STUDENT CURRICULUM

ACT

ESM PREP

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ENGLISH CURRICULUM

ACT

ESM PREP

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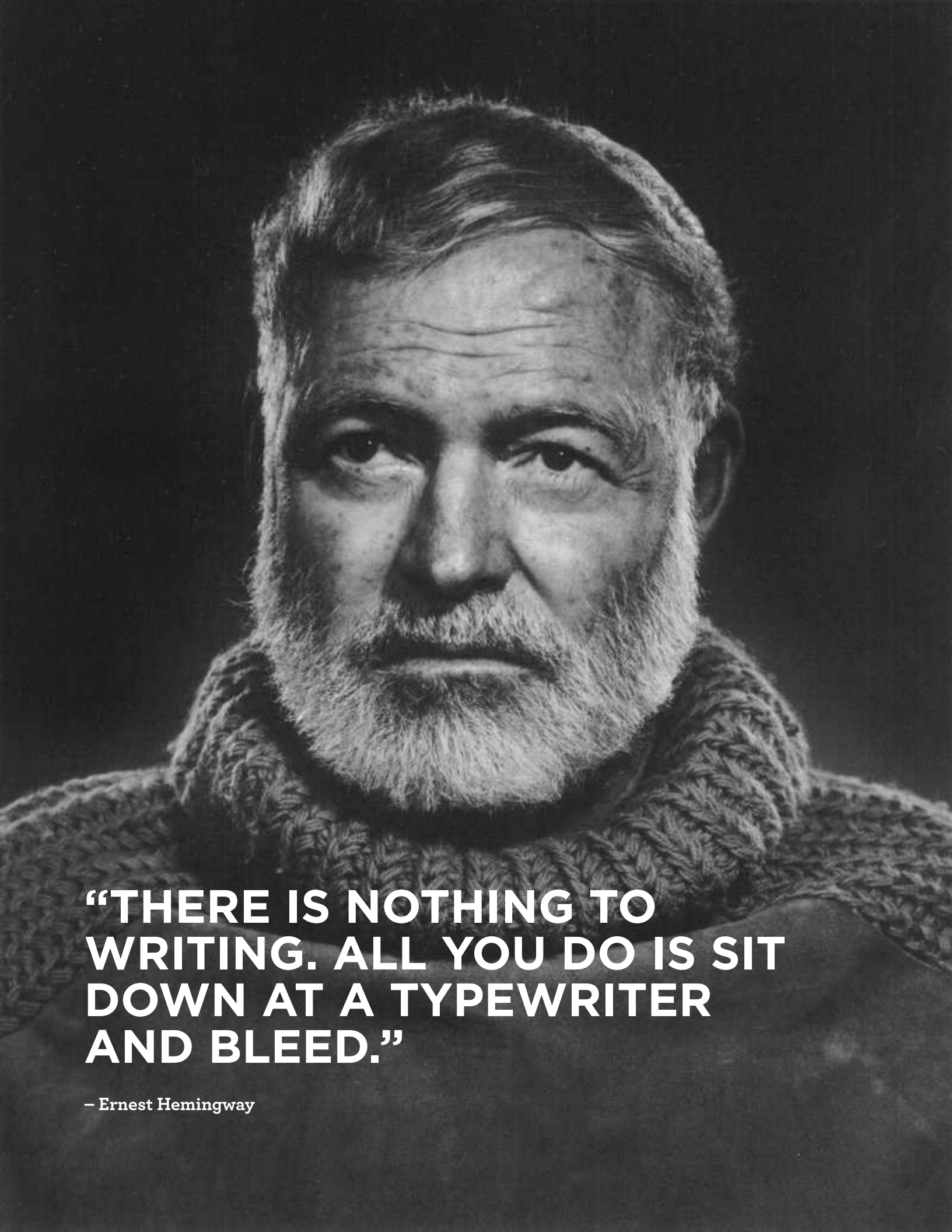

“THERE IS NOTHING TO WRITING. ALL YOU DO IS SIT DOWN AT A TYPEWRITER AND BLEED.”
– Ernest Hemingway

INTRODUCTION

The English Test is the first test of the ACT. Over the course of 45 minutes, you are tasked with answering 75 questions. While that may seem difficult, the English Test is generally considered to be the least time-sensitive test on the ACT. High school juniors and seniors who are strong readers (especially those who read often for fun) will usually score highly on this test, as the majority of answers will be obvious to students who read consistently. However, even the strongest readers will need to review grammar rules and familiarize themselves with the format if they want to ace the test.

The English test requires you to read five "passages" (essays or stories). Each passage contains fifteen questions, and each question falls into one of three categories:

Usage/
Mechanics

Rhetorical
Skills

Main Ideas /
Editing

IDENTIFYING PARTS OF SPEECH

There are eight parts of speech in the English language. In order to have success on the ACT English Test, you will need to be able to identify seven of them. This document contains a brief guide to locating parts of speech within a sentence. Notice that the sample questions here are not ACT English questions. They are simply questions designed to test your ability to recognize parts of speech, which is a fundamental skill needed on the ACT. This section provides background knowledge for those looking to start at square one in their ACT English Test preparation.

Beyond simply identifying the parts of speech, each of the following sections contains more advanced grammar rules that will also be crucial if you're hoping to score highly on the ACT English Test.

VERBS

Verbs are the words that explain what is happening in a sentence.

There are two main categories of verbs: action verbs and state of being verbs.

Examples of **action** verbs

RUN

Delivered

Spoke

Examples of **state of being** verbs

WAS

Is

Are

Verbs take different forms depending on their subject and their tense. For example, a verb that is being done by a singular subject (such as "she") will be slightly different from a verb that is being done by a plural subject (such as "they").

She **wants** to eat a grasshopper.
They **want** to be eaten by a grasshopper.

Verbs are the most commonly tested part of speech on the ACT English Test. The verbs "to be" and "to have" are tested with extra frequency. These verbs are irregular (meaning that their actual wording changes depending on their subject and tense), and thus many students have a harder time spotting when they have been used improperly.

Here is the proper wording for **to be**:

Singular	**Plural**
I am	We are
You are	You are (plural)
He/she/it is	They are

Here is the proper wording for **to have**:

Singular	**Plural**
I have	We have
You have	You have (plural)
He/she/it has	They have

In the following sentences, identify the verbs and their corresponding subjects. If one has been used improperly, identify its proper form.

1. I was eight feet tall before the Rapture.
2. That dog spoke to me in Hebrew.
3. The government are the most dangerous organization known to man.
4. Italian doctors from the ghetto often makes me recite the Declaration of Independence.
5. Banks don't have money for Teddy Bears.

Mentor Notes

"Some of the worst writing around suffers from inert verbs."

- Constance Hale

NOUNS

Nouns are people, places, ideas, objects, or gerunds (verbs with -ing at the end), and can always be preceded by “a”, “an”, or “the”.

Proper nouns are specific nouns that are always capitalized.

Subject Noun: is a noun that performs the action of the verb.
Object Noun: is a noun that receives the action of the verb.

Examples of **nouns**

a clown (person)
the tundra (place)
slavery (idea)
an eraser (object)
running (gerund)

Examples of **proper nouns**

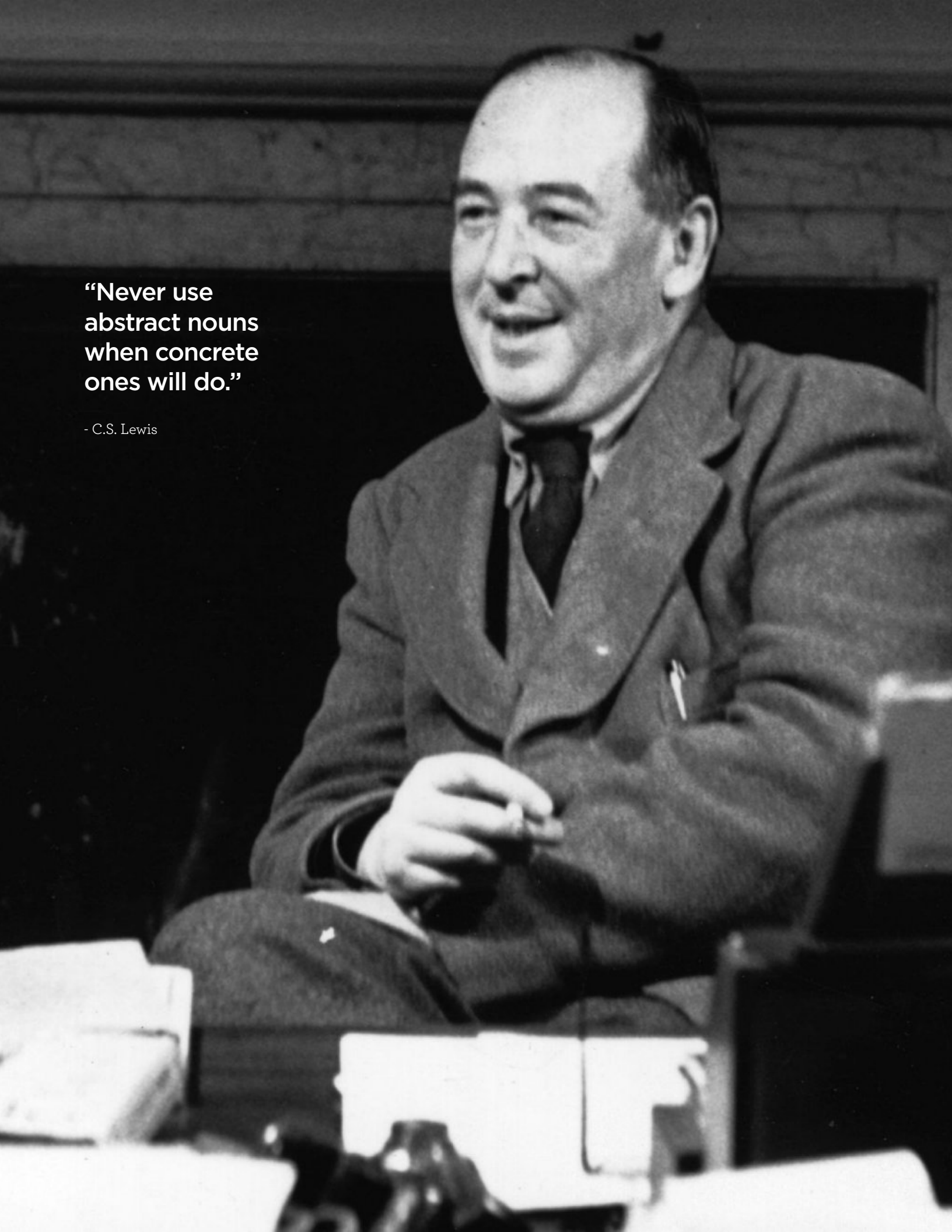
Ben Franklin (person)
Mars (place)
Communism (idea)
The Rosetta Stone (object)
Saving Private Ryan (gerund at the beginning of a title, and thus capitalized... yes, that’s us cheating.)

Nouns can be either the subject of the sentence or the object of the sentence. If a noun is doing something in the sentence, it is the subject; if something is being done to it, it is the object. Understanding subject and object is crucial for many different question types on the ACT English Test.

In the following sentences, identify all of the nouns, then identify which are subjects and which are objects:

1. Ducks shouldn’t swim in volcanoes.
2. That hat would look better on a chimpanzee than on you.
3. I don’t have any use for Communism now that I’m rich.
4. After the Elephant Scandal, South Africa decided to secede from the African Union.
5. Thomas Jefferson wrote many love letters, but only some were addressed to humans.

Mentor Notes


“Never use abstract nouns when concrete ones will do.”
- C.S. Lewis

PRONOUNS

Pronouns are words that take the place of nouns.

Subjective pronouns: used as the subject of the sentence or as a predicate noun.

Objective pronouns: is a personal pronoun that is used typically as a grammatical object.

Possessive pronouns: indicating possession, for example mine, yours, hers, theirs.

Common Singular Pronouns

I
you
he/she/it

Common Plural Pronouns

we
you
they

After verbs, pronouns are the second most commonly tested part of speech on the ACT English Test. Like verbs, they take different forms depending on whether they are replacing a singular noun or a plural noun.

Identify all of the pronouns in the following sentences. If one is used improperly, identify its proper form:

1. Eskimos are fearless. They attack anything with eyes.
2. My hamster is growing. Soon it will be bigger than you.
3. Ford is Ashley's favorite car company. She met their founder when she went back in time.
4. There is only one Batman. He reminds me of my grandmother.
5. For me, purple is an awful color. We shouldn't wear it.

As with nouns, pronouns can be either subjects or objects, and you must be able to tell the difference on the ACT English Test.

Examples of Subjective Pronouns

I	we
you	you
he, she, it	they

Examples of Objective Pronouns

me	us
you	you
him, her, it	them

Pronouns that show possession are called possessive pronouns. They can either be used as nouns that act alone or as possessive adjectives that describe a noun.

Mentor Notes

Examples of Possessive Pronouns (Nouns)

mine	we
yours	you
his, hers, its	they

Examples of Possessive Adjectives

mine	our
your	your
his, her, its	their

In the following sentences, identify the pronouns. If one is being used subjectively when it should be used objectively (or vice versa), correct it:

1. He likes to eat sloppy joes in her apartment.
2. They were howling at either us or the moon, though I'm not sure which.
3. The band came out late, but they really blew Alex and I away.
4. With whom do you want to look at monkeys?
5. Liza and I like to pickpocket tourists who have nice shoes.

PREPOSITIONS

Prepositions indicate where a noun is or when a noun happened.

The most common prepositions

Of
to
on
in
for
at
from
with
as
about
into
by
through
after
before.

On the ACT, prepositions will usually be tested in how they pair with a particular verb. You will have to study these pairings and then rely on your instincts when you don't recognize a pairing on the test. Here is a table with many of the most common preposition-verb pairings used on the ACT. Note that some words can properly be matched with several prepositions.

angry at
arrive at
stare at
accuse of
afraid of
approve of
capable of
consist of
approve of
hatred of
love of
opinion of
in charge of
ask for
apologize for
blame for
desire for
excuse for
hope for
need for
pray for
sorry for
abide by

absorbed in
engrossed in
believe in
interested in
participate in
succeed in
trust in
disintegrate into
able to
according to
agree to
attribute to
respond to
prefer [one thing]
to [another thing]
subscribe to
compare to
complain to [someone]
object to
equal to
encourage to
introduce to
regard as

hide from
differ from
protect from
escape from
recover from
different from
discourage from
discriminate against
insist upon
rely on
count on
depend on
argue with
familiar with
popular with
ask about
care about
complain about
happy about
talk about
think about

In the following sentences, identify the preposition. If it is incorrect, replace it with the correct preposition.

Mentor Notes

1. Sarah was accused of being too high maintenance.
2. Edgar insisted on throwing a house party.
3. Dante proved capable in writing a complete sentence.
4. Marco was discouraged in joining the hockey team.
5. You cannot escape against Emilia's dungeon.

Outside of testing for correct prepositional usage like in the sentence above, there's another common way that prepositions are involved in the ACT English Test: as a distraction. Consider the sentence:

The monkeys sit in the tree.

"In" is a preposition in this sentence, and it can be said that "in the tree" is a prepositional phrase in this sentence. On the ACT, prepositional phrases are often used to distract you from other errors in the sentence. Consider:

The monkeys in the tree likes when it rains.

The prepositional phrase is the same: "in the tree". However, it's now separating the subject of the sentence, "monkeys", from the verb in the sentence, "likes". And since "monkeys" is plural, the verb needs to be plural- "like" is correct. Some students may see "tree", a singular noun, and think that it is the subject of the sentence, which would make "likes" correct. But "tree" is part of the prepositional phrase, and a prepositional phrase can never be the subject of a sentence.

In the following sentences, identify the prepositional phrase, the subject, and the verb. If there is an error in the sentence, correct it. Don't be distracted by the prepositional phrases!

1. The rocks on the seashore is really slippery.
2. Andre from the Hawaiian Islands blew up a factory farm last week.
3. The wolverines in Lake Michigan are ruining everyone's fun.
4. The criminals on my street makes everyone lock their doors at night.
5. Mike Tyson, the best interviewee in the world, likes to nibble on earlobes.


“A preposition is a terrible thing to end a sentence with.”
- Winston Churchill

ADJECTIVES

Adjectives modify nouns and pronouns.

On the ACT, you will be tested on your ability to identify whether a word should be used as an adjective or as an adverb.

The big, dumb dog slept on the train tracks.

Your mother's cooking is delightful.

She's amazing but infuriating.

ADVERBS

Adverbs modify verbs, adjectives, and other adverbs. They almost always end in -ly.

He speaks loudly and carries a small stick.

The bomb is amazingly small, considering how much damage it does.

Beaver meat is delectable, but only when it's extremely carefully cooked.

I don't hear well.

Mentor Notes

That last example is one of the few adverbs that doesn't end in -ly. You've probably had people correct you before on your usage of "good" vs. "well", and now you know why: "good" is an adjective, while "well" is an adverb (usually).

In the following sentences, circle all adjectives and underline all adverbs. If one is improperly being used as the other, correct it.

1. There are a shocking small number of green pigs in the world today.
2. That humongous pizza delivery man is rapidly confused.
3. Tiger Woods has a large golf bag, but his shoes are incredible undersized.
4. I write hastily whenever I have a horrible idea in my head.
5. He plays well defense because he moves his feet good.
6. Most smartly students will quick see the glaring obvious issues with this sentence.

CONJUNCTIONS

Conjunctions indicate relationships between words, phrases, and clauses (parts of sentences).

There are two main kinds of conjunctions: coordinating conjunctions, which usually occur in the middle of a sentence, and subordinating conjunctions, which can occur at the beginning or in the middle of a sentence. The main grammatical difference is that a coordinating conjunction is used to relate two independent clauses (sentences that could stand on their own). Subordinating conjunctions provide a transition between the ideas of a dependent (or subordinate) clause and an independent clause. The dependent clause usually provides more information about the independent clause, such as the time, place, or the cause of an effect.

Examples of Coordinating Conjunctions (remember "FANBOYS")

for	or
and	yet
nor	so
but	

Examples of Subordinating Conjunctions

after	once	until
although	rather than	when
as	since	where
because	than	wherever
even though	though	while
if	unless	why

Conjunctions are generally tested based on their logic on the ACT. For instance, consider:

The boys chose to wear the same t-shirt, although they had different styles.
The boys chose to wear the same t-shirt, although they had the same style.

The second sentence doesn't make logical sense, because the conjunction "although" indicates that there should be a contradictory relationship between the two parts of the sentence. The first sentence is properly includes two contradictory parts, joined by "although".

In the following sentences, identify the conjunction. If the conjunction is being used illogically, identify a conjunction that would be more appropriate:

1. The astronauts were anxious about takeoff, because everyone assured them it was ok.
2. While I like Sundays, I don't like going to church.
3. The sun was extremely bright, however I wore my sunglasses while driving.
4. Everyone thinks she's beautiful, yet I think she's too manipulative.
5. There are hundreds of people like me at my school, so I'm still a loner.

Mentor Notes

THE RECOMMENDED STRATEGY

Now that you have familiarized yourself with the seven parts of speech tested on the ACT, let's take a look at the strategies you should when you approach any ACT English question.

There are three rules that you should follow to begin every question on the English Test:

1. Read to the end of the sentence.

Don't stop reading at the end of the underlined portion, as the meaning of the sentence could change. This is true even when a sentence has two underlined portions. In fact, some questions may require you to read the sentence before and/or after the underlined portion in order to determine the correct answer.

[2] It seems that it came down to the perceptions in 7

7. **A.** NO CHANGE
   B. perceptions of
   C. perceptions to
   D. perceptions which

In this example, you may be tempted to stop reading at the end of the underlined portion, or at the comma. However, this isn't enough to answer the question. All options are grammatically correct, so we must read to the end of the sentence to figure out which answer choice is best.

[2] It seems that it came down to the perceptions in 7 the leadership board at the National Association for the Advancement of Colored People (NAACP).

7. **A.** NO CHANGE
   B. perceptions of
   C. perceptions to
   D. perceptions which

Reading to the end of the sentence reveals that answer choice B is correct. We are dealing with the "perceptions of" the NAACP.

2. Use the answer choices to anticipate what English knowledge the question is testing you on.

For Usage/Mechanics questions, identifying which part of speech or grammar rule is being tested can be a huge help to determining the correct answer. Once you know that a question is testing your ability to use a comma correctly, for instance, you're much more likely to remember your comma rules and choose the right answer than you would be on a question where you're unable to tell what is being tested. If you recognize that two different grammar rules are being tested in a question, you should start by evaluating the easier grammar rule and then eliminating the answer choices that incorrectly follow that rule.

Students shouldn't spend much time attempting to exactly pinpoint what category a question falls under. However, it is useful in helping students identify what skills they need to use to answer a question correctly. With enough repetition and practice, this recognition will become second nature.

3. If there's an actual question being asked, pay close attention to it.

Take a look at these two English questions:

At just the age of 15, she

stood up to not only a white bus driver and police

officer, and[5] the entire system of racial segregation.

Furthermore, though within a year the Montgomery

Bus Boycott would be in full swing, she acted alone,

bravely facing[6] an injustice that she knew needed to

be eliminated.

5. **A.** NO CHANGE
B. officer. Also
C. officer, but also
D. officer but also

6. Which of the following choices most closely maintains the style and tone of the essay?
F. NO CHANGE
G. saying "no thanks" to
H. channeling the power of God to destroy
J. going kamikaze against

Notice anything? Question 5, like all Usage/Mechanics questions, is composed of an underlined portion of text and four answer choices. Question 6, like all Rhetorical Skills questions, asks an actual question and then offers four answer choices. When working with the latter, you must pay close attention to what the question specifically tells you to look for. test will often try to throw you off by asking you to choose the answer that is NOT correct rather than the best answer. Students who don't pay attention to the question itself will often find themselves making silly mistakes.

If after reading the question and eliminating bad answer choices the correct answer doesn't jump right out, students should make their best guess based on all of the critical thinking they've done. Remember, there's no guessing penalty on the ACT. Students should never leave any question blank.

4. Know when you need to read more than just the sentence with an underlined portion.

Questions involving verb tense often require reading the sentences before or after to see if the events occurred in the past or will occur in the future. Pronoun use is another time you may have to read another sentence; if you can't find which noun the pronoun is replacing in the current sentence, you need to read the sentence before. Another time you may have to read more than just the sentence with the underlined portion would be if the question or an answer choice involves another part of the passage such as the context of a paragraph, scope, or conclusion. For example, if one of the answer choices in the middle of the passage is "foreshadowing of the conclusion", you cannot correctly answer that question until you finish reading the passage.

