, and all the adopted processes were based on well-accepted and widely applied rock mechanics techniques and theories. A key conclusion of this paper is that if the key controlling mechanisms of the rock damage process that have been well-researched and understood over the past years are included in a model that uses a unified approach, then the rock damage process can be predicted much more accurately, without the need for introducing any new theories or criteria.

The key components and features of the IUCM are:

- For the peak failure envelope of the rock matrix, the IUCM uses the generalised Hoek–Brown [35] failure criterion to determine the instantaneous Mohr–Coulomb parameters (C and Φ) at each level of confining stress (± 0.001 MPa tolerance). These instantaneous parameters are updated in real-time as the model runs and as new phases of confinement are formed from nearby damage or geometrical changes.
- For the residual state of the rock matrix, the IUCM assigns a linear Mohr–Coulomb envelope. Properties of completely broken and crushed rock are applied by default for the residual state of the material using cohesion and tensile strength values of 0, and a friction angle of 45° .
- The critical strain in this model is chosen based on equations suggested by Lorig and Varona [42]. In this method, critical strain values are determined based on model zone size and the GSI value of each rock unit.
- The dilation angle in this model is determined using the relationship proposed by Alejano and Alonso [2]. This relationship gives an estimate of the peak dilation angle based on confinement, friction angle, and UCS. The dependency of this relationship on confinement levels results in higher dilation angles at lower confinements, and lower dilation angles at higher confinements. Additionally, it causes a degradation of dilation angle with increasing damage, and ultimately the dilation angle is reduced to zero when the maximum porosity of 40% is reached in the model zones. This behaviour is similar to that observed in laboratory rock testing results.
- The IUCM accounts for the confinement dependency of the rock damage process at various stages from peak to residual. Confinement dependency is an important factor controlling rock damage and seismicity in high stress conditions; however, it is largely ignored in most conventional constitutive models. In the IUCM, at low confinement levels, the linear nature of the residual envelope replicates cohesion and friction softening. At high confinement levels, it replicates cohesion softening and friction hardening.
- Most conventional constitutive models assume a constant modulus of elasticity for the rock mass, irrespective of its damage state. In real-life situations, when rock undergoes failure and continuous loading, more voids are generated within the rock mass. The rock mass porosity is subsequently increased. The greater the porosity in the rock mass, the smaller its elastic modulus. The drop in rock mass modulus can significantly affect the redistribution of stresses around a failed area and the

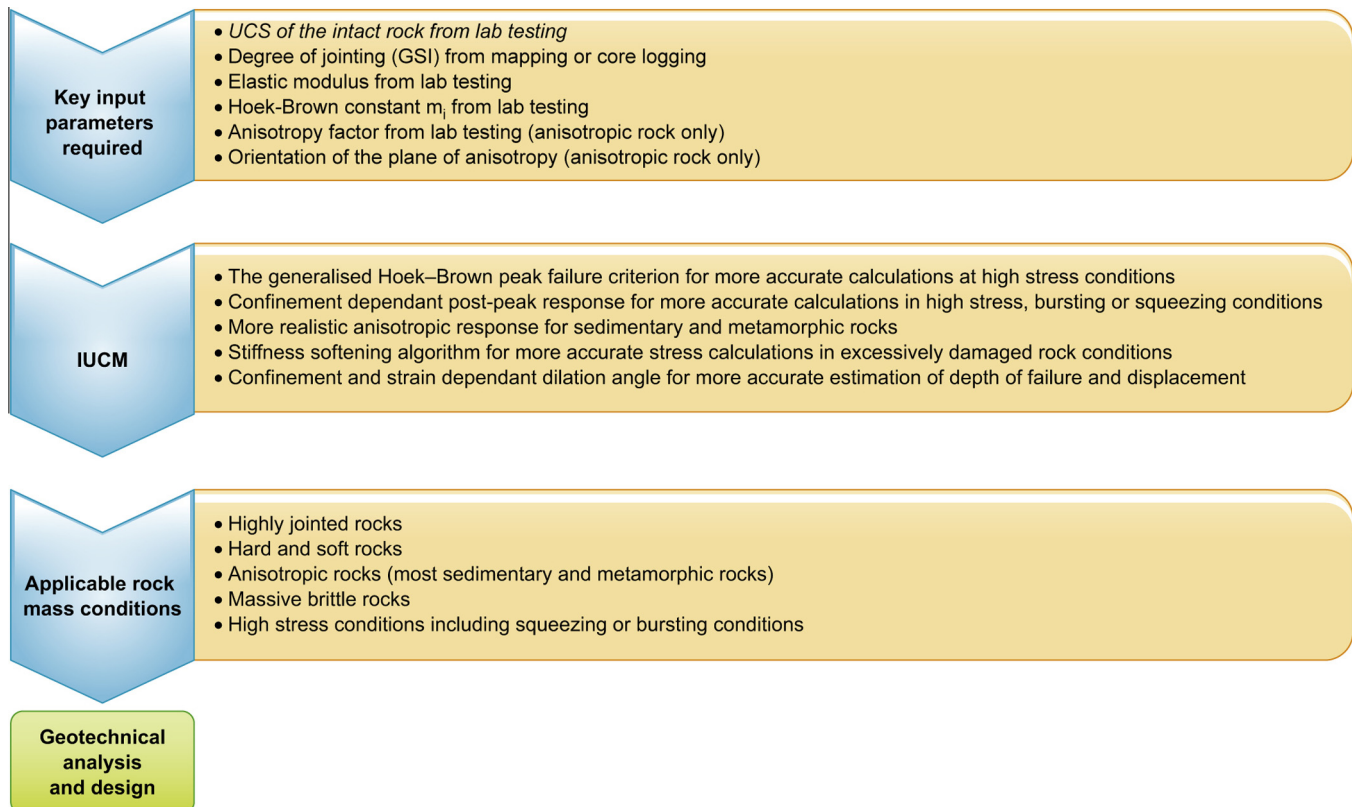


Fig. 6.1. A summary flowchart for the IUCM.

subsequent phases of induced confinement. The IUCM uses this relationship to update the elastic modulus values according to new porosity levels. The porosity is calculated using the model volumetric strain outputs.

- The strength anisotropy in the IUCM is explicitly included through a ubiquitous joint model, which accounts for both rock matrix strength and the lower strength associated with the existence of an anisotropy plane. For anisotropic rocks, the model uses two non-linear Hoek–Brown failure envelopes. One envelope defines the maximum strength and is related to the rock matrix strength. The other defines the minimum strength associated with the anisotropy plane.
- This constitutive model is implemented in the explicit finite difference code FLAC3D and in 3DEC, and therefore uses a time-stepping solution for calculations. As a result, progressive and time-dependent failures can be replicated in this model by updating the material properties as a function of new confinement and strain levels.

Fig. 6.1 shows a summary flowchart for this model.

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