Now, let's talk more in-depth about what grammar and punctuation rules you will have to know so that you can correctly answer the vast majority of questions on the ACT English Test.

GRAMMAR AND PUNCTUATION QUESTIONS

SUBJECT-VERB AGREEMENT

The Problem: The number of a verb does not match up with the number of the subject to which it refers.

The Solution: Identify what subject the verb refers to, and then ensure that the verb is in the right form. This means matching a singular verb with a singular subject, and a plural verb with a plural subject. Note that unlike plural nouns, plural verbs do not often have an 's' on the end. In contrast, singular verbs do often have an 's' on the end.

For the first time in my life, the challenges of high school was not the focus of my daily energy.

A. NO CHANGE
B. were not
C. was never
D. were never

The correct answer is B. The subject of the second half of the sentence is "challenges", which is a plural noun. Therefore, the verb of the sentence must also be plural: "was not" should be changed to "were not". Note that the prepositional phrase in between the subject and the verb "of high school" works to distract you from the true subject at hand.

Try It Out!

Each of my friends dances like a fool on New Year's Eve.

A. NO CHANGE
B. dance
C. dancing
D. to dance

PRONOUN-ANTECEDENT AGREEMENT

The Problem: The pronoun does not agree with its antecedent (the noun that the pronoun is taking the place of).

The Solution: First, identify which noun the pronoun is taking the place of (this may require you to look at previous sentences in the paragraph). Then, determine what the correct pronoun should be. Remember that, like a subject verb agreement question, a plural pronoun must take the place of a plural noun, and a singular pronoun must take the place of a singular noun.

The frustrating thing about wizards is that he doesn't always use magic on the correct frog.

A. NO CHANGE
B. he don't
C. they don't
D. they doesn't

The correct answer is C. First, you must observe that "he" is a pronoun. Next, you must identify the antecedent: "wizards". This is a plural noun, so you must use a plural pronoun. "They" is appropriate. Answer choice D is incorrect because it establishes a subject-verb disagreement.

Try It Out!

King Arthur ruled the realm with an iron fist. Despite this, people loved him.

A. NO CHANGE
B. it
C. them
D. us

COMMAS

The Problem: Proper comma use is probably the most difficult question type on the ACT, simply because it involves so much knowledge. There are many different ways a comma can and should be used, and students will have to understand at least five if they want to ace the ACT English Test.

Comma Problem 1: Descriptive Phrases

The Solution: Descriptive or introductory phrases (that could not be a complete sentence on their own) need to be separated from the rest of the sentence by a comma.

In the shadows of the city he watches over us all.

A. NO CHANGE
B. In, the shadows of the city, he
C. In the shadows, of the city he
D. In the shadows of the city, he

The correct answer is D. "In the shadows of the city" is an introductory phrase that is dependent on the second half of the sentence, "he watches over us all". Therefore, we need a comma between the two halves of the sentence.

Try It Out!

My Spanish teacher is scary. His nose, by far the longest facial feature I've ever encountered points to the left of where he's looking.

A. NO CHANGE
B. His nose by far the longest facial feature I've ever encountered, points
C. His nose, by far the longest facial feature I've ever encountered, points
D. His nose by far the longest facial feature I've ever encountered points,

Comma Problem 2: Conjunctions

The Solution: Two parts of a sentence that could stand alone as sentences on their own need to be separated by a comma followed by a conjunction (remember "FANBOYS").

Michael Jordan was an amazing basketball player, but lately his reputation as a businessman has been called into question.

A. NO CHANGE
B. Michael Jordan was, an amazing basketball player, but lately
C. An amazing basketball player, Michael Jordan, but lately
D. Michael Jordan was an amazing basketball player but lately

The correct answer is A. The original sentence correctly separates two independent clauses ("Michael Jordan was an amazing basketball player" and "Lately his reputation as a businessman has been called into question") with a comma and a conjunction ("but").

Try It Out!

I'll have to give up on my dreams of becoming a football player for my brain injuries are too severe.

A. NO CHANGE
B. player; for
C. player, for
D. player. For

Comma Problem 3: Lists

The Solution: Items in a list of three or more need to be separated by commas.

The three things that I hope to do on vacation are to eat to sleep and to read.

A. NO CHANGE
B. is to eat, to sleep, and to read.
C. are to eat; to sleep; and to read.
D. are to eat, to sleep, and to read.

The correct answer is D. We must separate each of the items in the list with a comma. B correctly uses the commas, but creates a subject-verb disagreement ("things" and "is").

Try It Out!

My first boss, though a great man, always seemed to think that he was landing a big contract, going out of business or both.

A. NO CHANGE
B. he was: landing a big contract, going out of business, or both.
C. he was landing a big contract going out of business or both.
D. he was landing a big contract, going out of business, or both.

Comma Problem 4: Dates and Locations

The Solution: Dates and locations need to have their parts separated by commas. For instance: I went to Albany, New York in January, 1946.

George Washington's famous crossing of the Delaware River began on December 25 1776.

A. NO CHANGE
B. began, on December 25 1776.
C. began on December 25, 1776.
D. began, on December 25, 1776.

The correct answer is C. No comma is necessary after "began" - a good way of knowing this is to recognize that "on" is a preposition, and we never have a comma directly before or after a preposition. Answer choice C also correctly separates the day and year with a comma.

Try It Out!

On February 4, 1902 in Atlanta, Georgia, Elroy Simpson was shot dead after a dispute over a broken saxophone.

A. NO CHANGE
B. On February 4 1902, in Atlanta Georgia,
C. On February 4 1902 in Atlanta Georgia
D. On February 4, 1902, in Atlanta, Georgia,

Comma Problem 5: No Comma Needed

The Solution: On top of all four of those rules, there are many ACT English problems that test your ability to recognize when no comma is needed. If the sentence doesn't fall under one of the first four categories, you can be confident that it does not require a comma.

For more practice on commas and other types of punctuation, please review the "ACT Punctuation Tips" and "ACT Punctuation Quiz" with your mentor

COLONS

The Problem: Knowing when to use a colon instead of a period, comma, or semicolon.

The Solution: Use a colon to introduce an idea or list. The colon should come after a clause (part of a sentence) that could stand alone, and before the idea or list being introduced. Do not use a colon in situations where the sentence would make perfect sense without it.

You can also use a colon in between independent clauses if the independent clause after the colon is directly describing or elaborating upon something mentioned in the independent clause that precedes the colon.

My mother told me to pick up three things from the store, garlic, wooden stakes, and crucifixes.

A. NO CHANGE
B. store
C. store;
D. store:

The correct answer is D. Just by reading the sentence, you should be able to tell that some sort of punctuation is necessary after "store". In this case, because a list of items comes directly after that, it makes sense for us to use a colon.

Try It Out!

With binoculars, there's only one safe way to see a lion in the wild.

A. NO CHANGE
B. There's only one safe way to see a lion in the wild: with binoculars.
C. With binoculars: there's only one safe way to see a lion in the wild.
D. There's only one safe way to see a lion in the wild, with binoculars.

SEMI-COLONS

The Problem: Knowing when to use a semi-colon instead of a period, comma, or colon.

The Solution: Grammatically, semi-colons function the same way as periods. They separate two complete thoughts. Stylistically, semi-colons imply an added level of connection between the two thoughts as compared to just using a period. Commas cannot connect two complete thoughts on their own - remember that they need to partner with a conjunction to do so. Note that on the ACT English Test, students will never be asked to choose between these three grammatically correct options (a semi-colon, a period, or a comma with a conjunction). Only one grammatically correct option will be present. You can also use a colon in between independent clauses if the independent clause after the colon is directly describing or elaborating upon something mentioned in the independent clause that precedes the colon.

If the guys on the soccer team were nervous about playing in the championship game, they didn't show it, they played flawlessly and patiently, like a well-oiled machine.

A. NO CHANGE
B. it;
C. its,
D. its;

The correct answer is B. The clauses before and after "it" are both independent. Therefore, we have the choice of separating them with a period, a semi-colon, or a comma with a conjunction. Only the semi-colon is given as an answer choice.

Try It Out!

My mother was of a long line of scientists; the fact that she became so religious never sat well with my grandparents.

A. NO CHANGE
B. Being of a long line of scientists; my mother became religious, which didn't sit well with my grandparents.
C. Not sitting well with my grandparents, my mother became religious; despite being from a long line of scientists.
D. Despite being from a long line of scientists; the fact that my mother became so religious never sat well with my grandparents.

APOSTROPHES

The Problem: Knowing how to correctly use an apostrophe to designate ownership of one noun over another.

The Solution: Know when and how to use apostrophes. The ACT English Section tests two types of apostrophes: to show the possession of a noun and contractions. To determine if the apostrophe is possessive, ask yourself: "does (blank) belong to (blank)?" If so, then an apostrophe is required.

For possessive apostrophes, follow these rules:

1. If the owning noun is singular, add 's (even if it ends in s)

the girl's purse
the bass's lake

2. If the owning noun is plural but doesn't have an s at the end, then you add 's.

women's soccer team
children's toys

3. If the owning noun is plural and already has an s at the end, then you only need to add an apostrophe after the s.

Two dogs' tails
The companies' employees

Some possessive pronouns do not use an apostrophe at all, so be careful with these!

her
his
their
its
whose

Apostrophes are also used to show where a letter was omitted from a contraction. To check if it's a contraction, try to replace the contraction with the two words that make up that specific contraction. The ACT loves to test students on the different between its/it's, whose/who's, their/they're, so make sure you know the difference! Let's look at an example problem:

My cats' tongue feels like it's made of hardened mashed potatoes.

A. NO CHANGE
B. cats
C. cat's
D. cat

The correct answer is C. “Tongue” belongs to “cat”, so we know that we need an apostrophe. Furthermore, we can tell that the sentence is referring to only one cat by looking at the verb; “is” is singular, so A must be incorrect.

Try It Out!

We’ll be spending the afternoon in James dad’s secret treehouse.

A. NO CHANGE
B. James dads’
C. James’s dads
D. James’s dad’s

STYLE AND WORD CHOICE QUESTIONS

WORDINESS

The Problem: Some sentences or answer choices contain extra words that make the meaning of the sentence hard to understand.

The Solution: Choose the answer that is the most clear. Avoid flowery or redundant language. Note that the correct answer to a "wordiness question" will almost always be the shortest answer. plural verbs do not often have an 's' on the end. In contrast, singular verbs do often have an 's' on the end.

My brother has been a police officer for an indescribably long time, over 40 years, what an accomplishment.

A. NO CHANGE
B. what an accomplishment - over 40 years
C. an indescribably long 40 years.
D. over 40 years.

The correct answer is D. The other three answer choices all contain extra words that don't add to the meaning or clarity of the sentence.

Try It Out!

Astronomers have been staring out at the wide, empty and beautiful sky for centuries, wondering what distant stars may hold.

A. NO CHANGE
B. the expansive sky
C. the endless sky
D. the sky

REDUNDANCY

The Problem: There is a redundant phrase in the sentence.

The Solution: Identify the redundant phrase, and change the sentence to eliminate it. Often, the correct answer choice for a redundancy problem will be to "OMIT the underlined portion".

Annually, more than 1,000 monkeys are killed by poachers who want to sell their fur every year.

A. NO CHANGE
B. each year.
C. every 12 months.
D. OMIT the underlined portion and insert a period after "fur"

The correct answer is D. "Every year" is redundant because the sentence has already said "annually".

Try It Out!

The bomb exploded with an explosive force greater than that of the bomb that destroyed Hiroshima.

A. NO CHANGE
B. a concussive force
C. a destructive force
D. a force

TRANSITIONS

The Problem: A transitional word or phrase is ineffective or inaccurate. This transition can occur between two sentences or between two paragraphs.

The Solution: Because Transition questions always occur between two entities (either two sentences or two paragraphs), it's very important that students understand the main idea of each entity. Carefully read both sentences and determine their relationship - are they agreeing with each other? Disagreeing with each other? Seemingly unrelated? - and use that knowledge to choose the right transitional word or phrase.

Astronauts know that when they leave the Earth's surface, they may never return. <u>However,</u> there have been multiple examples of fatal space accidents.

A. NO CHANGE
B. Indeed,
C. Contrastingly,
D. On the other hand,

The correct answer is B. By carefully reading both sentences, we can tell that the second sentence is confirming an idea presented in the first sentence. Therefore, "indeed" is appropriate. All of the other answer choices are contradictory.

Try It Out!

I haven't seen my family since I left for college, <u>so</u> I'm very excited to go home for Thanksgiving.

A. NO CHANGE
B. yet
C. but
D. for

MISPLACED MODIFIERS

The Problem: A phrase is misplaced within the sentence, making it unclear what the phrase is intended to modify.

The Solution: Choose the proper placement for the modifying phrase. Often, these questions will have you consider what word the phrase should be placed after. Remember that the subject must always follow the introductory or modifying phrase.

Rolling around in the mud, I just shook my head as my dog looked up at me with a goofy smile.

A. NO CHANGE
B. I smiled as my dog shook his head at me
C. my dog, smiling at me, and I just shook my head.
D. my dog looked up at me with a goofy smile, and I just shook my head.

The correct answer is D. It's clear that the dog is rolling around in the mud, not the narrator. Therefore, the first subject after the modifying phrase ("rolling around in the mud"), needs to be the dog. That leaves either C or D, but C presents a verb tense error.

Try It Out!

Snowboarders use their lower bodies while avoiding rocks and trees to glide through the snow.

The best placement for the underlined portion would be:

A. where it is now
B. after the word snowboarders
C. after the word bodies
D. after the word while

LOGICAL COMPARISON

The Problem: hings being compared must be similar entities. It's the classic 'apples and oranges' scenario.

The Solution: Whenever you see comparison key words such as "like" or "than", make sure that the two things being compared are actually comparable. Note that this is one of the few question types that often result in a longer answer choice being correct.

Many scientists say that we know less about the ocean floor than we know about the moon.

A. NO CHANGE
B. the surface of the moon.
C. the lifeforms on the moon.
D. the craters of the moon.

The correct answer is B. Though you can probably tell what is being compared, the sentence needs to be clarified. It does not make sense to compare an entire moon to the "ocean floor". It makes sense to compare one surface to another.

Try It Out!

Selling a work of art that you made is like a person selling a part of your body.

A. NO CHANGE
B. someone
C. an artist
D. OMIT the underlined portion

WHO VS. WHOM

The Problem: Identifying whether to use who or whom in a sentence.

The Solution: While this is a rare question on the ACT, it does come up occasionally. Students must identify whether the word that who/whom is taking the place of is the main subject of the sentence. If it is, use "who". If it is not, use "whom". For more, see the pronouns section of ACT - Parts of Speech.

The Mario Brothers, whom have come under fire recently for their treatment of endangered turtles, remain steadfast in their plea of not-guilty.

A. NO CHANGE
B. which
C. that
D. who

The correct answer is D. The underlined portion is taking the place of "Mario Brothers", which is the subject of the sentence. Therefore, "who" is the correct pronoun.

Try It Out!

If you could meet anyone, dead or alive, whom would you choose?

A. NO CHANGE
B. which
C. that
D. who

IDIOMS

The Problem: "Idioms" deal with the proper way of saying a phrase. More specifically, they deal with matching the correct preposition with a verb.

The Solution: This is a problem where students will do best to trust their ear. If it sounds right, it probably is. For more practice, see the prepositions section of ACT - Parts of Speech. "whom". For more, see the pronouns section of ACT - Parts of Speech.

Though I've always admired thrill seekers, I've always had a deathly fear in skydiving.

A. NO CHANGE
B. fear of
C. fear with
D. fear for

The correct answer is B. "Of" is the proper preposition to match with "fear".

Try It Out!

We hadn't seen each other in years, yet as soon as we were together we slipped into our familiar habits, talking about old friends and making jokes about our hometown.

A. NO CHANGE
B. fell upon
C. slipped on
D. fell in

MAIN IDEA AND EDITING QUESTIONS

FUNCTION ("DELETE A PHRASE")

The Problem: These questions ask whether or not the author should delete a phrase, sentence, or paragraph.

The Solution: Determine how the phrase, sentence, or paragraph functions within the passage as a whole. Then decide whether or not it should be kept or deleted, and why. This is a great question type on which to anticipate the correct answer before reading the choices.

If scientists are able to devise a plan to send a rover to Europa, one of the largest moons of Jupiter, they have reason to believe that they may find liquid water and signs of life deep under its icy crust.

The writer is considering deleting the phrase "one of the largest moons of Jupiter," from the previous sentence. Should this phrase be kept or deleted?

A. Kept, because knowing that Europa is a moon of Jupiter is relevant information for evaluating how difficult the mission would be.
B. Kept, because Europa is one of the largest moons, but not the largest moon, of Jupiter.
C. Deleted, because knowing that Europa is a moon of Jupiter is not relevant information for evaluating how difficult the mission would be.
D. Deleted, because Europa is one of the largest moons, but not the largest moon, of Jupiter.

The correct answer is A. Note, however, that this assumes that the main idea of the passage is the mission. If this were a totally unrelated anecdote in the passage, then the answer would probably be C, or something different. Context matters a great deal on these questions.

Try It Out!

Christopher Columbus, though long celebrated in America for his famous voyage, has lately come under greater scrutiny. His actions against the native populations of the Americas have led many people to believe that he was an imperialist, racist murderer. Some have gone so far as to call for an end to celebrating Columbus Day.

The writer is considering deleting the phrase "His actions against the native populations of the Americas have led many people to believe that he was an imperialist, racist murderer" from the previous paragraph. Should this phrase be kept or deleted?

A. Kept, because the writer might believe that this is true.
B. Kept, because it provides information on why Columbus has come under scrutiny.
C. Deleted, because it does not consider the views of his defenders.
D. Deleted, because it is impossible to know if any natives were murdered by Columbus or not.

INTENT OR GOAL

The Problem: We are told that the writer has a goal he or she is trying to accomplish, and we must determine which answer choice best accomplishes that goal.

The Solution: Pay very close attention to the specific goal. Keeping that in mind, eliminate answer choices that do not accomplish the goal or only connect to it tangentially.

By the end of the opening speech, I was feeling poorly about my choice to attend the week-long seminar.

The writer is attempting to indicate that at this point she felt extremely bad about attending the seminar. Which of the following choices best accomplishes the writer's goal?

A. NO CHANGE
B. horrified by
C. unsure about
D. non-committal regarding

The correct answer is B. The goal is to use language that shows that the narrator is feeling "extremely bad", and "horrified by" accomplishes this the best.

Try It Out!

The World Cup draw is actually one of the most exciting parts of the tournament. Slowly, teams are drawn from different categories in order to construct the final eight groups that will face off in the upcoming tournament.

The writer would like to give a sense of the drama involved in the World Cup draw. Given that all of the following choices are true, which best accomplishes the writer's goal?

A. NO CHANGE
B. yet
C. but
D. for

REPHRASING FOR TONE

The Problem: Determining what version of the sentence or paragraph fits with the overall tone of the rest of the passage.

The Solution: These problems require the student to pay close attention to what style of writing is prevalent in the essay. Is it a personal narrative, written in a casual tone? Is it a scientific article, written in a formal tone? Or somewhere in between? Choose the version that most closely matches the rest of the passage.

These questions cannot be taken out of the context of the passage, but here is an example of a question you might encounter:

Which of the following choices most closely maintains the style and tone of the essay?

A. NO CHANGE
B. saying “no thanks” to
C. channeling the power of God to destroy
D. going kamikaze against

ADDITIONAL STATEMENTS

The Problem: Students are tasked with deciding whether or not an additional statement should be added to the passage.

The Solution: These questions almost always require a significant amount of reading around the area in question. Pay close attention to the main idea of the surrounding sentences.

These questions cannot be taken out of the context of the passage, but here is an example of a question you might encounter:

At this point, the author is considering adding the following true statement:

Her mother went so far as to slap her across the face when she found Claudette holding hands with a white child at a department store.

Should the writer include this sentence?

A. Yes, because child abuse is a serious issue.
B. Yes, because this story illustrates how engrained racial segregation was in Montgomery.
C. No, because this story has little to do with Claudette's childhood.
D. No, because this story is not related to the Civil Rights Movement.

SEQUENCE OF IDEAS

The Problem: These questions require students to identify the correct sequence of a series of sentences or paragraphs.

The Solution: Pay close attention to the main idea if each sentence or paragraph. Choose the answer choice that presents the main ideas in the most logical order. Note that these problems will require a good deal of reading around the area(s) in question.

Pronouns are also a great way to help locate the correct placement for a sentence. If the sentence contains a pronoun, figure out what its antecedent should be based on your knowledge of the passage, and then find the answer choice that places the sentence after the mention of that antecedent.

These questions cannot be taken out of the context of the passage, but here is an example of a question you might encounter:

What is the best placement for sentence 5 in the paragraph above?

A. Where it is now
B. Before sentence 1
C. Before sentence 3
D. Before sentence 4

SAMPLE ENGLISH PASSAGE

Claudette Colvin: Unknown Hero

On March 2nd, 1955 in Montgomery Alabama,[1] an African-American woman refused to give her seat on a public bus to a white passenger. After a minor confrontation with the bus driver and police, the woman was arrested. Her resistance added fuel to the Civil Rights Movement that was rapidly expanding in 1950's America. You'd be forgiven for thinking that this is the story of Rosa Parks. However,[2] Rosa Parks wouldn't have her confrontation with the police for another nine months. Instead, this is the story of a brave 15-year-old girl named Claudette Colvin.

Claudette Colvin, adopted daughter of C.P. and Mary Anne Colvin, were born[3] in one of the poorest neighborhoods of Montgomery. From an early age, the realities of racial segregation were apparent to her. She recalled that one of her earliest memories was of her mother telling her that she couldn't spend time with white children.[4] In Montgomery, racial segregation ran deep: African Americans had to eat at different restaurants, attend different schools, and even drink from different water fountains.

1
- **A.** A. NO CHANGE
- **B.** On March 2nd, 1955 in Montgomery, Alabama,
- **C.** On March 2nd 1955 in Montgomery Alabama,
- **D.** On March 2nd, 1955, in Montgomery, Alabama,

2
- **A.** NO CHANGE
- **B.** Indeed,
- **C.** Accordingly,
- **D.** J. Against popular opinion,

3
- **A.** NO CHANGE
- **B.** born
- **C.** being born
- **D.** was born

4 At this point, the author is considering adding the following true statement:

Her mother went so far as to slap her across the face when she found Claudette holding hands with a white child at a department store.

Should the writer include this sentence?

- **A.** Yes, because child abuse is a serious issue.
- **B.** Yes, because this story illustrates how engrained racial segregation was in Montgomery.
- **C.** No, because this story has little to do with Claudette's childhood.
- **D.** No, because this story is not related to the Civil Rights Movement.

Despite growing up in such a harsh environment, Claudette was an ambitious and intelligent child. She spoke about becoming President someday. Perhaps it was that inherent pride that led her to her fateful confrontation on the bus. At just the age of 15, she stood up to not only a white bus driver and police officer, and [5] the entire system of racial segregation. Furthermore, though within a year the Montgomery Bus Boycott would be in full swing, she acted alone, bravely facing [6] an injustice that she knew needed to be eliminated.

[1] Considering Claudette's amazing story and Rosa Parks' fame, why is it that so few people know Claudette's name? [2] It seems that it came down to the perceptions in [7] the leadership board at the National Association for the Advancement of Colored People (NAACP). [3] At the time of Claudette's individual protest, they [8] may not have been ready to act against the Montgomery bus system. [4] Another factor that likely prevented Claudette from becoming a figurehead was her age, when [9] the NAACP considered whom they could use as the face of its movement, it was less comfortable with the idea of a rebellious 15-year-old

5 **A.** NO CHANGE
B. officer. Also
C. officer, but also
D. officer but also

6 Which of the following choices most closely maintains the style and tone of the essay?

A. NO CHANGE
B. saying "no thanks" to
C. channeling the power of God to destroy
D. going kamikaze against

7 **A.** NO CHANGE
B. perceptions of
C. perceptions to
D. perceptions which

8 **A.** NO CHANGE
B. she
C. we
D. it

9 **A.** NO CHANGE
B. age, yet when
C. age; when
D. age when

than a seasoned 42 year-old. [5] This group was extremely
10
influential in organizing the protest movement that
brought about the end of legalized racial segregation. 11

Regardless of the reason behind her lack of fame at the
time, we should all now know the name of Claudette
12
Colvin. She should serve as a reminder of the countless
heroes whom risked their personal safety to combat the
13
oppressive system of racial segregation in America.

It's time that we remember these heroes.
14

10 **A.** NO CHANGE
B. than it was with the idea of
C. compared to
D. like it felt about

11 What is the best placement for sentence 5 in the paragraph above?

A. Where it is now
B. Before sentence 1
C. Before sentence 3
D. Before sentence 4

12 **A.** NO CHANGE
B. the unforgettable name of
C. the beautiful and inspiring
D. the one, the only,

13 **A.** NO CHANGE
B. who
C. which
D. that

14 **A.** NO CHANGE
B. These heroic heroes were amazing
C. How could we forget these heroes?
D. OMIT the underlined portion

Question 15 asks about the preceding passage as a whole.

15 Suppose the writer's goal had been to write an essay illustrating the importance of the Montgomery Bus Boycott to the American Civil Rights Movement. Would this essay accomplish this goal?

A. Yes, because the essay discusses the founder of the Montgomery Bus Boycott
B. Yes, because without Claudette Colvin, the Montgomery Bus Boycott never would have been successful.
C. No, because the essay focuses on the actions of Claudette Colvin.
D. No, because the essay never mentions the Montgomery Bus Boycott.

ACT ENGLISH TEST SECTION OVERVIEW

TP

KEY RESOURCES:

ACT English Curriculum

ACT English Quizzes

ACT Punctuation Tips

BASICS:

5 Passages and 75 Questions
Time – 45 minutes
Average time per passage – 9 minutes
Average time per question – 36 seconds

Question Types:

Usage/Mechanics (~40 questions) – deal with grammar, punctuation and sentence structure

Rhetorical Skills (~35 questions) – deal with writing strategies, style and organization

GENERAL STRATEGY:

1. Read the ENTIRE passage, but stop to answer a question after fully reading the sentence in which it is located.
2. Use the answer choices to determine which grammar rule is tested.
    a. If two or more rules are tested, start by evaluating the easiest one.
3. Eliminate wrong answer choices based on the rule or rules tested.
4. If you can't eliminate three answer choices, then read the sentence with each remaining answer choice and select the one that sounds the best to you.
5. If you are unsure of the answer choice you selected, circle the question and MOVE ON! If you reach the end of the test and have time remaining, review all of the Rhetorical Skills questions and then check over any question that you circled.

EXTRA TIPS:

1. Take your time! This is the one section of the ACT that most students finish with time remaining, but it is very hard to catch mistakes when reviewing questions at the end of the section.
    a. If you are finishing your practice tests with more than 5 minutes remaining, try to:
        i. Read each passage more carefully for its content – this will help you increase your accuracy on Rhetorical Skills questions, which are some of the hardest on the test.
        ii. Commit to Step 4 of the Basic Strategy!
2. Use your leftover time wisely by:
    a. Reviewing every Rhetorical Skills question.
    b. Reviewing any questions that you circled in your booklet.
    c. Thinking about math formulas in your head to get ready for the next section!
3. Read extra sentences surrounding the sentence of the question when the question is testing:
    a. Pronoun Usage
    b. Verb Tenses
    c. Transition words
4. If two or more answers to a question are grammatically correct, the question is probably testing wordiness or redundancy.
    a. When in doubt, go with the shortest answer!
5. NEVER choose an answer choice with the word "being", and be very careful when choosing "having"!
    a. These cannot stand alone as verbs, and are used incorrectly 99% of the time.
6. For questions testing punctuation usage, evaluate the answer choice that contains a semi-colon first.
7. For comma usage questions, take a long pause when reading a comma in an answer choice. This will help you determine if a break in the sentence is necessary.
8. Memorize "there" vs. "their", "who's" vs. "whose", "its" vs. "it's", "who" vs. "whom", and "could of" (NEVER correct) vs. "could have". These are quick and easy points!
9. For questions that ask you to re-arrange or insert additional sentences, use pronouns and/or words that are repeated in sentences to help you locate their correct place in the passage.
10. Words such as and, or, also, and than, as well as sentences that contain a list of different actions, often indicate that parallel structure is needed. Make sure that you keep verb and noun forms consistent!
11. If two answer choices are grammatically identical, then neither of them can be the correct answer.

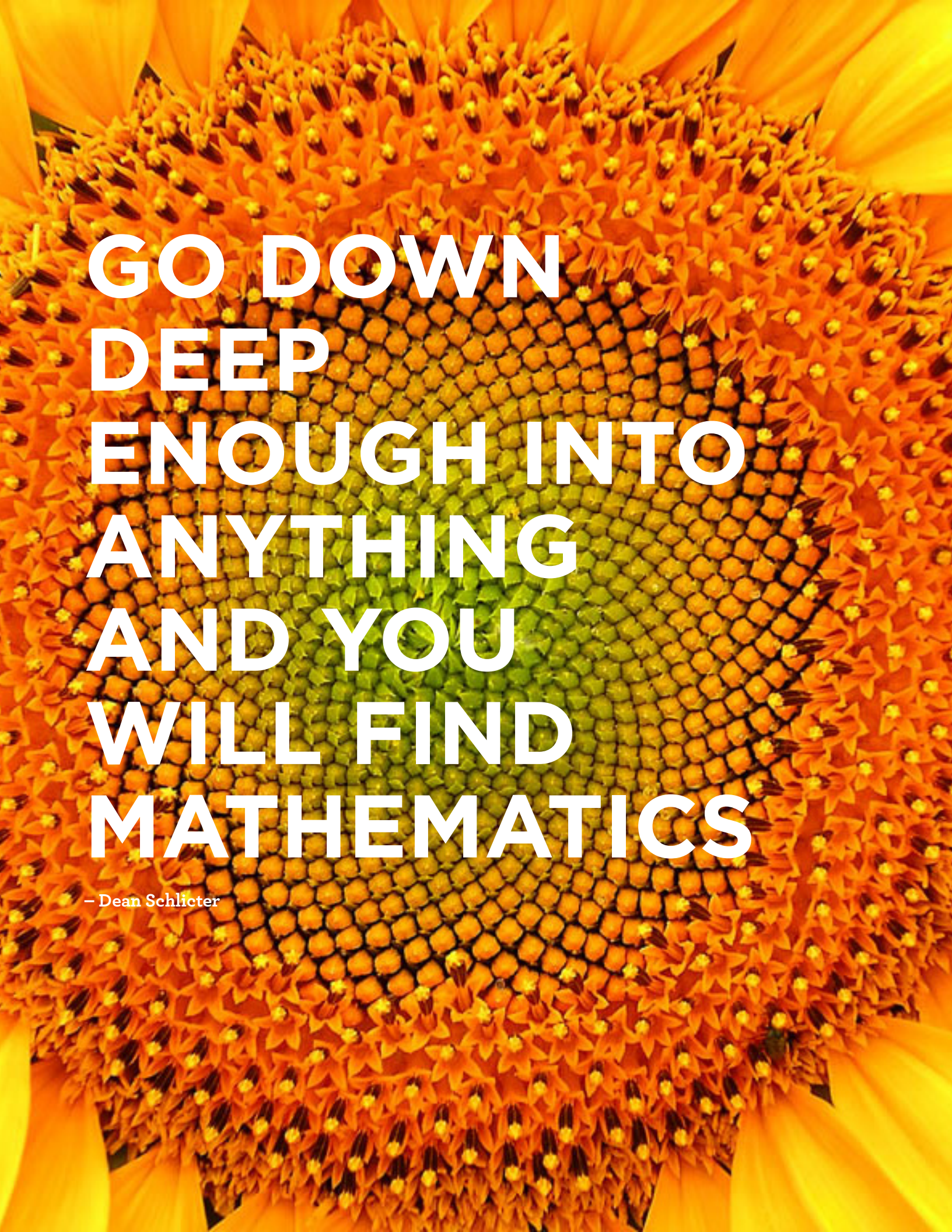
ESM
PREP

MATH CURRICULUM

ACT

ESM PREP

Contents


GO DOWN
DEEP
ENOUGH INTO
ANYTHING
AND YOU
WILL FIND
MATHEMATICS
– Dean Schlicter

INTRODUCTION

What's on the Test?

The ACT Math Test consists of 60 multiple-choice questions that test your knowledge in six different subject areas. The subject area breakdown is the same for every test:

Subject Area	Number of Questions	Percent of Test
Pre-Algebra	14	23%
Elementary Algebra	10	17%
Intermediate Algebra	9	15%
Coordinate Geometry	9	15%
Plane Geometry	14	23%
Trigonometry	4	7%
Total	**60**	**100%**

The six main subject areas cover the following topics:

1. **Pre-Algebra:** order of operations; basics of exponents and roots; ratios and proportions; percentages; factors and multiples of integers; absolute values; linear equations; simple probability; interpreting charts, tables, and graphs; basics of mean, median, and mode.

2. **Elementary Algebra:** expressing relationships by using variables; substitution; basic operations of polynomials; factoring polynomials; solving simple quadratics; solving linear inequalities; properties of exponents and roots.

3. **Intermediate Algebra:** using the quadratic formula; radical and rational expressions; inequalities and absolute value equations; sequences; systems of equations; inequalities involving quadratics; functions; matrices; roots of polynomials; and complex numbers.

4. **Coordinate Geometry:** using graphs of number lines as well as points, lines, polynomials, circles, and other curves in the standard coordinate plane; relationships between equations and graphs; slope; parallel and perpendicular lines; distance; midpoints; transformations; and conics.

5. **Plane Geometry:** properties of triangles, rectangles, parallelograms, trapezoids, circles; angles; parallel and perpendicular lines; geometric translations, rotations and reflections; simple 3-D geometry; measurement of perimeter, area, and volume.

6. **Trigonometry:** trigonometric ratios for right triangles (i.e. SOHCAHTOA); values, properties and graphs of trigonometric functions; trigonometric identities; trigonometric equations; modeling with trigonometric functions.

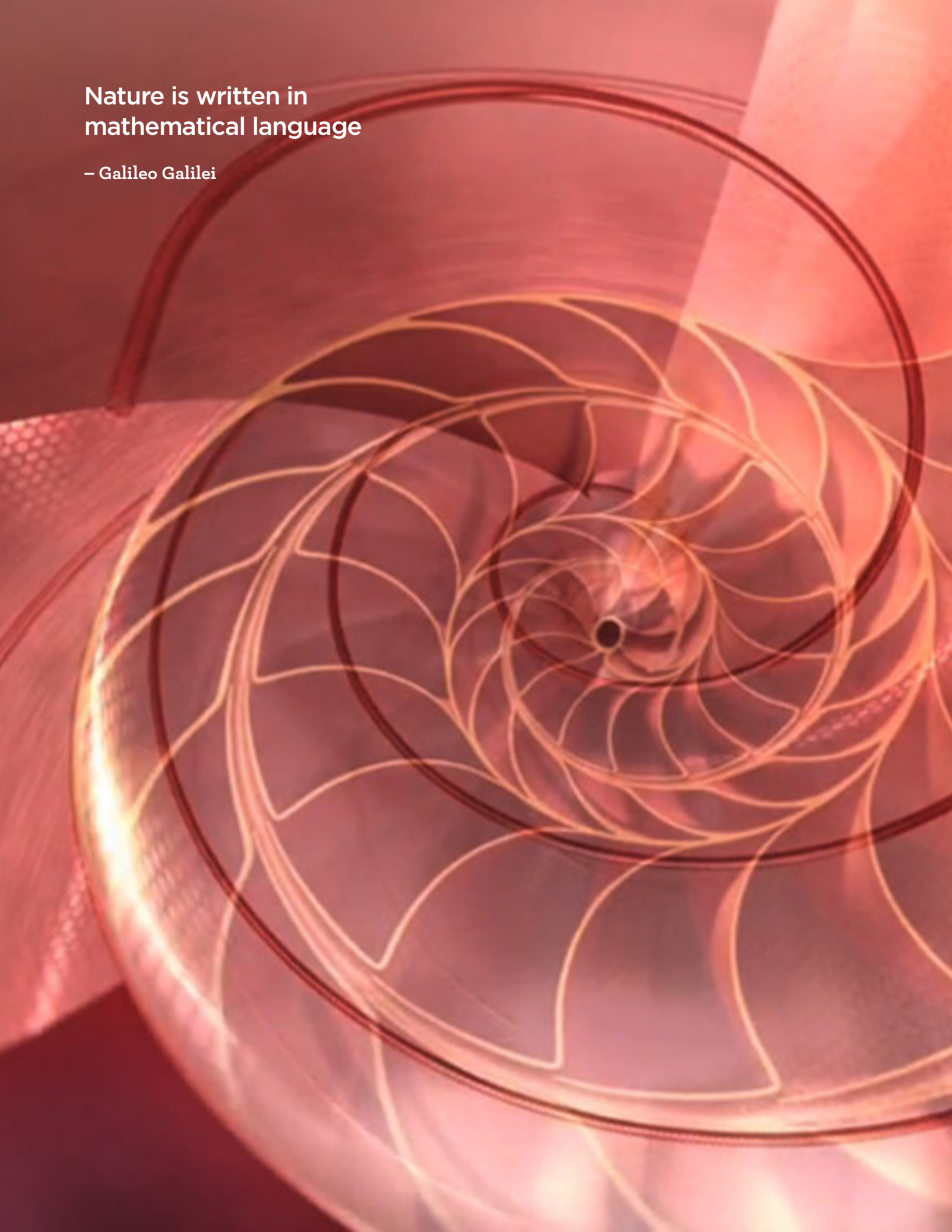
Why is this breakdown important?

Knowing the breakdown by subject allows you and your academic mentor to effectively focus your prep time on the areas that will help increase your score the most. For example, it does not make sense for you to begin your preparation for the Math Test by reviewing trigonometry questions if you have significant room to improve in algebra or geometry, as those questions make up 93% of the test!

General Subject Area	Number of Questions	Percent of Test
Algebra	33	55%
Geometry	23	38%
Trigonometry	Just 4!	7%

How long is the ACT Math Test?

The test is sixty minutes long, which means that students have an average time of exactly one minute for each question. This may not seem like a lot of time, and for most students it isn't, but if you prep effectively and efficiently for the test, it can be more than enough.


Nature is written in
mathematical language
– Galileo Galilei

THE GENERAL APPROACH

Because there is a set breakdown of questions for each test, and time is such an issue, students should answer the questions they know right away, and then come back to questions they are not sure how to solve to at least arrive at an educated guess:

THE BASIC STEPS

Below is a general outline of the steps to finishing the ACT Math Test. While the smaller details of these steps vary depending on skill level and pacing, the basics should generally be followed by all students.

Step 1. Read the question CAREFULLY to determine what it is asking you to find.

This step may seem like common sense, but do not underestimate its importance! After all, you cannot answer questions if you don't know what to solve for. If you are unsure what a question is asking for after you read it the first time, skip it and come back to it later. Your time is best used answering all the questions you immediately understand, and then coming back to the ones that were unclear to you. MAKE SURE YOU MARK ON YOUR ANSWER SHEET ANY QUESTIONS SKIPPED.

Step 2. What are you given?

You want to extract as much information from every question as you can and jot it down in your workspace. While you may not be able to see the importance of all of the information immediately, writing down EVERY piece of information can help you see the relationships much more clearly.

Does an algebra question give you an equation to work with or the values of any variables? Write them down!

Writing down given information is especially important in geometry problems. If a problem provides you with a figure, such as a triangle, and information that is relevant to the figure, such as the length of sides or angle measures, write this information directly on the figure! Doing so can point you towards other info like side lengths or angle measures. If a geometry problem does not give you a figure, draw one yourself and add all of the information you can.

If a geometry problem involves geometric keywords such as area, perimeter, circumference, midpoint, or distance, write down the formulas that define those words (which you will have memorized by test day!) Doing so can help you to easily determine what variables you need.

Step 3. What do the answers look like?

After reading the question and writing down all relevant information, take a peek at the answer choices. Knowing what form the answers are in can point you in the right direction. Answer choices can also offer hints about the problem if you look closely. For example, if a geometry question involving triangles has answer choices containing $\sqrt{2}$ or $\sqrt{3}$, you might be able to realize that using the properties of special right triangles is needed.

Step 4. Can you solve it?

If you read a problem, know what it's asking for, and immediately know how to solve for the answer, jump right in and solve it! These are the questions that we want to complete FIRST, no matter where they are in the test.

If you do not how to solve the problem right away, check to see if you can use any of the more unorthodox strategies we will practice in the next lesson.

If you do not know how to solve a problem, do not waste time trying to figure out how to do so! Just circle the question in your test booklet, make an educated guess using one or more of the guessing strategies we have practiced, and move on!

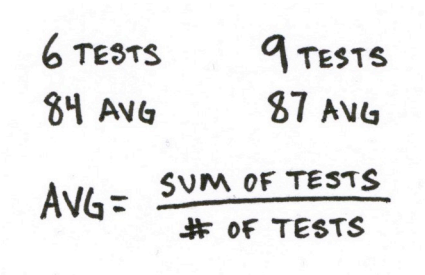
Once you have gone through each of the 60 questions and completed the questions you knew how to do immediately, return to the ones you were unsure of and try to solve them using other methods. The perplexities you initially faced when reading the question the first time often disappear after you have gotten into the flow of the section and answered most of the questions!

EXAMPLES

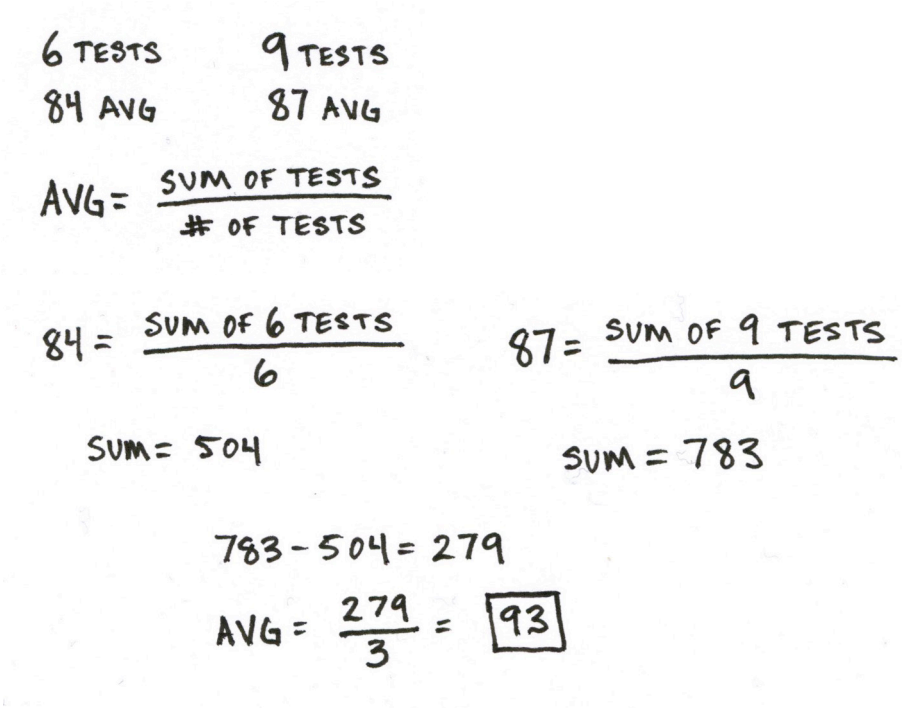
Now that we have reviewed the basic approach to any math problem on the ACT, let's practice putting it into action by looking at some examples. We'll start with a tough algebra question:

> John has taken 6 of the 9 equally weighted tests in his Algebra class this semester, and he has an average score of exactly 84.0 points. How many points does he need to average on the final 3 tests to raise his average score to exactly 87.0 points?
>
> **A.** 99
> **B.** 93
> **C.** 90
> **D.** 87
> **E.** 84

After carefully reading the question, we must first determine for what it is asking. Here, we are asked to find the average score for the 7th, 8th, and 9th tests that will bring Tom's average score up to exactly points. Next, go back into the question to extract all of the important information from the question, and jot down any formulas that might be useful.



The phrase "average score" is used twice in the question, so let's write down the equation for average below the answer choices. Even if we don't know how to solve the problem right away, we can use this equation to help point us in the right direction. We are also given that John has an average score of 84.0 point after 6 tests, and needs an average score of 87.0 point on 9 tests. We can plug these values into the equation we wrote down to see what information we can derive:



Just by using the one equation we wrote down, we were able to find the sum of the first six tests and what the sum needs to be for the 9 tests. We can subtract the sum of the first 6 test from the desired sum of the 9 tests to find the total points needed from the remaining three tests. By using the average equation one more time, and dividing the points needed by the three tests, we arrive at the correct answer of B!

Next, lets take a look at a sample geometry question:

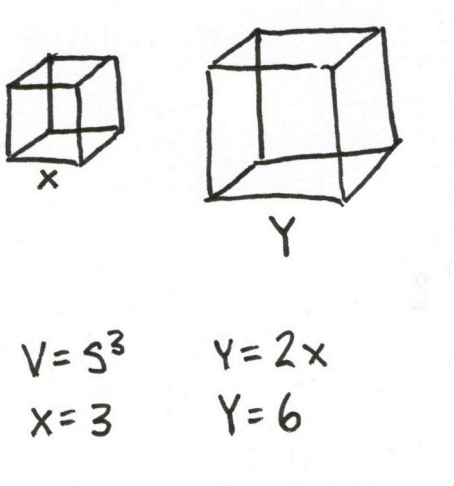
Cube X has an edge length of 3 inches. Cube Y has an edge length double that of Cube X. What is the volume, in cubic inches, of Cube Y?

A. 6
B. 9
C. 27
D. 36
E. 216

This plane geometry question is asking us to find the volume of Cube Y. Since we are asked to find the volume of a cube, let's jot down the equation under the equation. We can also draw a quick sketch of the two cubes if we are having trouble conceptualizing the problem. We are given that the length of the edges of Cube Y is double that of Cube X, which has an edge length of 3. Let's jot that down as well:

Cube X has an edge length of 3 inches. Cube Y has an edge length double that of Cube X. What is the volume, in cubic inches, of Cube Y?

A. 6
B. 9
C. 27
D. 36
E. 216



Next, check the answer choices to see if they tell us anything about how to approach the problems. Well, they are all multiples of 3, but there isn't anything that helps us to determine how to approach the problem. Finally, can we solve the problem? By looking at the equation for volume, we can see that the only piece of information we need is the length of the sides of a cube. We know that the length of the sides of Cube Y is double the length of the sides of Cube X, which the question tells us is equal to 3. Therefore, the length of the sides of Cube Y is 6, and the volume of Cube Y is equal to or 216. Answer choice E is the correct answer.

SPECIFIC ACT MATH STRATEGIES

As we mentioned, the ACT is fairly straightforward in how it asks questions, but the questions themselves can be complex. At times you will be asked to solve for a variable or understand the relationships between two quantities, but it will appear impossible to solve for these values. Don't worry. **There is no one correct way to solve any problem on the ACT.**

If you don't know how to solve a math problem on the ACT the "right" way, try using one of the test-taking strategies outlined below.

Use your own numbers to get rid of variables!

We can often simplify complex problems by plugging in our own numbers and then evaluating the problem. This works well for problem with variables in both the question and answer choices. Let's practice this by choosing numbers and plugging them in on several questions.

The sum of 3 consecutive odd integers is k. In terms of k, what is the sum of the 2 smaller of these integers?

A. $\frac{2k}{3} - 2$

B. $\frac{2k}{3}$

C. $\frac{2k}{3} + 2$

D. $k - 2$

E. $k - 3$

In the problem above, we can pick any three consecutive odd integers and evaluate their sum to find k. Let's choose 3, 5, and 7 as our three odd integers. The problem states that their sum is k, so we take 7 and find that . Now we just have to find out which answer choice equals 8, the sum of the 2 smaller of our three chosen integers.

A. $\frac{2(15)}{3} - 2 = 8$

B. $\frac{2(15)}{3} = 10$

C. $\frac{2(15)}{3} + 2 = 12$

D. $15 - 2 = 13$

E. $15 - 3 = 12$

The answer is A. While we used 3, 5, and 7 because they were easy odd numbers to use for this problem, it usually doesn't matter which numbers you choose. The exception there is that you want to avoid 0, 1, or 2 when picking numbers. These can be problematic. Otherwise, the correct answer will always equal the value you found when evaluating the problem.

The "If/Which of the Following" Situation

This type of ACT problem consistently shows up in medium and hard level questions, as they seem extremely abstract at first glance. These questions involve a parameter, such as an "if" or "for" statement ("if x is a positive integer", "for all x>0"), and the phrase "which of the following", which means that you will have to use the answer choices to find the correct answer. For questions like these, the plugging in values strategy works extremely well. Let's take a look at an example:

If n is a positive integer, which of the following expressions must be an odd integer?

A. 3^n

B. n^3

C. $3n$

D. $\frac{n}{3}$

E. $3 + n$

In this example, we have an if statement that tells us that our variable n must be a positive integer. All we have to do to evaluate this problem is to pick a number that is greater than 0, and then plug that into the answer choices to see which answer choice gives us our desired result. Let's use the number 3 for n, and evaluate each answer choice:

A. $3^3 = 27$; this is an odd number, but don't stop here! When you plug-in numbers, you must evaluate ALL answer choices!

B. $3^3 = 27$; we have a duplicate, but don't panic!

C. $3(3) = 9$; Another odd number. Still, don't panic.

D. $\frac{3}{3} = 1$; odd number

E. $3 + 3 = 6$; finally, we've narrowed it down.

We actually have four answer choices that give us an odd value here! If this happens to you on the test, do not panic. Simply use another number and go through the process again. In this case, the word must indicates that you are looking for a choice that will be odd no matter what! Let's set n equal to 4 instead.

A. $3^4 = 81$

B. $4^3 = 64$

C. $4(3) = 12$

D. $\frac{4}{3} = 1.33$

Answer choice A is the only answer that gives us an odd integer during our second try, so it must be the correct answer!

Tips and Tricks for choosing your values

- Be sure to test ALL of the answer choices. There is a possibility that two answer choices could give you the same result based on the value you chose to evaluate them. If this is the case, pick a second number with which to test them again.
- Don't make it hard on yourself! Choose small numbers that make doing arithmetic as easy as possible.
- Avoid using 0, 1, and 2 for questions involving exponents or multiplication/division. 2 creates problems because squaring it is the same as doubling it.
- For questions that involve variables but ask you to take percents in the question, use 100!

TRANSLATING FROM ENGLISH TO MATH

Word problems are everywhere on the ACT. Many students find these problems to be difficult and intimidating. However, these problems can be made easier by learning how to translate English into Math. Many of the words in these word problems have direct translations into mathematics. The following table will provide you with these translations and tell you how to use them on the test.

Key Words	Meaning	Example
Increased by, more than, combined, together, total of, sum, plus, added to	Addition	"Five more than thirty" 30 + 5
Decreased by, minus, less, difference between/of, less than, fewer than	Subtraction	"8 less than 12" 12 – 8
Of, times, multiplied by, product of, increased by a factor of	Multiplication	"Half of 12" ½*12
Per, a, out of, ratio of, quotient of, percent, decrease by a factor of	Division	"50 decreased by a factor of 10" $\frac{50}{10}$
Squared, cubed, square root, cube root, to the power of	Exponential	"The square of 8" 8^2
Is, are, was, were, will be, gives, yields, sold for, result is	Equality	"Five plus five is ten" 5 + 5 = 10
A number, what number, what percent, another number	Variable	A number is 3 less than 11 x = 11 - 3

*When you have a subtraction problem written in English, it can be confusing as to which number to put first. See the following examples of different ways to say 8 – 5:

"Eight **minus** five"

"Eight **is decreased by** five"

"Five **subtracted from** eight"

"Five **less than** eight"

Example

An integer, n, is added to 4. That sum is then multiplied by 8. This result is 10 less than twice the original integer. Which of the following equations represents this relationship?

A. $8(n + 4) = 2n - 10$
B. $8(n + 4) - 10 = 2n$
C. $8(n + 4) = 10 - 2n$
D. $n + 4 \times 8 = 2n - 10$
E. $4 + 8 = 2n - 10$

We start here with "An integer, n, is added to 4", which should tell us that we are going to simply write:

$n + 4...$

Next, the phrase "That sum is then multiplied by 8" tells us we have to group together and multiply the entire thing by 8. We do that with parentheses:

$8(n + 4)...$

The phrase "the result is" signifies that we have completed one side of our equation and are completing an operation that equals another:

$8(n + 4) = ...$

and that is equivalent to "10 less than twice the original integer", which means that we double n and subtract 10. We write:

$8(n + 4) = 2n - 10$

Now check your answer choices! The correct answer is A.

THE "PLUG 'N CHUG" METHOD

Plugging in answer choices can be a very effective strategy on the ACT, especially when a question asks you to find the value of a variable. This strategy is very similar to the "Guess and Check" strategy you have been told not to use in math class for years. The main difference is that, instead of having to "guess" a value to use, you are instead given five choices from which to choose.

Almost every question that asks you to find a value can be solved with this method, it can often take a long time. However, there is a way to find the correct answer by only plugging in two answer choices. Let's look at an example.

For what value of x is th equation $2(x - 6) + x = 36$

A. 24
B. 16
C. 14
D. 10
E. 8

This is a rather simple example. The best-case scenario is that you'll be able to solve this directly. For example's sake, let's pick the second smallest answer choice and plug it in for x.

$x = 10$
$2(10 - 6) + 10 = 18$

This is incorrect. Since this result is less than our desired result of 36, we know that we need to pick a larger answer choice. Since answer choice E. is smaller than answer choice D., we know in this case that it will give us a smaller result when plugged into the equation. That allows us to cross out both answer choices D and E.

Next, try answer choice B. It's important to note that we are now choosing to plug in the second largest answer choice.

$x = 16$
$2(16 - 6) + 16 = 36$

In this case, answer choice B is correct! But even if it wasn't, we would still be able to find the correct answer just by plugging in choice B. If it gave us a value that was bigger than 36, then answer choice C would have to be correct. If it gave us a smaller value, then answer choice A would have to be correct.

You should be careful using this method when plugging into absolute value functions or polynomials of even degree. Smaller integers, when plugged into these types of functions, will not always produce a smaller value for the function.

For example:

f(x) = x^2
f(2) = 2^2 = 4
f(-3) = -3^2 = 9

Notice that even though -3 is smaller than 2, it produces a bigger result when plugged into f(x). Be sure to test all answer choices when you are dealing with either type of function.

THE GRIP 'N RIP METHOD

Sometimes the easiest thing to do is to just use your calculator! If you can't figure out how to do a problem the right way, but know that you can find it "the long way" by entering a large number of equations into your calculator, go ahead and do it, but only after you have gone through the section and completed every problem that you know how to do quickly.

Among the following arthmetic operations, which could be the symbol ◊ represent given that the equation $(2 \lozenge 1)^4 + (6 \lozenge 3)^2 = 10$ is true?

I. Addition
II. Subtraction
III. Division

A. I only
B. II only
C. III only
D. I and II only
E. I, II, and III

This funky symbols question asks us to choose which operations, when plugged in for the diamond symbol, make the above equation true. We may know how to take a shortcut to arrive at the answer, but if not, we can always plug in each operation on our calculator and check to see if it works:

I. Addition: **Eliminate all answer choices that contain option I.**

II. Subtraction: **Answer II is correct. Notice that we can cross out all answer choices not containing option II, which means that our answer is G. We're done.**

III. Division: **No need to test that. But if we did, we would do it like that.**


Mathematics is the door
and key to the sciences
– Roger Bacon

ESTIMATING METHOD

When To Estimate

1. When you don't know how to solve a problem.

Estimating is simply not as surefire a strategy as solving a problem correctly! Don't take shortcuts at the risk of answering questions incorrectly.

2. On geometry questions that include a figure.

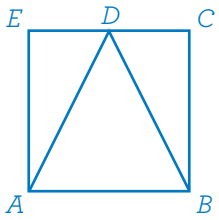
The directions at the beginning of the ACT Math Test state "Illustrative figures are NOT necessarily drawn to scale". This is NOT true! All of the figures on the ACT are drawn close enough to scale that you can estimate. You can also estimate on questions that require you to draw your own figure, but you must draw it as close to scale as possible.

Estimating is not as useful for algebra questions, as we can always use our calculators to find exact answers in a shorter amount of time.

Examples

In square $ABCE$ shown below, D is the midpoint of $\overline{CE}$. Which of the following is the ratio of the area of ΔADE to the area of ΔADB?

A. 1:1
B. 1:2
C. 1:3
D. 1:4
E. 1:8



This geometry question asks us to find the ratio of the areas between two triangles. If you don't know how to solve, eyeball the figure to estimate the relationship between the areas of the two triangles. Triangle *ADB* looks to be approximately twice as big as triangle *ADE*, so we can eliminate answer choices based on that estimation. They do not look like they are the same size, so answer choice F can be eliminated. Triangle *ADB* doesn't look four times or eight times bigger than triangle *ADE*, so eliminate answer choices J and K. That leaves us with two answer choices and a 50% chance of guessing correctly! Here, the correct answer is 1:2, or choice G.

In the figure below, ABCD is a square. Points are chosen on each pair of adjacent sides of ABCD to form 4 congruent right triangles, as shown below. Each of these has one leg that is twice as long as the other leg. What fraction of the area of the square ABCD is shaded?

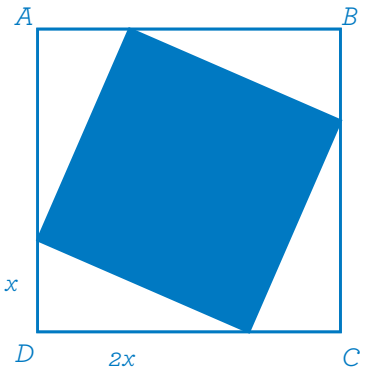
A. $\frac{1}{9}$

B. $\frac{2}{9}$

C. $\frac{4}{9}$

D. $\frac{5}{9}$

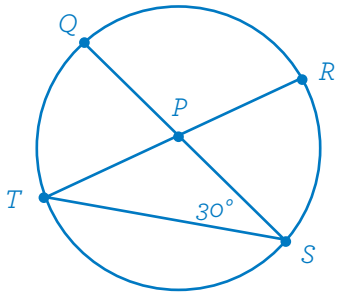
E. $\frac{8}{9}$



This question asks us to determine what fraction of the area of the larger square is made up of the shaded square. If we aren't sure how to find the exact answer, we can try to estimate. It looks like the shaded figure takes up more than half of the larger square, but not much more than that. Answer choice D is the only fraction that fits that description, so choose that answer and move on!

In the circle shown below, chords $\overline{TR}$ and $\overline{QS}$ intersect at P, which is the center of the circle, and the measure of $\angle PST$ is 30°. What is the degree measure of minor arc $\overset{\frown}{RS}$?

A. 30°
B. 45°
C. 60°
D. 90°
E. Cannot be determined from the given information



This question asks us to find the angle of an arc that is created by the intersection of two diameters of the circle. The only angle we are given is , which is equal to 30 degrees. The angle opposite arc RS looks to be bigger than , but smaller than 90 degrees. This only leaves answer choices G and H as potential answer choices, and gives us a 50% chance of guessing correctly! The correct answer here is 60 degrees, or answer choice H.

(Note: Cannot be determined from the given information is almost always a trap! Never guess this answer if you do not know how to solve a problem.)

ACT MATH TEST SECTION OVERVIEW

TP

BASICS:

60 Questions in 60 minutes
Average time per question: 1 minute

Topic	*Approx. # of Questions*
Pre-Algebra	14
Elementary Algebra	10
Intermediate Algebra	9
Plane Geometry	14
Coordinate Geometry	9
Trigonometry	4

KEY RESOURCES:

ACT Math Curriculum
ACT Math Quizzes
ACT 'Need to Know' Formula Sheet

ACT Math Strategies to Use:
Choose Own Values – when there are variables in the question or the answer choices. Use small values!
Plug In Answers (B/D Method) – when you have to find the value of a variable.
Estimating – when you have a geometric figure, or when you have no other options!

GENERAL STRATEGY:

1. Read the question carefully to determine for what it is asking you to find.
2. Write down any information you are given in the question and any formulas that may be helpful.
3. Take a peek at the answer choices for fractions, variables, π or radicals (e.g. $\sqrt{2}$ or $\sqrt{3}$).
4. If you know how to solve the question, solve it immediately!
5. If you don't know how to solve a question, try one of ESM's ACT Math Strategies.
6. If you can't use a strategy, circle the question in your booklet, eliminate any answer choices you can, and make your best guess.
7. Once you have gone through every question, go back to the circled questions if you have time.

EXTRA TIPS:

1. Keep It Moving – Staring idly at a question is the biggest detriment to maximizing your score. If you don't know a question, circle it in your booklet and move on! There are easier points ahead.
2. Stay away of "Math Class Mode" – showing every step of a question in school is important to get an A on your math test. On the ACT, where efficiency is key, doing so can be a big inhibitor to your score!
3. Memorize your formulas – knowing all relevant math formulas saves time, turns random guesses into educated guesses, and changes geometry questions into simple algebra question. Spend time reviewing math formulas each week!
4. Be aware of your tendency to solve for x – the ACT has lots of questions that try to prey upon your natural tendencies by asking for a value other than the main variable (such as asking for $2x$ instead of x). The value of the variable itself will always be an answer choice, so be careful!
5. Use decimals – convert fractions and percentages into decimals to use in equations and on your calculator. Memorizing the easy conversions (1/2, 1/4, 1/5, etc.) can help you save time and minimize small mistakes.
6. Find the triangle – triangles are often the key to geometry questions that don't seem to test them. Drawing triangles in these figures can help you easily find angles and the lengths of lines or segments.
7. Learn to love word problems – they make up over 75% of the test, so your ability to work through them quickly and correctly is hugely important. Remember how certain key words translate into mathematical terms.
8. Be careful with your calculator – put all negative numbers in parentheses, and watch your order of operations. The ACT often creates their four incorrect answer choices by making a common arithmetic mistake, so it is hard to catch mistakes after the fact!

Now that you know the basic strategies for the ACT math section, it's time to improve your understanding of the content covered. Check out the Annotated Index or ACT Math quizzes later in the book.

ESM
PREP

MATH INDEX

ACT

ESM PREP

Contents

PRE-ALGEBRA AND ELEMENTARY ALGEBRA QUESTIONS

NUMBERS

Integer:	Any number that is not a decimal or a fraction.
Examples:	-99, -50, 0, 6, 15
Whole number:	Any number that is not negative and not a fraction.
Examples:	0, 2, 37, 455
Odd Integer:	Any integer that cannot be divided by 2 without a remainder.
Examples:	-111, -57, -1, 1, 67
Even Integer:	Any integer that can be divided by 2 without a remainder (including zero!)
Examples:	-34, -2, 0, 4, 10, 12
Consecutive Integers:	Numbers that directly follow each other on a number line.
Examples:	*-4, -3, -2, -1 ... or 3, 4, 5, 6 ...*
Variable Form: ...	*n, n + 1, n + 2, n + 3...*
Consecutive Odd Integers:	Odd numbers that follow each other on a number line.
Examples:	*-5, -3, -1, 1... or 3, 5, 7, 9...*
Variable Form:	*n, n + 2, n + 4, n + 6...*
Consecutive Even Integers:	Even numbers that follow each other on a number line.
Examples:	*-6, -4, -2, 0... or 2, 4, 6, 8...*
Variable Form:	*n, n + 2, n + 4, n + 6...*

Real Number: Any number that can be found on a number line. Excludes infinity and imaginary numbers.

Examples: *All integers, rational numbers, and irrational numbers*

Rational Number: Any number that can be written as a fraction (ratio of integers).

Examples: $\frac{2}{3}$, 4 , $\frac{37}{99}$, -5, .20

Irrational Number: Any number that cannot be written as a fraction.

Examples: $\sqrt{3}$, π, e (Euler's Number)

Prime Number: A positive number that can only be divided by 1 AND itself.

Examples: *2, 3, 5, 7, 11, 13, 17, 19, 23, 2 9, 31, 37..*

Tips: *is NOT a prime number. is the smallest prime number, and prime numbers cannot be negative or even.*

Remainder: The amount left over when a quantity is divided by another number.

Examples: $\frac{7}{7}$ = 1, remainder 0

$\frac{8}{7}$ = 1, remainder 1

Factors: The numbers that divide evenly *into* a given number without a remainder.

Examples: *The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.*

Multiples: The numbers that divide evenly by a given number without a remainder.

Examples: *Multiples of 30 are 30, 60, 90, 120...*

Digit Cycles Patters: A fraction that has a repeating set of numbers.

Example: The fraction 3/7 is equivalent to $0.\overline{428571}$. What is the digit in the 103 rd place of $0.\overline{428571}$? (Note: The digit in the third place is 8.)

A) 1

B) 4

C) 2

D) 5

E) 8

To do this problem you want to note that there are 6 values being repeated, which means that the cycle starts over every 6 numbers. If you can find out how many times 6 goes into 103, you can see which number you will land on.

Example Problems

1. What is the least common multiple of 20, 40, and 60?

A. 20
B. 60
C. 80
D. 120
E. 240

2. What rational number is halfway between $\frac{1}{5}$ and $\frac{1}{3}$?

A. $\frac{1}{2}$
B. $\frac{1}{4}$
C. $\frac{2}{15}$
D. $\frac{4}{15}$
E. $\frac{8}{15}$

3. Which of the following is a rational number?

A. $\sqrt{2}$
B. $\sqrt{\pi}$
C. $\sqrt{7}$
D. $\sqrt{\frac{5}{25}}$
E. $\sqrt{\frac{64}{49}}$

4. How many prime factors does the number 210 have?

A. 2
B. 3
C. 4
D. 8
E. 10

5. Five consecutive integers add up to 115. What is the smallest of these integers?

A. 15
B. 17
C. 19
D. 21
E. 23

6. What is the greatest integer smaller than $\sqrt{120}$?

A. 4
B. 8
C. 10
D. 11
E. 12

7. Which of the following is a rational number?

A. $\sqrt{3}$

B. π

C. $\sqrt{\frac{25}{2}}$

D. $\sqrt{\frac{36}{16}}$

E. $\sqrt{\frac{100}{80}}$

8. Each of the following is a factor of 80 EXCEPT:

A. 8
B. 10
C. 15
D. 20
E. 40

9. If x and y are positive integers such that the greatest common factor between x^3y and x^2y^2 is 63, then which of the following could be the value of x?

A. 9
B. 6
C. 4
D. 3
E. 2

ORDER OF OPERATIONS

Parentheses

Exponents

Multiplication

Division

Addition

Subtraction

Example: Simplify the following expression: $2(x + 3)^2 - (2x + 3^2)$

$2(x^2 + 6x + 9) - (2x + 9)$

$2x^2 + 12x + 18 - 2x - 9$

$2x^2 + 10x + 9$

Example Problems

10. What is the value of the expression when $2z - 3(2z - 1)^2$ when $z = 2$?

A. -117
B. -17
C. -23
D. -5
E. 5

11. If, $2(x - 2)^2 + 4 = 4$, then x =?

A. -4
B. -2
C. 2
D. 3
E. 4

PERCENTAGES

Percent: Means "divided by 100"

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}$$

Percent Change:

$$\frac{\text{(new value-origin value)}}{\text{original value}} \times 100$$

Example: If rent went from $1,380 to $1,630, it increased by 18 percent.

$$\frac{1630 - 1380}{1380} = \frac{250}{1380} = .18 \times 100 = 18\%$$

Percent Change Lesson

In everyday talk, we usually refer to the difference between two amounts in terms of addition or subtraction. For instance, if Timmy had 10 apples and Cindy only had 8, we might say that Timmy has 2 more apples than Cindy. Or if Sarah has lived in Scotland for 3 years and Manolo has lived there for 6 years, we might say that Sarah has lived there for 3 years less than Manolo.

However, another completely valid—and often much more informative—way of representing the difference between two amounts is by percent change. For example, instead of saying that Timmy has 2 more apples than Cindy, we might have said that the number of apples Timmy has is 25% more than Cindy. Or instead of saying that Sarah has lived in Scotland for 3 years less than Manolo, we might have said that the number of years Sarah has lived in Scotland is 50% less than the time Manolo has lived there.

This way of speaking might not be too popular, but it does appear in real life a lot. Taxes, sales, interest rates, stocks, and a bunch of other real life things are measured with percentages. So, it's extremely helpful to understand how to differ between amounts by percent change.

Luckily, percent change really isn't that hard to master! The idea is as follows:

To increase by a certain percentage, just multiply the original number by 1 plus the percent (in decimal form).

EX: What is a 20% increase of 140?

First, let's convert 20% to its decimal form: 0.20

Then, the math is easy!

$140 \times (1 + 0.20) = n$
$140 \times 1.2 = n$
$168 = n$

Now you might have your own way of doing this math, but trust me when I tell you that none will be faster or more useful when the numbers get bigger and scarier than this simple one step method.

Decreasing by a certain percentage is just as easy. The only change is instead of adding to 1, we will subtract our percent (in decimal form) from 1.

Ex: What is a 20% decrease from 140?

$140 \times (1 - 0.20) = n$
$140 \times 0.8 = n$
$112 = n$

It's as easy as that!

Now if someone were to ask you to go in the opposite direction, for instance asking, "140 is a 20% decrease from what number?", we already know how to set up our equation:
$n \times (1 - 0.20) = 140$
$n \times 0.8 = 140$
$n = 140 / .08$
$n = 175$

Similarly, for the question, 140 is a 20% increase from what number:
$n \times (1 + 0.20) = 140$
$n \times 1.2 = 140$
$n = 140 / 1.2$
$n = 116.666...$

These calculations above represent a single instance of percent change. However, in the real world, we are often interested in a series of percent changes that occur regularly over a period of time: for instance, the percent interest that accrues on a savings account year after year or the percent decrease of the population an endangered species each year.

Solving these problems might seem daunting at first, but it really just breaks down to doing the math we did above multiple times. We take our starting number and multiply by 1 plus/minus our percentage but instead of doing that just once, we do it as many times as the example dictates.

EX: Tom invests $140 with an expectation that his investment will increase by an incredible 20% every year. If his expectation is correct, how much money will he have after 8 years?

(We start the same way as before)

$140 \times (1 + 0.20) = n$
$140 \times 1.2 = n$

BUT HOLD ON!

We don't want to do this just once, we want to do it 8 times, right?

So, we add an exponent!

Exponents represent multiplying by the same number multiple times, and that's exactly what we want to do here. We want to multiply 140 by 1.2 not just once, but 8 times!

Therefore, by adding an exponent of 8 to the 1.2 in our equation, we have a simple calculation to find our answer:

$140 \times 1.2^8 = n$
$601.97 = n$

So, Tom will have about $600 in his investment after 8 years. The same would hold true for a percent decrease over many years, but I'll leave that one up to you!

Example Problems

12. If 125% of a number is 530, then what is 50% of the number??

A. 662.5
B. 424
C. 331.5
D. 265
E. 212

13. If 60 percent of n is equal to v percent of 30, where $v > 0$, then what is the value of $\frac{n}{v}$?

A. $\frac{1}{2}$
B. $\frac{2}{3}$
C. $\frac{1}{2}$
D. $\frac{3}{4}$
E. 2

14. 30 percent of one-half of a number is 21. What is the number?

A. 7
B. 42
C. 63
D. 70
E. 140

15. If 22 is added to one-half of a certain number, the result is 64. What is the original number?

A. 42
B. 84
C. 106
D. 128
E. 150

16. The string group of a classical orchestra typically uses 22 violins, 8 violas, 8 violoncellos, and 6 double basses. Approximately what percent of the string group are violas or double basses?

A. 18%
B. 30%
C. 32%
D. 37%
E. 50%

17. The string group of a classical orchestra typically uses 22 violins, 8 violas, 8 violoncellos, and 6 double basses. If a circle graph were made of the data, what would be the central angle of the sector that represents the number of double basses?

A. 14°
B. 18°
C. 49°
D. 60°
E. 98°

18. An amusement park increased its revenue by 12% from 2010 to 2011 and by 18% from 2011 to 2012. By approximately what percent did revenue increase from 2010 to 2012?

A. 28%
B. 30%
C. 32%
D. 34%
E. 36%

SEQUENCES

Arithmetic Sequence: Each term is equal to the previous term plus d

Examples: $t_1 , t_1 + d , t_1 + 2d , t_1 + 3d \ldots$ OR $t_1 , t_1 + d , t_2 + d , t_2 + d \ldots$

If $t_1 = 2$ and $d = 7$, then the sequence would be: 2, 9, 16, 23…

If $t_1 = 5$ and $d = -3$, then the sequence would be: 5, 2, -1, -4…

Geometric Sequence: Each term is equal to the previous term *multiplied* by r

Examples: $t_1, (t_1 * r) , (t_1 * r^2) , (t_1 * r^3) \ldots$

If $t_1 = 3$ and $r = 3$, then the sequence would be: 3, 9, 27, 81…

If $t_1 = 12$ and $r = \frac{1}{4}$, then the sequence would be: 12, 3, $\frac{3}{4}$, $\frac{3}{16}$

If this is your sequence: 3, 6, 12, 24, 48, _____, 192 and the question asks for the sixth number in the sequence, you simply want to:

- look at the multiplier: what do you multiply 3 by to get 6? 2
- multiply the fifth number by 2 to obtain the sixth: 48x2=96
- double check that this is correct by multiplying 96x2=192, which is the seventh number in the sequence, so you know that 96 is the correct answer.

It is important to note that geometric sequences can involve both multiplication of an integer, and multiplication of a fraction, which is the same as dividing. It can also involve multiplying by positive numbers or negative numbers.

Example Problems

19. What is the sum of the first 4 terms of the arithmetic sequence in which the 6th term is 8 and the 10th term is 13?

A. 10.5
B. 14.5
C. 18
D. 21.25
E. 39.5

20. What is the sum of the first 3 terms of the arithmetic sequence in which the 5th term is 8 and the 8th term is 20?

A. –12
B. –8
C. 0
D. 8
E. 28

TRANSLATING WORD PROBLEMS

Of: Multiply (x)

Examples: One half of 50 = $\frac{1}{2}$ * 50, A third of the circle's area = $\frac{1}{3}$* πr^2

Per: Divide (÷)

Examples: 75 miles per hour = $\frac{75 \text{ hours}}{\text{hour}}$, 3 apples per orange = $\frac{3 \text{ apples}}{\text{orange}}$

Percent: Divide by 100 or ($\frac{}{100}$)

Examples: 35 percent = $\frac{35}{100}$, 700 percent = $\frac{700}{100}$

Is: Equals (=)

Examples: 20% of x is 4 means $\frac{20}{100}$ x $x = 4$

Diameter is twice the radius means d = 2r

A number: Variable (typically x or y)

Examples: Half of a number is twice another number = $\frac{1}{2}x = 2y$

Example Problems

21. 20 percent of one half of a number is 13 . What is the number?

A. 6.5
B. 13
C. 26
D. 65
E. 130

22. When the sum of a number and 5 is multiplied by 3, the result is the number divided by 2. What is the number?

A. $-\frac{15}{2}$

B. -6

C. -3

D. 3

E. 6

23. The total price for two slices of pizza and half of a salad is $8. The total price for one slice of pizza and one full salad is $5.50. How much does a salad cost?

A. $1.50
B. $2.00
C. $2.50
D. $3.25
E. $3.50

24. Monica wants to drive 300 miles in 6 hours to San Francisco. She is considering towing a trailer, but will be driving an average of 10 miles per hour slower if she does. How many hours longer will it take her to drive 300 miles with the trailer attached to her car?

A. 1
B. 1.5
C. 2
D. 4
E. 7.5

AVERAGES, COUNTING, STATISTICS

Average (Arithmetic Mean):

$$\frac{\text{sum of terms}}{\text{number of terms}}$$

Average Speed:

$$\frac{\text{total distance}}{\text{total time}}$$

Weighted Average:

$w_1a_1 + w_2a_2 + \ldots + w_{n-1}a_{n-1} + w_na_n$

w_1 = weight of object 1(decimal between 0 and 1)

a_1 = average of object 1

Example:

If your class grade was composed of 70 percent classwork and 30 percent final test grade, and you had a 92% going into the final, what do you need on the final for an A?

.70(.92) + .3(x) = .90

x = .853 or 85.3%

Mode:

Value(s) that occurs most frequently

Example:

Modes of [2, 5, 5, 6, 32, 32, 37] = 5, 32

Median:

Middle point of an ordered list

For set with odd number of elements: Middle number

For set with even number of elements: Average of two middle numbers

Examples:

median of [2,5,6,32,37] = 6

median of [2,5,6,32] = $\frac{5+6}{2}$ = 5.5

Fundamental Counting Principle:

If an event can happen *M* ways and another, independent event can happen *N* ways, then both events can happen in *N* * *M* ways.

Example:

If you flip a coin and roll a die there are 2×6 = 12 possible results. (If three independent events, N*M*O)

Example Problems

25. In a town of 600 people, 200 females have an average age of 30 and 400 males have an average age of 45. To the nearest year, what is the average age of the town's entire population?

A. 33
B. 35
C. 37
D. 40
E. 42

26. Alisha's math class receives scores of 97 93, 78, 83, 76, 78, 84, 83, and 78 on their most recent test. What is the mode of the test scores for the entire class?

A. 73.3
B. 76
C. 78
D. 83
E. 97

27. Mario is bowling in a tournament and has the highest average after 5 games, with scores of 210, 225, 254, 231, and 280. In order to maintain this exact average, what must be Mario's score in his 6th game?

A. 200
B. 210
C. 231
D. 240
E. 245

28. A certain type of pizza costs $15 before sales tax is added. When you buy 5 pizzas you receive 1 additional pizza for free. What is the average cost per pizza for the 6 pizzas before sales tax is added?

A. 12
B. 12.5
C. 13
D. 15
E. 18

29. If Jimmy owns one suit, four shirts, and five ties, and two belts, how many outfits can he put together?

A. 11
B. 18
C. 20
D. 24
E. 40

30. A hiking group will go from a certain village to a certain town by bus on 1 of 4 roads, from town to a mountain by riding on 1 of 2 bicycle paths, and then from the mountain to their campsite by hiking on 1 of 6 trails. How many routes are possible for the hiking group to go from the village to the town to the mountain to their campsite?

A. 6
B. 12
C. 24
D. 48
E. 220

31. Any 2 points determine a line. If there are 7 points in a plane, no 3 of which lie on the same line, how many lines are determined by pairs of these 7 points?

A. 12
B. 14
C. 20
D. 21
E. 28

32. In a small town in Alaska, the low temperatures, in degrees Fahrenheit, for each of 7 recorded days in February were 2°F, 3°F, 1°F, 0°F, 3°F, −4°F, and 2°F. What was the median of these low temperatures?

A. −3.0°F
B. 0.0°F
C. 0.5°F
D. 1.0°F
E. 2.0°F

33. Eleanor and Juanita own a pizza shop: They offer 2 kinds of crust, 6 meat toppings, and 4 kinds of cheese. Each pizza is a combo of exactly: 1 crust, 1 meat, and 1 cheese. How many possible types of pizzas are there?

A. 14
B. 24
C. 30
D. 48
E. 60

34. Stephen has scored an average of 24.4 points per game in his last 10 games. How many points does he have to score in his next game to average 25 points per game overall?

A. 25
B. 29
C. 30
D. 31
E. 35

35. The local deli sells a fixed menu in which customers are allowed to pick one each of 4 sandwiches, 2 soups, 2 salads, and 3 drinks. How many different meal combinations are possible?

A. 2
B. 4
C. 12
D. 48
E. 72

PROBABILITY & RATIOS

Probability: $\frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$

Example:

Likelihood of rolling a six = $\frac{1}{6}$

One of Two Independent Events: Probability of A or B happening: P (A or B) = P(A) + P(B)

Likelihood of rolling a five or six = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Both of Two Independent Events: Probability of A and B both happening: P (A and B) = P(A) x P(B)

Example: Likelihood of rolling two sixes = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Ratios:

Tips: Represented as $\frac{\text{part}}{\text{part}}$ or $\frac{\text{part}}{\text{whole}}$ (part: part and part: whole will also be used)

Example: "part:part" questions are usually meant to confuse you. Turn them into part:whole by adding all parts together to form a denominator.

If a fruit bowl has only apples and oranges and the ratio of apples to oranges is 3: 2, then total number of fruit will be divisible by 5 (3 + 2)

Ratios Lesson:

Ratios are a way of expressing the relative amounts of two or more things. A common use for ratios is to express the relative make up of something that consists of various subgroups. For instance, a ratio could be used to express the relative number of boys and girls in a math class. One might say that the ratio of boys to girls in a class is 6:5, which simply means that for every 6 boys, there are 5 girls.

While it's not too hard to handle an example like the one above in your head, when the numbers are large or the ratio compares multiple subgroups instead of just two, similar problems can become quite tricky.

Luckily, there's an easy 3 step process that will allow you to solve any ratio problem that presents you with the subgroup ratio of a larger whole.

STEP 1: Add up the numbers in the ratio
STEP 2: Divide the total number of the group by this sum
STEP 3: Multiply each number from the ratio by this quotient.

EX 1:
Q: A class of 40 students has a 3:5 ratio of boys to girls. How many girls are in the class? How many boys?

STEP 1: 5 + 3 = 8
STEP 2: 40 students / 8 = 5
STEP 3: 5 x 3 boys = 15 boys
5 x 5 girls = 25 girls

EX 2:
Q: A tapestry is made up of 3 consecutive segments: one of silk, one of madras, and one of cashmere. The ratio of the lengths of these segments is 3:8:5. The total length of the tapestry is 304 inches.

What is the length of the longest segment of the tapestry? Which segment is it?

STEP 1: 3 + 8 + 5 = 16
STEP 2: 304 inches / 16 = 19
STEP 3: 19 x 3 silk = 57 inches of silk
19 x 8 madras = 152 inches of madras
19 x 5 cashmere = 95 inches of cashmere

OTHER RATIOS?
You should realize that ratios do not only compare subgroups within a larger whole. Ratios can also simply express the relative amounts of two independent things. For instance, in a cake there may be 1 candle for every 2 inches of cake. The ratio of candles to inches of cake is 1:2, but these are not subgroups of some larger whole. If you were asked, how many candles are on a 24 inch cake, you would not use the 3 step method above.

Instead, we should set up an equivalence:
Solving for x will give us our answer.

BUT WAIT!
It's important to understand that ratio problems are NOT the same as proportion, probability, or percent problems. These problems (the 3 P's) differ from ratios in that they do not relate the amounts of the individual subgroups that make up a larger whole, but rather relate a single subgroup to the whole.

Let's look at an example.

At a watering hole, there are 40 animals; 15 elk and 25 whitetail deer. If I were to give the ratio of elk to deer, I would simplify 15 elk to 25 deer to give my final ratio of 3:5.

However, one could also describe this group of animals using the 3 P's. The proportion of the animals that are deer is 25 deer / 40 total animals. Simplified, this gives 0.625. Similarly, the percent of the animals that are deer is 25 deer / 40 total animals x 100%. Simplified, this gives 62.5%. Finally, the probability that a randomly selected animal is a deer is also 25 deer / 40 total animals, which simplified gives 5/8.

It's important to understand the difference between ratios of subgroups, ratios of independent groups, and the 3 P's, so be sure to look out for them on the ACT!

Example Problems

36. A bag contains 12 red marbles, 5 yellow marbles, and 15 green marbles. How many additional red marbles must be added so that the probability of randomly drawing a red marble is $\frac{3}{5}$?

A. 13
B. 18
C. 28
D. 32
E. 40

37. An integer from 100 through 999, inclusive, is to be chosen at random. What is the probability that the number chosen will have 0 as at least one digit?

A. $\frac{19}{900}$
B. $\frac{81}{900}$
C. $\frac{1}{10}$
D. $\frac{171}{900}$
E. $\frac{271}{1,000}$

38. A deck of cards contains 13 hearts, 13 spades, 13 clubs, and 13 diamonds. In a certain game, players take turns, each drawing a card a card at random from the deck and putting the card on the table. When it is the 7th player's turn, there are 3 spades, 2 clubs, and 1 diamond on the table. What is the probability that the 7th player will draw a spade?

A. $\frac{5}{26}$
B. $\frac{5}{23}$
C. $\frac{11}{46}$
D. $\frac{13}{46}$
E. $\frac{13}{52}$

39. Janelle cut a board 30 feet long into 2 pieces. The ratio of the lengths of the 2 pieces is 2:3. What is the length, to the nearest foot, of the shorter piece?

A. 5
B. 6
C. 12
D. 15
E. 18

40. On a finch farm, 100 pounds of birdseed are required to feed 60 finches per month. To the nearest pound, how many pounds of birdseed are required to feed 25 finches for a month?

A. 15
B. 35
C. 41
D. 42
E. 45

41. Karen has 6 sweaters, 5 shirts, and 3 pairs of pants. Her pant selection includes one pair each of khakis, jeans, and dress pants. If Karen randomly selects an outfit consisting of 1 sweater, 1 shirt, and 1 pair of pants, what is the probability that the pants are khakis?

A. $\frac{1}{90}$
B. $\frac{1}{30}$
C. $\frac{1}{3}$
D. $\frac{2}{3}$
E. $\frac{1}{15}$

42. The ratio of the radii of two circles is 3:8. What is the ratio of their circumferences?

A. 9:64
B. 3:16π
C. 6:16
D. 3:16
E. 1:2π

43. Priya, Sheldon, and Bernadette shared a pizza. Priya ate $\frac{1}{4}$ of the pizza, Sheldon ate $\frac{2}{3}$ of the pizza, and Bernadette ate the rest. What is the ratio of Priya's share to Sheldon's share to Bernadette's share?

- 9:64:**1**
- 3:16:**2**
- 6:16:**4**
- 3:8:**1**
- 1:2:**3**

44. Rob and Jay cut a 45 ft. long board into two pieces. The ratio of the long piece to the short piece is 7:2. What is the length of the short piece?

A. 9
B. 10
C. 12
D. 14
E. 15

INTERMEDIATE ALGEBRA AND COORDINATE GEOMETRY QUESTIONS

ABSOLUTE VALUE & INEQUALITIES

Absolute Value: The distance from 0 (always positive)

Example: $|-7| = 7$

Note: *When working with an equation or inequality with an absolute value, remember that all negative values inside of that absolute value will become positive. That said, it's important to consider exactly what is allowed inside of the absolute value bars. Then find those values of x that will produce the values that are allowed.*

Example: $|3x + 5| \leq 11$

$3x + 5 \leq 11$ and $3x + 5 \geq -11$

$x \leq 2$ and $x \geq \frac{-16}{3}$

Inequalities: The alligator is hungry and always eats the BIGGER value

Example: *1 is less than 2, $1 < 2$*

Tips: *When you divide an inequality by a negative number, reverse the inequality*

Example Problems

45. If $-8 \leq x \leq 8$, then which of the following must be true?

A. $x^2 \leq 8$
B. $8x > 64$
C. $x^3 \geq 8$
D. $|x+3| \leq 11$
E. $x^2 \geq 8$

46. Which of the following is equivalent to the inequality $4x - 8 > 8x + 16$?

A. $x < -6$
B. $x > -6$
C. $x < -2$
D. $x > 2$
E. $x < 6$

47. Which of the following inequalities defines the solution set for the inequality $18 - 3x \leq 10$?

A. $x \leq \frac{8}{3}$
B. $x \geq -\frac{8}{3}$
C. $x \leq -\frac{3}{8}$
D. $x \geq \frac{8}{3}$
E. $x \leq -\frac{8}{3}$

48. If $|3x - 5| \leq 4$, what is the value of x?

A. $\frac{1}{3} \leq x \leq 3$

B. $x \leq 3$

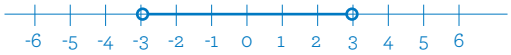
C. $x \geq \frac{1}{3}$

D. $x \leq \frac{1}{3}$

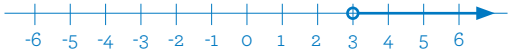
E. $x \geq 3$

49. Which of the following graphs represents the solution set of the inequality $|x| > 3$ on the real number line?

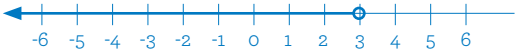
A.



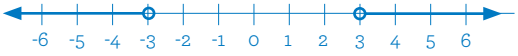
B.



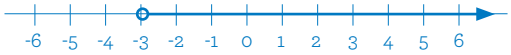
C.



D.



E.



50. Which of the following is **an** irrational number that is a solution to the equation $|x^2 - 13| - 3 = 0$?

A. 2

B. $\sqrt{10}$

C. 4

D. $5\sqrt{2}$

E. $2\sqrt{3}$

POWERS, EXPONENTS & ROOTS

Multiplying Exponents: When two powers have the same base, add the exponents

Example:

$x^a \times x^b = x^{a+b}$

$x^5 \times x^3 = x^8$

$(x \times x \times x \times x \times x) \times (x \times x \times x)$

$x^{5+3} \times x^8$

Dividing Exponents: When two powers have the same base, subtract the exponents

Example:

$$\frac{x^a}{x^b} = x^{a-b}$$

$$\frac{5^5}{5^2} = 5^3 = 125$$

$$\frac{5x5x5x5x5}{5x5} = 5^3 = 125$$

Negative Exponents: A negative exponent means that that the exponential term is on the wrong side of a fraction. When you see a negative sign in the exponent on the ACT, you will most likely just make the exponent positive and move the entire term into the denominator, leaving 1 in the numerator.

$$x^{-b} = \frac{1}{x^b}$$

Example:

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

Two exponents, one base:

When one exponent is raised to another exponent, multiply the exponents

$(x^a)^b = x^{axb}$

Example:

$(a^3)^5 = a^{15}$

$(a^{-2})^5 = a^{-10} = \frac{1}{a^{10}}$

Distributive Property:

If a base has more than one element, apply the exponent to each element.

$(xy)^a = x^a \times y^a$

Example:

$6^5 = (3x2)^5 = 3^5 \times 2^5$

Base raised to power of 0:

Any base to 0 power always equals 1

$x^0 = 1$

Example:

$100^0 = 1$

Positives, negatives:

Any negative number to an even power is positive

Any negative number to an odd power is negative

$(-1)^n = 1$, if n is even

$(-1)^n = -1$, if n is odd

Examples:

$(-1)^8 = 1$

$(-1)^{-1} = -1$

Fractional Exponents:

$a^{\frac{1}{n}}$ is the same as taking the nth root of a

Examples:

$\sqrt{49}^{\frac{1}{2}} = \sqrt{49} = 7$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

Example Problems

51. $(3x^3)^3$ is equivalent to:

A. x

B. $9x^6$

C. $9x^9$

D. $27x^6$

E. $27x^9$

52. For all $a > 1$ the expression $\frac{3a^4}{3a^6}$ is equivalent to what?

A. 12

B. $-a^2$

C. a^2

D. $-\frac{1}{a^2}$

E. $\frac{1}{a^2}$

53. In the set of real numbers, what is the solution of the equation $8^{2x+1} = 4^{1-x}$?

A. $-\frac{1}{3}$

B. $-\frac{1}{4}$

C. $-\frac{1}{8}$

D. 0

E. $\frac{1}{7}$

54. Which of the following equations expresses **c** in terms of ***a*** for all real numbers **a**, **b**, and **c** such that $a^4 = b$ and $b^3 = c$?

A. $c = a^6$
B. $c = a^{12}$
C. $c = a^7$
D. $c = \frac{2a}{3}$
E. $c = a$

55. Which of the following expressions is equivalent to $(2x^2)(x^2y^2 + 3xy)$?

A. $2x^3y^2 + 3x^3y$
B. $2x^4y^2 + 3x^3y$
C. $2x^4y + 6x^3y$
D. $2x^4y^2 + 6x^3y$
E. $2x^4y^2 + 2x^2y$

56. Which of the following expressions is equivalent to $(-x^3y^5)^7$?

A. $x^{10}y^{12}$
B. $7x^{21}y^{28}$
C. $-x^{10}y^{12}$
D. $-x^{21}y^{35}$
E. $x^{21}y^{35}$

57. The solution set of $\sqrt{x-3} > 7$ is the set of all real numbers **x** such that:

A. $x > 52$
B. $x > 49$
C. $x < 52$
D. $x > 8$
E. $x > 2$

FACTORING AND SOLVING QUADRATIC FUNCTIONS

"FOIL": $(x + a)(x + b) = x^2 + (b + a)x + ab$

Example: $(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$

Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$

Example: $(a - 3)(a + 3) = a^2 - 9$

Perfect Square Trinomials: $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$
$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Examples: $(x + 2)^2 = (x + 2)(x + 2) = x^2 + 4x + 4$
$(w - 3)^2 = (w - 3)(w - 3) = w^2 - 6w + 9$

Example Problems

58. When asked how many cats he had, Bashir said, "If you square the number of cats I have and then subtract 21 times the number of cats I have, the result is 46." How many cats does Bashir have?

A. 15
B. 23
C. 24
D. 30
E. 33

59. When $(3x - 2)^2$ is written in the form $ax^2 + bx + c$, where **a**, **b**, and **c** are integers, $a + b + c =$?

A. −4
B. 1
C. 7
D. 11
E. 19

60. What is the sum of the 2 solutions of the equation $x^2 + x - 10 = 0$?

A. −8
B. −4
C. −1
D. 5
E. 8

61. What are the values for **x** that satisfy the equation $(2x + a)(x - b) = 0$?

A. $-\frac{a}{2}$ and b

B. $\frac{a}{2}$ and −b

C. −a and b

D. $-\frac{a}{2}$ b

E. a and −b

FUNCTIONS

Functions are operations in which you input one value (usually represented by x) into an equation to get an output (usually represented by f(x) or y) . They can tell you to do anything.

Let's do a couple examples with this function $f(x) = 2x^2 + 3$:

What is the value of f (3)?

Whatever is inside the parentheses always goes in for x

$f(3) = 2(3)^2 + 3$

$f(3) = 21$

In other words, if , then x = 3, then y = 21.

If f (p) = 75, what is the value of p?

This gives us the result (the y-value) and asks us to solve for a variable (the x-value)

$f(p) = 75$

$2p^2 + 3 = 75$

p = 6 and – 6

In other words, if y = 75 , then x = 6 or x = – 6

What is the value of f (x + 3)?

Again, whatever is inside the parentheses always goes in for x

$f(x + 3) = 2(x + 3)^2 + 3$

$f(x + 3) = 2(x2 + 6x + 9) + 3$

$f(x + 3) = 2x^2 + 12x + 21$

In other words, if x = x + 3, then $y = 2x^2 + 12x + 21$.

What is the value of f (f (1))?

Now we have two functions to solve but the same rules apply. Start with the inside parentheses and we are going to have to solve it twice.

$f(1) = 2(1)^2 + 3 = 5$

Then $f(f(1)) = f(5)$

$f(5) = 2(5)^2 + 3 = 53$

In other words, if $x = 1$, then $y = 5$ and then if $x = 5$, then $y = 53$

Example Problems

62. A function f(x) is defined as $f(x) = -8x^2$. What is the value of $f(-3)$?

A. −576
B. −72
C. 57
D. 72
E. 192

63. If $\mathbf{f(x) = x^2 - 2x + 1}$ and $\mathbf{g(x) = 2\sqrt{x}}$, then what is the value of $\mathbf{g(9) \cdot f(2)}$?

A. 0
B. 1
C. 3
D. 6
E. 12

64. If $\mathbf{f(x) = x^2 + 5x - 6}$, what is the value of $\mathbf{f(x + 2)}$?

A. $7x + 8$
B. $x^2 + 9x + 8$
C. $x^2 + 9x + 4$
D. $x^2 + 5x - 8$
E. $x^2 + 4x - 6$

65. If $\mathbf{f(x) = 2x^2 + 3x - 5}$, then $\mathbf{f(-3)} =$?

A. −32
B. −5
C. 0
D. 4
E. 22

"FUNKY" FUNCTIONS

Example: *If you are given the equation x y = 2x – y and are asked to solve 3 2, you would just substitute based on the symbols and position of the numbers.*

3 2 = 2(3) – (2)

3 2 = 4

Example Problems

66. Let the function **f (a,b)** be defined as **f (a,b) = $a^2 - b^2$**
For all x and y, **f((x + y),(x – y)) =** ?

A. 4xy
B. $2x^2 + 2y^2$
C. $2x^2 + 4xy + 2y^2$
D. $x^2 + 2x + y^2$
E. $2x^2 - 2y^2$

67. Let the operation a•b be defined as $a^2 + 2b + 3$.
What is the value of 2•4?

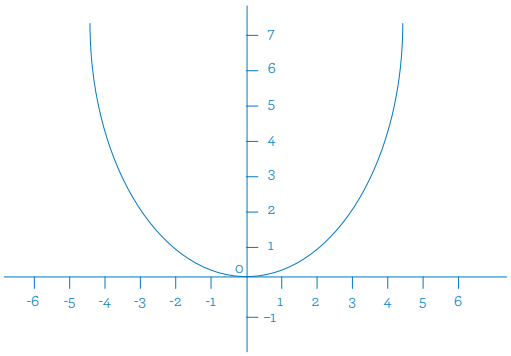
A. 3
B. 9
C. 11
D. 15
E. 23

GRAPHICAL TRANSFORMATIONS OF FUNCTIONS

Base Function:	$y = f(x)$
Amplitude Increase:	$y = 3f(x)$
(Vertical Stretch)	All y values multiplied by 3
Amplitude Decrease:	$y = 1/2f(x)$
(Horizontal Stretch)	All y values multiplied by ½
Horizontal Shift Right:	$y = f(x - 2)$
	All points shifted two to the right
Horizontal Shift Left:	$y = f(x + 1)$
	All points shifted one to the left
Vertical Shift Up:	$y = f(x) + 2$
	All points shift up two
Vertical Shift Down:	$y = f(x) - 4$
	All points shifted down four

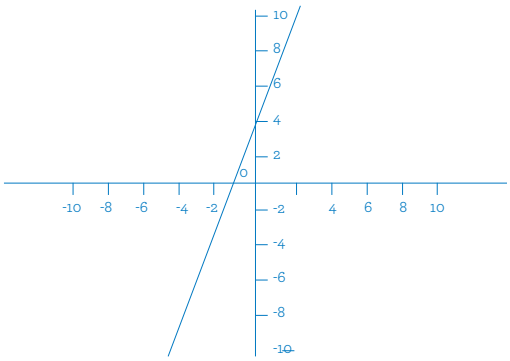
Example Problems

68. The graph of $\mathbf{y = x^2}$ is shown in the standard **(x,y)** coordinate plane below. For which of the following equations is the graph of the parabola shifted 2 units to the left and 5 units up?



A. $y = (x - 2)^2 + 5$
B. $y = (x + 2)^2 - 5$
C. $y = (x + 2)^2 + 5$
D. $y = (x + 5)^2 - 2$
E. $y = (x - 2)^2 - 5$

69. One of the following is an equation of the linear relation shown in the standard **(x,y)** coordinate plane below. Which equation is it?



A. $y = 4x + 3$
B. $y = -4x$
C. $y = 3x - 2$
D. $y = 3x + 2$
E. $y = 3x + 4$

MATRICES

Matrix Addition:

Matrices of the same dimensions can be added by adding the numbers that have the same position in the matrices.

Equation form:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$$

Matrix Multiplication:

In order to multiply two matrices, the number of rows of the left matrix must equal the number of columns in the matrix to the right.

For example, a 3x2 matrix can multiply a 2x3 matrix, but not a 3x2 matrix since there would be 3 rows in the left matrix and 2 columns in the right matrix.

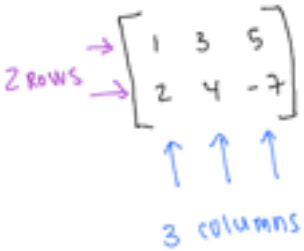
Matrix Multiplication is an operation completed as follows:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}\ b_{11} + a_{12}\ b_{21} & a_{11}\ b_{12} + a_{12}\ b_{22} \\ a_{21}\ b_{11} + a_{22}\ b_{21} & a_{21}\ b_{12} + a_{22}\ b_{22} \end{pmatrix}$$

All other matrix operations will be defined on the test.

Matrix Multiplication Lesson:
The most important thing to remember about multiplying matrices is that humans made this up. It is simply a way of combining and organizing data, with a simple set of rules for construction.

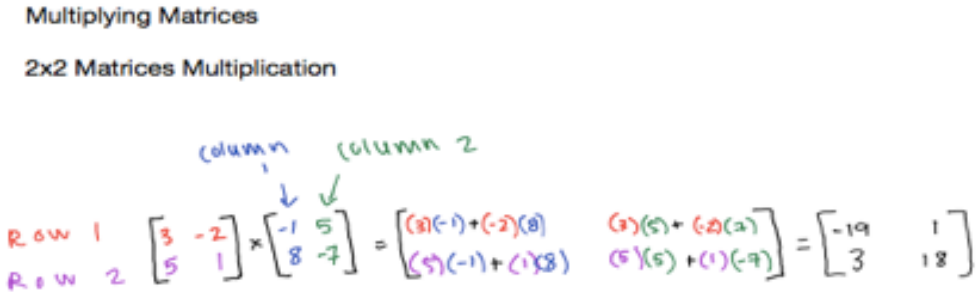
A matrix is constructed in a row by column fashion. To name a matrix, you state: number or rows, by number of columns. Examples: 3 rows x 2 columns= 3x2 matrix 2 rows x 3 columns=2x3 matrix

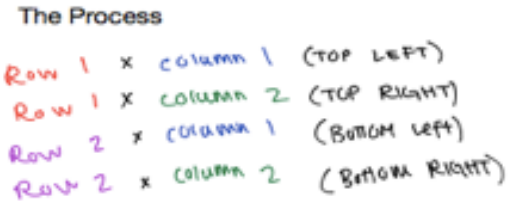


One can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

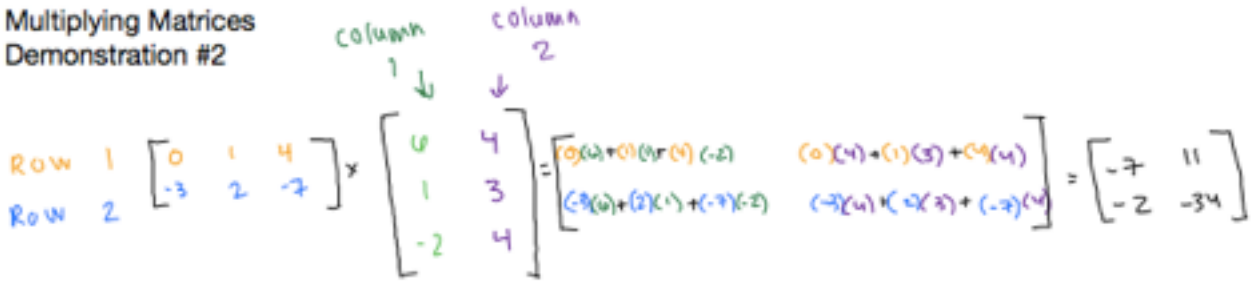
As you can see by the highlighted "3s" above, you can multiply these two matrices.

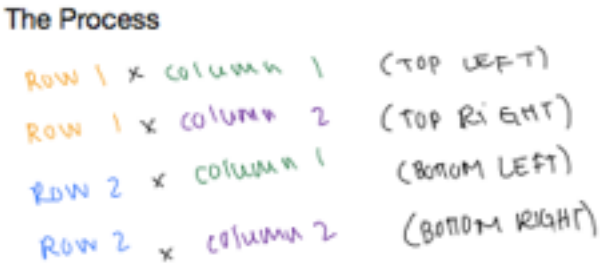
If the matrices were 2x2 and 3x2, you could not multiply them. Example 1:





Example 2:

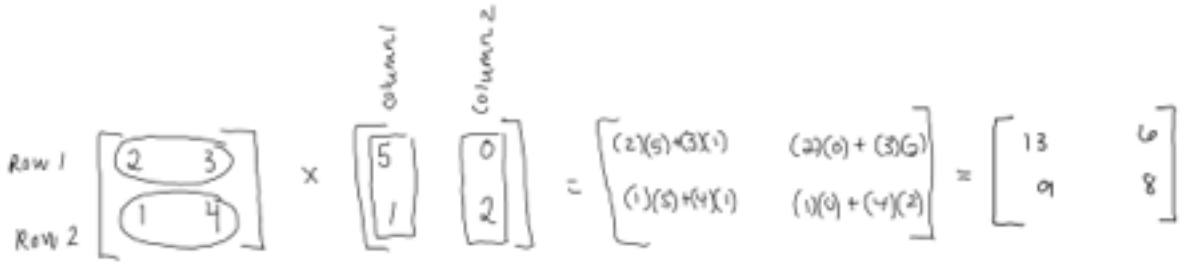




If using colors helps you master this process, practice a few of the examples with colored pencils or pens. Once you get the hang of it, practice without colors, since you won't be using color differentiation on the official exam.

Here's a good method to organize the process:

Draw circles around each row, and draw boxes around each column. As you go through the process, cover the row and column that you are not using, to focus on the row and column that you are using. When you are entirely done using row 1, cross it off and move on to row 2.



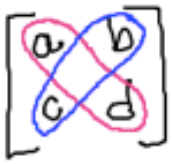
On the ACT, you may see a question that presents a matrix and asks you to calculate the determinant. It may also give you a matrix with a missing variable and the matrix's determinant, asking you to solve for the variable.

First of all: What is the determinant?

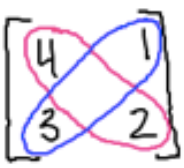
The determinant is a numerical value that proves useful in any field involving multiple interacting variables. People who work in data analytics, statistics, engineering, and other math-based professions, use determinants to help manage large sets of interacting data.

In our case, the determinant can be useful in either finding missing values in a matrix or simply answering an ACT question correctly!

To calculate the determinant of a 2x2 matrix, you cross multiply the top left term by the bottom right term and subtract the product of the top right term and the bottom left term. These variables are labeled a, b, c, and d in the example below. The first example uses variables, and the second example demonstrates the process with numbers.



$(ad) - (bc)$



$(4 \cdot 2) - (3 \cdot 1) = 8 - 3 = \boxed{5}$

In the case of the second example, you can conclude that the determinant of this 2x2 matrix is 5.

Example Problems

70. By definition, the determinant $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ equals **ad – bc**. What is the value of $\begin{pmatrix} 3x & 4y \\ 5x & 3y \end{pmatrix}$ when **x = -2** and **y = 1** ?

A. –58
B. 0
C. 6
D. 22
E. 32

71. What is the matrix product $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \end{pmatrix}$

A. $\begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 8 & 0 & -4 \end{pmatrix}$

B. $\begin{pmatrix} 2 & 0 & -4 \end{pmatrix}$

C. $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

D. $\begin{pmatrix} 0 \end{pmatrix}$

E. $\begin{pmatrix} 2 & 4 & 8 \\ 0 & 0 & 0 \\ -1 & -2 & -4 \end{pmatrix}$

LOGARITHMS

Logarithms are the inverses of exponential functions.
Put more simply, $\log_b a = c$ asks: b to what power c equals a?

So $\log_b a = c$ can be rewritten as $b^c = a$

Rules of Logarithms

Power Rule $\log_b a^c = c \cdot \log_b a$

Product Property $\log_b(ac) = \log_b a + \log_b c$

Quotient Property $\log_b \frac{a}{c} = \log_b a - \log_b c$

Logarithms Lesson

Having nothing to do with either logs or rhythm, logarithms can be a source of confusion for many students on the ACT. While you've likely seen them before, chances are you might not remember exactly how they work. Luckily, the log problems on the ACT are all relatively simple, and they only test a basic understanding of the log concept. So, while you might be initially put off by a problem containing the not-so-familiar "log" symbol, these questions are often some of the easiest on the test and should be considered easy improvements you can make to your score. By understanding a few simple facts about logs, you'll be able to get these questions right 100% of the time and help boost your math section score!

WOOHOO!
The most essential thing to remember about logarithms is the following idea:
$\log_3 9 = 2$
$3^2=9$

The two equations above represent the same idea, but one is in log form while the other is in exponent form. You can always switch between log and exponent form in the way shown above, and that's more or less all you need to know for the ACT.
For instance:
$\log_2 8 = 3$
$2^3=8$
$\log_4 16 = 2$
$4^2=16$

And so on and so forth. Note that you need not understand exactly what the log function does; just think of it as another way of expressing an exponent. On the test, you should almost always convert any logs you see into exponent form, as these should be more familiar to you and will make the problems easier.

Now let's see an example with a variable in it!

If $\log_x 81 = 2$, what is the value of x?

In log form this might look intimidating, so let's convert it to the more familiar exponent form:

$\log_x 81 = 2$
$x^2=81$
Now using some simple algebra, you can solve for x to find $x = \pm 9$.

NOTE: Log bases (the little subscript number next to the "log") can never be negative numbers. For this reason, the solution x = -9 for the above equation is not valid. Thus, the answer is only x = 9.

Sometimes the variable might have a coefficient or be part of a parenthetical term. Let's see examples.

$\log_{2x} 64 = 3$
$(2x)^3=64$
$8x^3=64$
$x^3=8$
$x=2$

OR
$\log_{x-4} 9=2$
$(x-4)^2=9$
$(x-4)(x-4)=9$
$x^2-8x+16=9$
$x^2-8x+7=0$
$(x-7)(x-1)=0$
$x-7=0$ and $x-1=0$
$x=7$ and $x=1$

NOTE: Plugging in the solution x = 1 gives us a negative log base, which (as we discussed above) is a big no-no. Therefore, the only valid solution is x = 7. And that's the gist of it! That's how you solve log problems on the ACT!

Before you're off solving log questions left and right, there are a few other important things for you to know:

You might run into a log that doesn't have a base. It'll look like this:
log100=2
When the log has no base number, we assume it to be base "10". So the equation above actually says $\log_{10} 100=2$

Also, you can evaluate logs using your calculator. For instance, if a question asked you the following:
If log27=x, what is the value of x?

Use your graphing calculator: On a TI-84 plus, click the MATH button, scroll down to the function "logBASE" and input the number to easily solve the problem. Try it out on your calculator!

Example Problems

72. Which of the following is a value of b that satisfies $\log_b 27 = 3$?

A. 3
B. 9
C. 10
D. 27
E. 81

73. If $x = 3$ and $y = 9$, what is the value of $\log_3 \left(\frac{x}{y}\right)$?

A. -1

B. $\frac{1}{2}$

C. 1

D. 2

E. 3

74. If $\log_b x = z$ and $\log_b y = w$, then $\log_b (xy)^2$?

A. $z + w$
B. z^2w^2
C. $2(z + w)$
D. $2zw$
E. zw

COMPLEX NUMBERS

Complex numbers are expressions that involve the imaginary number i. They are usually expressed in the general form $a + bi$, where $i = \sqrt{(-1)}$. Imaginary numbers are essentially used most frequently to rewrite a radical when there is a negative number inside.

For example, $\sqrt{(-4)} = \sqrt{4} \cdot \sqrt{(-1)} = 2i$.

On the ACT, it's important to remember that $i^2 = -1$.
Powers of i repeat in groups of 4.

That is:

$i = \sqrt{(-1)}$ $\quad i^2 = -1$ $\quad i^3 = -\sqrt{(-1)}$ $\quad i^4 = 1$

$i^5 = \sqrt{(-1)}$ $\quad i^6 = -1$ $\quad i^7 = -\sqrt{(-1)}$ $\quad i^8 = 1$

The ACT will sometimes ask you to find large powers of i, for example i^{23}. We use the above pattern and recognize that 4 goes into 23 five times with a remainder of 3. There are then 3 more i terms that must be multiplied in to reach 23. We then know that the 23rd i is the same as the 3rd i in the repeating pattern.
Thus, $i^{23} = -\sqrt{(-1)}$.

Example Problems

75. Which expression is equivalent to $(3+2i)(4-3i)$?

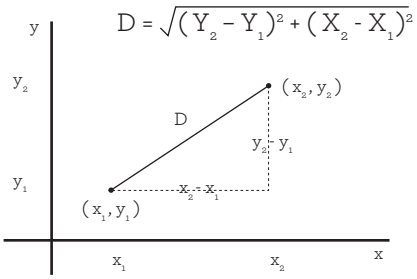
A. $18 - i$
B. $6 - i$
C. $6 + i$
D. $18 + i$
E. $12 - 1$

76. What is the value of $3i^{63}$?

A. $-3i$
B. $6 - i$
C. $6 + i$
D. $18 + i$
E. $12 - i$

COORDINATE GEOMETRY

Distance Formula: How to find the distance between two points (think of making a right triangle out of the two points and using the Pythagorean Theorem).

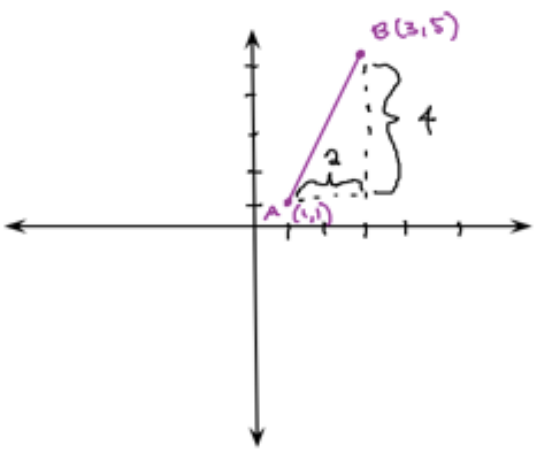


Common mistakes include:

1) Mixing up y1 and y2 or x1 and x2, which is why it is important to establish your coordinate points, as shown in the example above.

2) Making sign errors, so be sure to keep track of your positive and negative signs.

There is another easy way to look at these problems. Notice that the distance we are looking for makes the hypotenuse of a right triangle if you draw in the other lines. We can easily measure the change in the x direction and y direction, and then use the Pythagorean theorem to calculate our distance "c".



Pythagorean Theorem:

$a^2 + b^2 = c^2$

we are looking for the hypotenuse "c"

So...

$2^2 + 4^2 = c^2$

$4 + 16 = c^2$

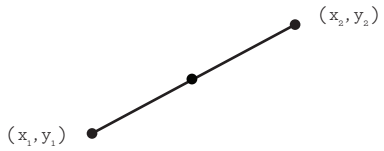
$20 = c^2$

$\sqrt{20} = c$

$2\sqrt{5} = c$

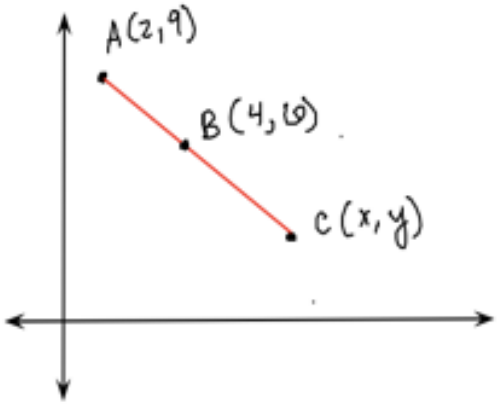
Midpoint Formula: How to find the midpoint of a line segment

$$\left(\frac{x_1 + x_2}{2} \quad \frac{y_1 + y_2}{2}\right)$$



Sometimes the ACT will give you an initial point and a midpoint, and ask you to find the end point. Your question could look like this:

Find the endpoint of line segment AC, given point A (2,9) and midpoint B (4,6)



midpoint formula

$\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

we know that B is the midpoint, and we know the coordinates of B, so we can do the following:

A = (2,9) B = (4,6) C = (x_2, y_2)

x_1 y_1 midpoint (x,y)

$\frac{2+x_2}{2} = 4$

$2+x_2 = 8$

$x_2 = 6$

$\frac{9+y_2}{2} = 6$

$9+y_2 = 12$

$y^2 = 3$

$(x_2, y_2) = C$

$(6,3) = C$

Lines: Have a consistent slope - so the "rate of change" is constant. Therefore, a linear function will have the same slope between any two points.

Parallel lines have the same slope.

Perpendicular lines have opposite reciprocal slopes.

Horizontal lines have a slope of zero and are expressed as y = a, where a is a constant.

Vertical lines have an undefined slope and are expressed as x = a, where a is a constant.

Slope-Intercept Form: y = mx +, where m = slope and b = y-intercept.

Point-Slope Form: $y - y_1 = m(x - x_1)$, where m = slope

Standard Form of a Line: Ax + By, where A is a positive integer and B and C are integers.

Collinear Points: Points that lie on the same line.

Example Problems

77. In the standard (x,y) coordinate plane, point M with coordinates (5,4) is the midpoint of $\overline{AB}$, and B has coordinates (7,3) . What are the coordinates of A?

A. (17,11)
B. (9,2)
C. (6,3.5)
D. (3, 5)
E. (−3, −5)

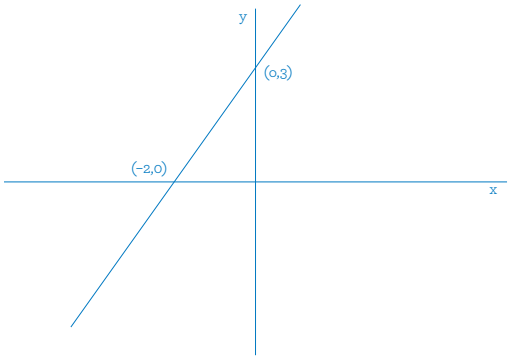
78. In the standard (x,y) coordinate plane, point A has coordinates (2,3) and point B has coordinates (5,7). What are the coordinates of the midpoint of AB?

A. (2, 5)
B. (3.5, 5)
C. (3.5, 7)
D. (5,7)
E. (7, 10)

79. In the xy-coordinate plane, lines l and q are perpendicular. If line l contains the points (0,0) and (1,2), and line q contains the points (1,2) and (0,t), what is the value of t?

A. −2
B. 0
C. 2.5
D. 3
E. 4

80. Line n (not shown) is perpendicular to line m and shares a y-intercept with line m. Which of the following points is on line n?



A. (-6, 8)
B. (-3, 5)
C. (2, 6)
D. (3, 0)
E. (6, -2)

81. The points (1,3) and (7,11) are endpoints of a diameter of circle O in the standard (x,y) coordinate plane. What is the length of the radius of circle O?

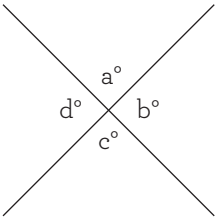
A. 3
B. 4
C. 5
D. 10
E. 14

PLANE GEOMETRY AND TRIGONOMETRY QUESTIONS

ANGLES

Vertical Angles:

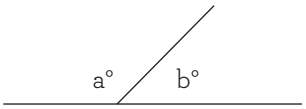
Vertical angles are formed by two intersecting lines or segments and are always congruent.



a = c and b = d

Linear Pair: (Supplementary Angles)

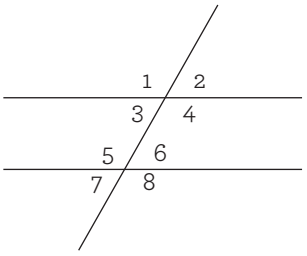
Two angles that form a line are equal to 180 degrees.



a + b = 180°

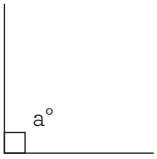
Angles formed by Parallel Lines:

Angles that are congruent:
Alternate Interior Angles (Ex. 3 & 6)
Alternate Exterior Angles (Ex.1 & 8)
Corresponding Angles (Ex. 1 & 5)
Angles that add up to 180 degrees:
Same Side Interior Angles (Ex. 3 & 5)



Right Angle:

A right angle measures 90 degrees.



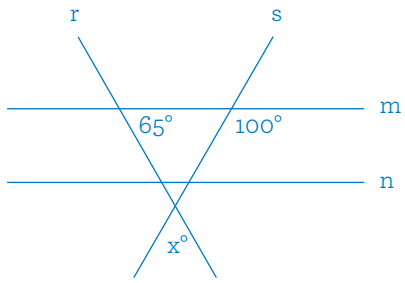
a = 90°

Complimentary Angles:

Two angles that add up to 90 degrees.

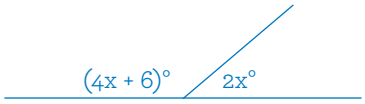
Example Problems

82. In the figure below, lines m and n are parallel, transversals r and s intersect to form an angle of measure x°, and 2 other angle measures are as marked. What is the value of x?



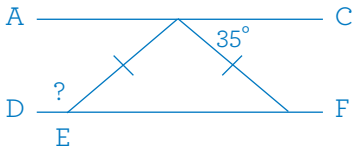
A. 15
B. 25
C. 35
D. 65
E. 80

83. What is the degree measure of the smaller of the 2 angles formed by the line and the ray shown in the figure below?



A. 14°
B. 28°
C. 29°
D. 58°
E. Cannot be determined from the given information

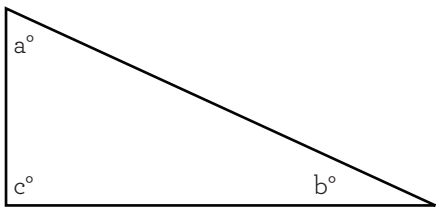
84. In the figure below, B is on AC, E is on DF, AC is parallel to DF, and BE is congruent to BF. What is the measure of ∠DEB?



A. 35°
B. 135°
C. 145°
D. 155°
E. 215°

TRIANGLES

Angles: The inside angles of a triangle always add up to 180 degrees.



$a + b + c = 180°$

Area: $A = \frac{1}{2}$ (base * height), where is the length of a side.

Area of Equilateral Triangle: $A = \frac{\sqrt{3}}{4} s^2$, where is the length of a side.

Triangle Inequality Theorem: The sum of the two shortest sides of a triangle is always greater than the length of the third side.

Equilateral Triangle: All three sides are equal and all three interior angles are 60 degrees.

Isosceles Triangle: Two equal sides. Base angles (angles across from the congruent sides) are equal.

Proportionality in Triangles: In every triangle, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle.

Pythagorean Theorem: $a^2 + b^2 = c^2$, where a and b are legs of a right triangle, and is the hypotenuse.

Pythagorean Triples: Three integers that, as side lengths of a triangle, form a right triangle.

Examples:

3/4/5 or any multiple (6/8/10, 15/20/25, 30/40/50)

5/12/13 or any multiple

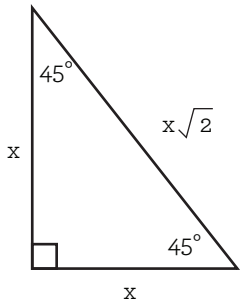
8/15/17or any multiple

7/24/25 or any multiple

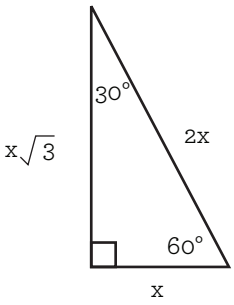
Similar Triangles: Triangles that have the same angle measures but different side lengths. If two corresponding angles are equal, the corresponding sides are proportional to each other.

Example: *A 3/4/5 and a 6/8/10 triangle must be similar because have the same ratios*

45-45-90 Triangles: (Isosceles Right Triangles) Have the following ratio of side lengths:



30-60-90 Triangles: Have the following ratio of side lengths:



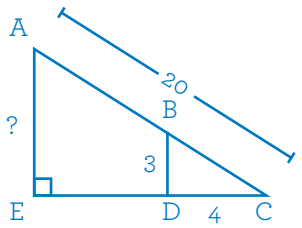
Example Problems

85. In ΔABC, the sum of the measures of $\angle A$ and $\angle B$ is 47 degrees. What is the measure of $\angle C$?

A. 43
B. 86
C. 94
D. 133
E. 223

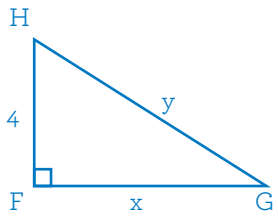
86. In the right triangle ΔACE below $\overline{BD}$, is parallel to $\overline{AE}$ and is perpendicular to $\overline{EC}$. What is the length, in feet, of $\overline{AE}$?

A. 10
B. 12
C. 15
D. 16
E. 18



87. For ΔFGH, shown below, which of the following is an expression for y in terms of x?

A. $x + 4$
B. $\sqrt{x^2 + 4}$
C. $\sqrt{x^2 + 8}$
D. $\sqrt{x^2 - 16}$
E. $\sqrt{x^2 + 16}$

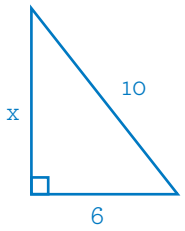


88. A triangle with a perimeter of 95 inches has one side that is 20 inches long. The lengths of the other two sides have a ratio of 2:3. What is the length, in inches, of the longest side of the triangle?

A. 20
B. 30
C. 40
D. 45
E. 50

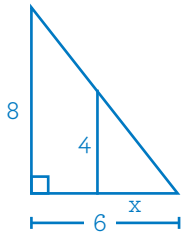
89. In the right triangle shown below, what is the value of x?

A. 4
B. 5
C. 7
D. 8
E. 9



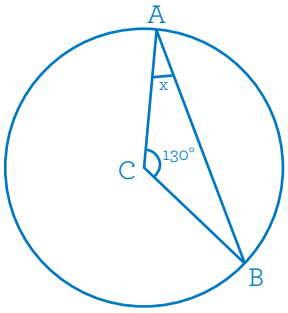
90. Two trees are planted next to one another on level ground such that at a certain time of the day, their shadows meet at the same place. One tree is 8 feet tall, the other is 4 feet tall. The length of the shadow of the taller tree is 6 feet, as depicted in the figure below. What is the length of the shadow of the shorter tree?

A. 1
B. 2
C. 3
D. 4
E. 5



91. Points A and B lie on the circle below, where central angle ∠ACB measures 130°. What is the measure of ∠BAC?

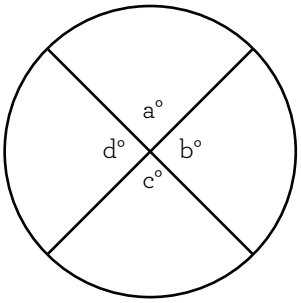
A. 15°
B. 25°
C. 45°
D. 50°
E. 60°



CIRCLES

Area of a Circle: $A = \pi r^2$

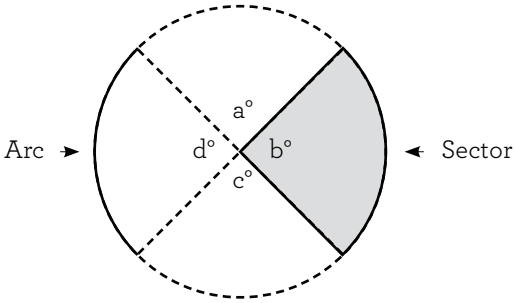
Central Angles: The central angles of a circle add up to 360 degrees.



Circumference of a Circle: $c = 2\pi r$ or $c = \pi d$

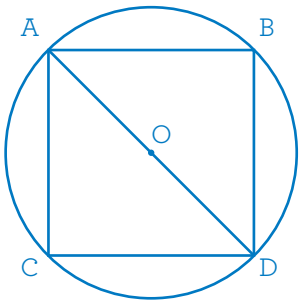
Area of a Sector: $\frac{n}{360} \times \pi r^2$, where n is the central angle

Arc Length: $\frac{n}{360} \times 2\pi r$ or $\frac{n}{360} \times \pi d$, where n is the central angle



Example Problems

92. Square ABCD has a perimeter of 8. What is the circumference of circle O?

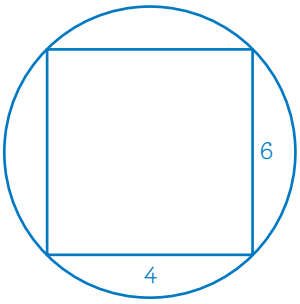


A. π
B. 2π
C. $2\pi\sqrt{2}$
D. 4π
E. Cannot be determined from the information given.

93. What is the arc length of a sector of a circle with a central angle of 45° and radius of 8?

A. π
B. 2π
C. 4π
D. 3π
E. 8π

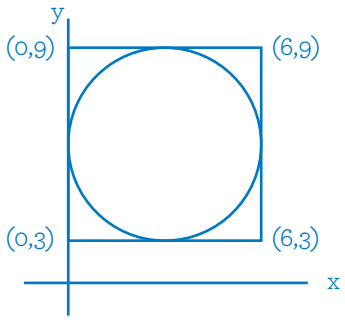
94. A 4-inch by 6-inch rectangle is inscribed in a circle as shown below. What is the circumference of the circle, in square inches?



A. $\pi\sqrt{10}$
B. $2\pi\sqrt{10}$
C. $2\pi\sqrt{13}$
D. 10π
E. 13π

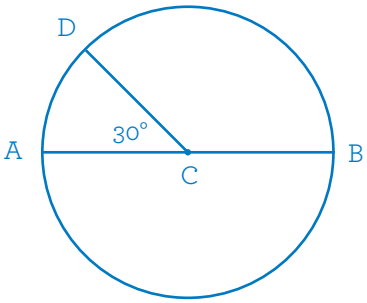
95. In the standard (x,y) coordinate plane below, the vertices of the square have coordinates (0,3), (6,3) (6,9), and (0, 9). Which of the following is the area of the circle that is inscribed in the square?

A. 3π
B. 6π
C. 9π
D. 12π
E. 36π



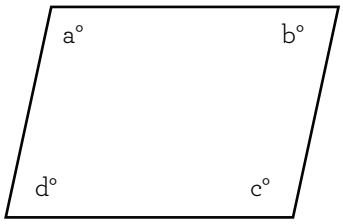
96. Points A and B are the endpoints of the diameter of a circle with center C, as shown below. Point D is on the circle, and measures 30°. The shortest distance along the circle from A to D is what fraction of the distance along the circle from A to B?

A. $\frac{1}{12}$
B. $\frac{1}{9}$
C. $\frac{1}{6}$
D. $\frac{5}{6}$
E. $\frac{11}{12}$



QUADRILATERALS

Interior Angles: Always add up to degrees



a + b + c + d = 360°

Area of Square: A = side x side, where is the length and is the width

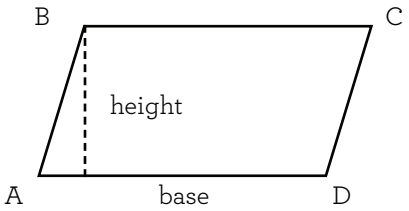
Diagonal of Square: side x$\sqrt{2}$

Area of Rectangle: A = length x width

Perimeter of a Rectangle: P = 2l + 2w, where l is the length and w is the width

Area of Parallelogram: A = base x height

(note: height is NOT equal to width, unless it is a rectangle, and the height must be perpendicular to the base)



Angles in Parallelogram: Opposite angles are equal

Area of a Trapezoid: Median x Height or $\frac{b_1 + b_2}{2}$ x h

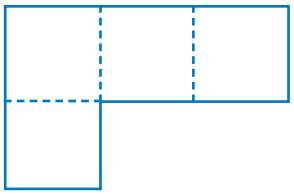
Example Problems

97. A rectangle has an area of 32 square feet and a perimeter of 24 feet. What is the shortest of the side lengths, in feet, of the rectangle?

A. 1
B. 2
C. 3
D. 4
E. 8

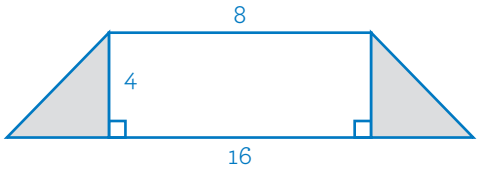
98. The 6-sided figure below is divided into 4 congruent squares. The total area of the 4 squares is 64 square inches. What is the perimeter, in inches, of the figure?

A. 20
B. 24
C. 36
D. 40
E. 64



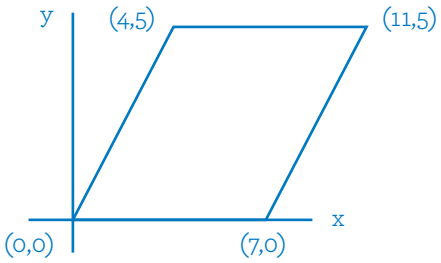
99. The trapezoid below is divided into 2 triangles and 1 rectangle. Lengths are given in inches. What is the combined area in square inches, of the 2 shaded triangles?

A. 8
B. 16
C. 24
D. 32
E. 64



100. In the standard (x,y) coordinate plane below, the points (0,0), (7,0), (4,5), and (11,5) are the vertices of a parallelogram. What is the area, in square coordinate units, of the parallelogram?

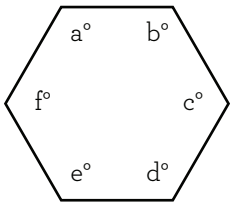
A. 15
B. 25
C. 30
D. 35
E. 55



POLYGONS

Area of a Polygon: $A = \frac{1}{2}aP$, where a is the apothem and P is the Perimeter.

Sum of Interior Angles of a Polygon: Sum = (n – 2)180, where n is the number of sides.



a + b + c + d + e + f = (n – 2)180

Interior Angle of a Polygon: Interior Angle = $\frac{(n-2)180}{n}$, where n is the number of sides

Example Problems

101. The measure of each interior angle of a regular polygon with n sides is $\frac{(n-2)180}{n}$ degrees. What is the measure of each interior angle of a regular polygon with n sides, in radians?

A. $\frac{(n-2)\pi}{4n}$

B. $\frac{(n-2)\pi}{2n}$

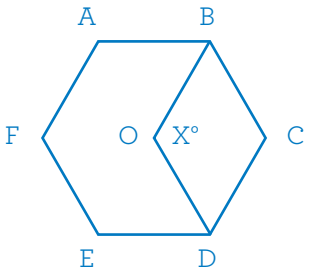
C. $\frac{(n-2)\pi}{n}$

D. $\frac{(n-2)2\pi}{n}$

E. $\frac{(n-2)4\pi}{n}$

102. In the figure below, ABCDEF is a regular hexagon. What is the value of x?

A. 60
B. 90
C. 100
D. 120
E. 160



103. The average of the measures of the interior angles of a polygon is 135°. Which of the following polygons has this property?

A. Triangle
B. Rectangle
C. Pentagon
D. Hexagon
E. Octagon

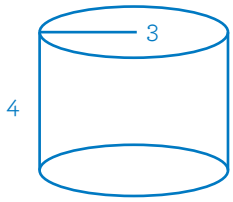
THREE-DIMENSIONAL SHAPES

Surface Area of a Sphere:	$SA = 4\pi r2$
Surface Area of a Cylinder:	$SA = 2\pi r2 + 2\pi rh$
Surface Area of a Prism:	Sum of the areas of all sides $SA = 2(lw + hl + hw)$ or: $SA = 2B + Ph$, where B is the area of the base, P is the perimeter of the base, and h is the height of the prism.
Volume of a Sphere:	$V = \frac{4}{3}\pi r^3$
Volume of a Cylinder:	$V = \pi r^2 h$
Volume of a Prism:	$V = l \times w \times h$
Volume of a Cube:	$V = side^3$
Diagonal of a Cube:	$side\ \sqrt{3}$

Example Problems

104. The height and radius of the right circular cylinder below are given in inches. What is the volume, in cubic inches, of the cylinder?

A. 9π
B. 12π
C. 18π
D. 36π
E. 72π



105. If the volume of a given sphere is equal to its surface area, what is the radius of the sphere?

A. 1
B. 3
C. 4
D. 6
E. 9

106. What is the radius of a sphere circumscribed around a cube, if the volume of the cube is 27?

A. $2\sqrt{2}$
B. $2\sqrt{3}$
C. $\frac{3\sqrt{3}}{2}$
D. $3\sqrt{2}$
E. $4\sqrt{3}$

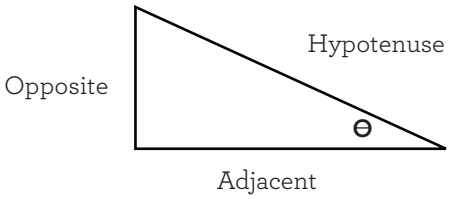
TRIGONOMETRY

Right Triangle Trigonometry (SOH CAH TOA)

SOH = sin Θ = $\frac{\text{opposite}}{\text{hypotenus}}$

CAH = cos Θ = $\frac{\text{adjacent}}{\text{hypotenus}}$

TOA = tan Θ = $\frac{\text{opposite}}{\text{adjacent}}$



Basic Trigonometric Identities

tan (Θ) = $\frac{\sin(\theta)}{\cos(\theta)}$

sec (Θ) = $\frac{1}{\cos(\theta)}$

csc (Θ) = $\frac{1}{\sin(\theta)}$

cot (Θ) = $\frac{1}{\tan(\theta)}$ or $\frac{\cos(\theta)}{\sin(\theta)}$

Pythagorean Identities

$\sin^2(\Theta) + \cos^2(\Theta) = 1$

$1 + \tan^2\Theta = \sec^2\Theta$

$1 + \cot^2\Theta = \csc^2\Theta$

Graphing Trigonometric Functions

$f(x) = A \sin(Bt - C) + D$

Amplitude = A

Period = $\frac{2\pi}{B}$ ($\frac{\pi}{B}$ for tangent)

Normal Period of Sine & Cosine: 2π

Normal Period of Tangent: π

Phase Shift = $\frac{C}{B}$

Vertical Shift = D

For our purposes sinusoidal functions are those that have the general form:

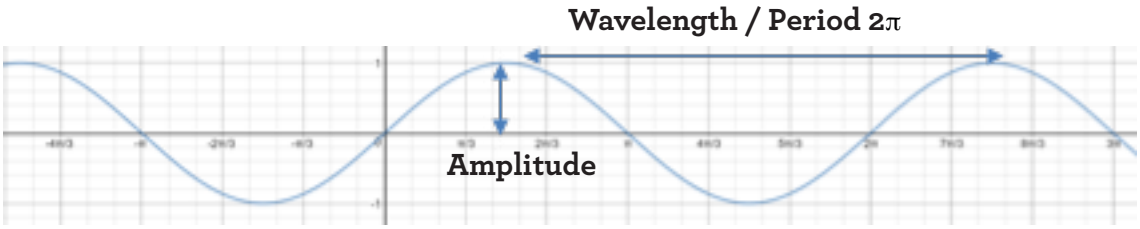
$f(x)=A \sin(Bt-C) + D$

When graphed on a coordinate plane, sinusoidal functions take the shape of a wave. There are only a few important things to remember about these functions.

First, the wavelength/period of the function can be measured as the distance from one peak of the wave to the next peak or from one trough to the next.

Second, the frequency of the function is the inverse of the wavelength: 1/wavelength.

Finally, the amplitude of the function is represented by the variable A, the number that appears directly in front of the sin function and is the height of the total function.



Example Problems

107. The dimensions of the right triangle shown below are given in meters. What is cos (Θ)?

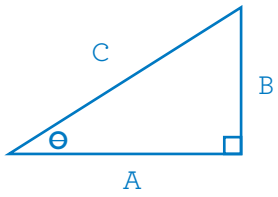
A. $\frac{a}{b}$

B. $\frac{a}{c}$

C. $\frac{b}{c}$

D. $\frac{b}{a}$

E. $\frac{c}{a}$



108. For the right triangle shown, sin (a) cos (ß) =

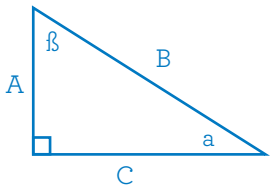
A. A

B. B

C. $\frac{a}{c}$

D. $\frac{A^2}{B^2}$

E. $\frac{A^2}{C^2}$



109. For the trigonometric function f(x) = 8tan (2πx 5) + 6, what is the period?

A. $\frac{1}{2}$

B. $\frac{\pi}{2}$

C. $\frac{1}{4}$

D. $\frac{\pi}{4}$

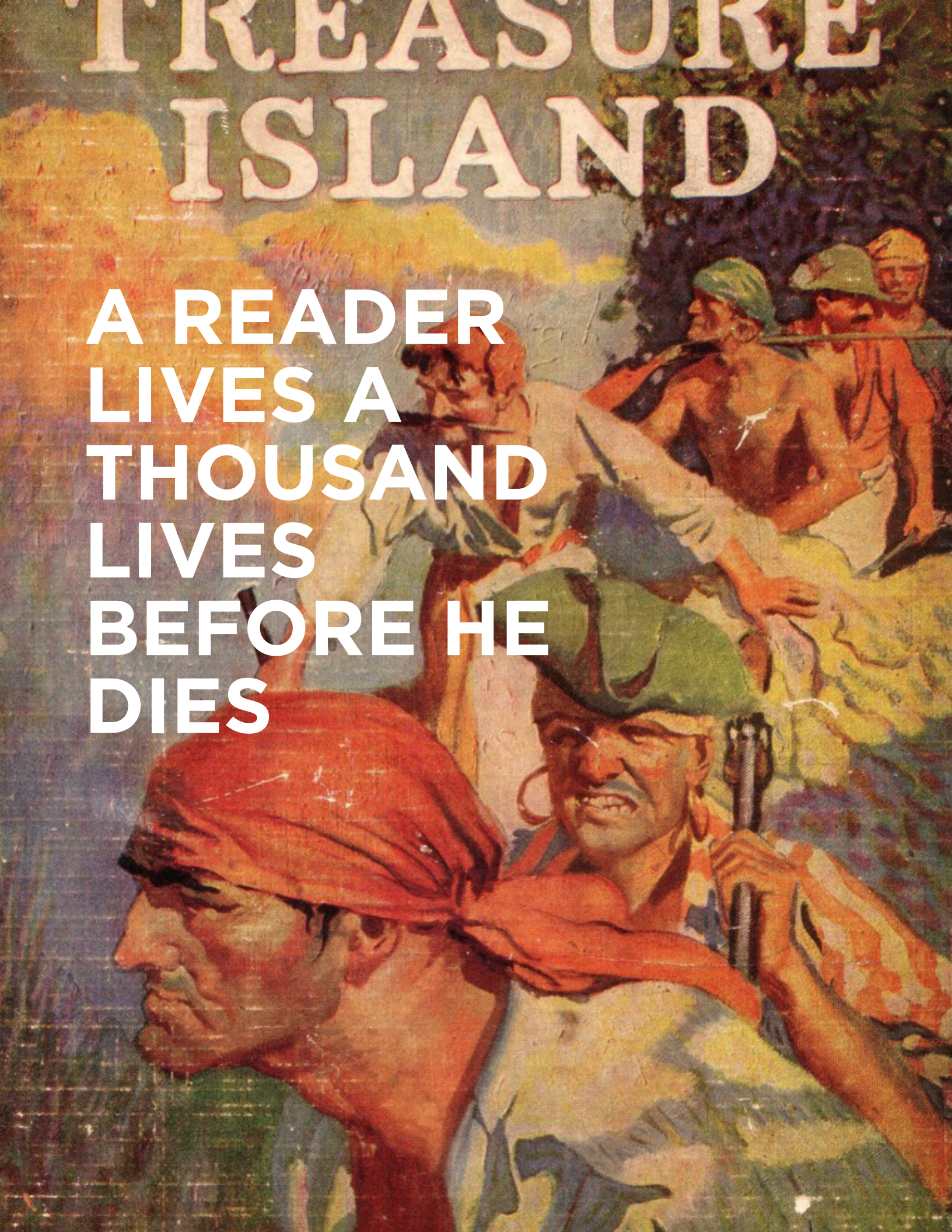
E. 2π

READING CURRICULUM

ACT

ESM PREP

Contents



A READER LIVES A THOUSAND LIVES BEFORE HE DIES

INTRODUCTION

The ACT Reading Test measures your reading comprehension and provides an example of content you might see during your first year of college. You will have 35 minutes to read 4 passages of approximately 750 words each and complete 40 questions. The Reading Test is broken down into four passage types: Literary Narrative, Social Science, Humanities, and Natural Science. Each reading test presents the four passages in this exact order, and each passage will be followed by 10 questions.

The questions will ask you to:

1. **Identify and interpret details**
2. **Determine the main idea of passages**
3. **Understand relationships between characters**
4. **Make generalizations**
5. **Analyze vocabulary in context**
6. **Draw conclusions about author's voice and method**

Note also that timing is demanding on the ACT Reading Test. To read and understand four passages then analyze and correctly answer 40 questions in 35 minutes is a challenging task. Any proper strategy shouldn't be focused simply reading the passages, but rather reading them efficiently in a way that will set you up to answer each question correctly.

If you read all of this and feel only dread or boredom, don't worry. That actually means you are exhibiting the best skill an ACT test taker can hone: active reading. You have already comprehended rather bland information, analyzed its implications, and predicted what will come next. (If, however, you have spent the last five minutes staring at the wall, we may have reason to be concerned). Over the course of your ACT preparation, you will focus on honing your active reading skills so that you can approach each passage like a test maker.

A note before we discuss our suggested strategy: there are several different reading approaches. No one reading strategy works best for ALL students and any strategy should be adaptable depending on the content in a particular passage and your relative strengths as a reader. We will teach the basic strategies that we feel are most effective, but you should work with your academic mentor to find and tailor the best strategy for you.

THE SUGGESTED STRATEGY

Imagine if you walked into a house you had never been in before and were given three minutes to explore it and draw a floor plan. Would you focus on the color of the wallpaper in the guest bedroom or the orientation of the toilet paper in the hall bathroom? Probably not. Instead, you would focus on the biggest items, such as the relative locations and sizes of each room in the house.

That is the same approach you should have for each ACT passage. Your goal should not to be to understand and interpret every fact, statistic, or character in your initial reading. In fact, doing so can actually hurt your score! Rather, you should determine the author's purpose(s) for writing the passage and a develop a general outline. Figure out the main ideas of the passage and of each paragraph so that you can always answer, "Why did the author write this?" and "What is the author's main argument?" Don't worry about the specifics yet -- If asked, you can always refer back to figure out whether the toilet paper unrolls upwards or downwards.

The Goal: After reading the passage, you should have a general understanding of the purpose of the passage, its main components (think main ideas of paragraph), and should have set yourself up to answer more detailed questions by leaving short notes in margins or circling key words!

Step 1: Read the introduction

A passage's introduction will always tell you who the author is and when it was written. In addition, it can often provide clues as to the setting or subject of the passage. While this information can help clear up some possible ambiguities in a passage, its greatest contribution is that it can serve as a buffer between passages. Reading the introduction allows you to clear your mind of the previous passage and refocus on the passage at hand. Here's an example of what an introduction looks like:

> **HUMANITIES**: This passage is adapted from the article "Wherever He Went, Joy Was Sure to Follow" by Stanley Crouch (©2000 by The New York Times Company). *Tin Pan Alley* is a district famous for its composers and publishers of popular music.

Step 2: Annotate the passage

This is the step that makes or breaks scores on the ACT Reading Test. It is crucial to keeping you focused on what you're reading, helping you rephrase the main ideas the passage in your own words, and setting you up to answer questions quickly and correctly. The question is, what should you look for in each type of passage?

Literary Narrative Passage

The first passage on every ACT Reading Test is a Literary Narrative passage. This passage is taken from a work of fiction and usually contains extended dialogue and significant characterization. **Annotate parts that reveal the character's traits and underline clues about the relationships between them.** Questions will usually center on the characters, tone, literary elements, and author's intention.

Humanities, Social Science and Natural Science Passages

The Humanities, Social Science and Natural Science can be grouped together because the best strategy involves a quick, investigative read before diving into the questions. Find and annotate the thesis and main ideas of each paragraph to guide your understanding of the passage. Look to topic and concluding sentences as they usually indicate the main idea of each paragraph. While literary elements will be tested, more questions center on the author's main idea(s) and the construction of those arguments.

Try jotting short phrases in the margins that point towards the answers to the following questions:

1. Who is the narrator/author?
2. What is the conflict?
3. What relationships are presented?
4. What is the main idea?
5. What is the purpose/significance of the passage?

Circle Proper Nouns!

ACT Reading passages will ask you "refer" questions, or questions that require you refer back to the passage to locate specific information. These questions are time-sucking traps, meant to trick you to waste precious time sifting through the passage to find information. To help you cut down the time you spend on these questions, **circle all proper nouns (people, places, dates, events) that are mentioned in the passage.** Doing so will create an index of key terms that will help you locate this information extremely quickly if one of the proper nouns are mentioned in a question! It is MUCH faster to find a term out of 10-15 circled terms rather than to search through a whole passage.

Step 3: Glance through annotations

Quickly reread your annotations before looking at the questions. You should only spend several seconds doing it but it can remind of you just what you took away from the passage before you dive into the questions and apply your takeaways.

Use the information you have pulled from the passage to help you work through the questions. While some may advocate going to the questions before reading the entire passage, that strategy is harder to implement on the ACT than on the SAT, as the ACT is less likely to give you line numbers for specific questions and doesn't always present its questions in sequential order. This means that reading the questions first is generally a waste of time.

Step 4: Eliminate Wrong Answer Choices

Read through the answers and eliminate clear incorrect choices. You can usually quickly cross off one or two choices. Then keep working. It is often easier and more accurate to proceed with a cynical eye and explain why an answer is wrong rather than why the correct answer is right. Look for specifics in the answer choices. For instance, if an answer choice says, "author introduces two theories on evolution" and you only found one theory, eliminate that answer choice.

Step 5: Answer questions correctly

Use the question type model to practice anticipating what is going to be asked on the ACT Reading Test. The more you know before you go into the test, the more comfortable you will feel on test day, and you will answer more questions correctly.

If you are stuck between a couple of answers, take your best guess and move on. You should circle the question on your booklet or note it on your answer sheet but you should not dwell on it. Time is of the essence and easier questions are usually waiting. If you have additional time later, come back to it.

Let's practice steps one through four on the Literary Narrative passage on the next page, and then examine the types of questions we will be faced with answering on the ACT Reading Test.

READING TEST

35 Minutes - 40 Questions

DIRECTIONS: There are four passages in this test. Each passage is followed by several questions. After reading a passage, choose the best answer to each question and fill in the corresponding oval on your answer document. You may refer to the passages as often as necessary.

Passage I

PROSE FICTION: This passage is adapted from the short story "The Threshold" by Cristina Peri Rossi (original Spanish version ©1986 by Cristina Peri Rossi; translation ©1993 by Mary Jane Treacy).

The woman never dreams and this makes her intensely miserable. She thinks that by not dreaming she is unaware of things about herself that dreams would surely give her. She doesn't have the door of dreams that opens every night to question the certainties of the day. She stays at the threshold, and the door is always closed, refusing her entrance. I tell her that in itself is a dream, a nightmare: to be in front of a door which will not open no matter how much we push at the latch or pound the knocker. But in truth, the door to that nightmare doesn't have a latch or a knocker; it is total surface, brown, high and smooth as a wall. Our blows strike a body without an echo.

"There's no such thing as a door without a key," she tells me, with the stubborn resistance of one who does not dream.

"There are in dreams," I tell her. In dreams, doors don't open, rivers run dry, mountains turn around in circles, telephones are made of stone. Elevators stop in the middle of floors, and when we go to the movies all the seats have their backs to the screen. Objects lose their functionality in dreams in order to become obstacles, or they have their own laws that we don't know anything about.

She thinks that the woman who does not dream is the enemy of the waking woman because she robs her of parts of herself, takes away the wild excitement of revelation when we think we have discovered some-thing that we didn't know before or that we had forgotten.

"A dream is a piece of writing," she says sadly, "a work that I don't know how to write and that makes me different from others, all the human beings and animals who dream."

She is like a tired traveler who stops at the threshold and stays there, stationary as a plant.

In order to console her, I tell her that perhaps she is too tired to cross through the doorway; maybe she spends so much time looking for her dreams before falling asleep that

she doesn't see the images when they appear because her exhaustion has made her close those eyes that are inside of her eyes. When we sleep we have two pairs of eyes: the more superficial eyes, which are accustomed to seeing only the appearance of things and of dealing with light, and dream's eyes; when the former close, the latter open up. She is the traveler on a long trip who stops at the threshold, half dead with fatigue, and can no longer pass over to the other side or cross the river or the border because she has closed both pairs of eyes.

"I wish I could open them," she says simply.

Sometimes she asks me to tell her my dreams, and I know that later, in the privacy of her room with the light out, hiding, she'll try to dream my dream. But to dream someone else's dream is harder than writing someone else's story, and her failures fill her with irritation. She thinks I have a power that she doesn't have and this brings out her envy and bad humor. She thinks that the world of dreams is an extra life that some of us have, and her curiosity is only halfway satisfied when I am finished telling her the last one. (To tell dreams is one of the most difficult arts; perhaps only author Franz Kafka was able to do so without spoiling their mystery, trivializing their symbols or making them rational.)

Just as children can't stand any slight change and love repetition, she insists that I tell her the same dream two or three times, a tale full of people I don't know, strange forms, unreal happenings on the road, and she becomes an-noyed if in the second version there are some elements that were not in the first.

The one she likes best is the amniotic dream, the dream of water. I am walking under a straight line that is above my head, and everything underneath is clear water that doesn't make me wet or have any weight; you don't see it or feel it, but you know it is there. I am walking on a ground of damp sand, wearing a white shirt and dark pants, and fish are swimming all around me. I eat and drink under the water but I never swim or float because the water is just like air, and I breathe it naturally. The line above my head is the limit that I never cross, nor do I have any interest in going beyond it.

She, in turn, would like to dream of flying, of slipping from tree to tree way above the rooftops.

EXAMPLES OF QUESTION TYPES

Details

It can reasonably be inferred from the passage that the woman most strongly desires to attain which of the following qualities from dreaming?

A. Relaxation
B. Self-awareness
C. Entertainment
D. Self-control

Detail questions ask you identify a characteristic of a person, place, thing or idea. This particular question also asks you to interpret and infer a detail about the idea of dreaming. Note that while this question refers to a very specific point in the passage, those line numbers are not given. This is common on the ACT.

The author says of the woman, "She thinks that by not dreaming she is unaware of things about herself that dreams would surely give her" (lines 2-5). She wants to receive this awareness.

We can quickly eliminate D because nowhere does the passage mention that dreaming increases self-control. We can also eliminate C, because, although the woman does listen to the narrator's dreams over and over, she doesn't do them for entertainment but so that she can then learn to dream too. Lastly, we can also eliminate A because as the narrator says, "a woman who does not dream robs her of parts of herself... [not dreaming] takes away the wild excitement of revelation when we think we have discovered something we didn't know before or had forgotten" (lines 26-30. This is not about relaxation, but finding something within oneself.

The correct answer is B, Self-awareness.

Relationships

Based on the passage, which of the following statements best describes the overall attitudes of the narrator and the woman?

A. The woman is frustrated and despairing, while the narrator is supportive and reassuring.
B. The woman is bitter and resentful, while the narrator is detached and uninterested.
C. The woman is lonely and resigned, while the narrator is optimistic and relaxed.
D. The woman is dismayed and miserable, while the narrator is discontented and angry.

Relationship questions will often ask how a character's actions cause another character to react a certain way. In this example, the woman is seeking answers to the questions she has about her own ability to comprehend dreams, while the narrator is providing the answers.

Now let's look at the answers and start with **D**. The woman may be dismayed, but the narrator doesn't seem discontented or angry. In fact he seems to try his best to help her by repeatedly explaining his dreams to the woman. **C** doesn't appear right either. The woman is not resigned as she keeps trying to learn to dream. **B** also looks incorrect as the narrator is far from detached and uninterested as he explains the women's issues with dreaming and his attempts to help her.

The correct answer is **A**. The author uses the phrase, "In order to her [woman] console her..." to establish the relationship where the narrator supports and reassures the woman.

Vocabulary in context

As it is used in line 55, the word humor most nearly means:

A. personality.
B. whim.
C. mood.
D. comedy.

Vocabulary in context questions ask you to determine what a word means from a specific context. Look at the sentences before and after the sentence in which the word is and this will clue you to the word in question. The specific sentence says, "She thinks I have a power that she doesn't have and this brings out her envy and bad humor" (lines 57-58). The passage is identifying the differences in characteristics between the woman and the narrator, specifically alluding to the woman's inability to interpret dreams. This inability frustrates the woman and puts her in a bad mood. When looking at the answers, we see that mood is present. None of the other ones would be appropriate. The correct answer is C.

Author's Voice and Method

Based on the narrator's account, the woman's approach to dreaming the narrator's dreams is best described as:

A. confrontational and powerful.
B. enthusiastic and playful.
C. precise and confident.
D. self-conscious and secretive.

Voice relates to author's point of view and attitude. ***Method*** focuses on the craft of writing—the main purpose and how each portion synthesizes to create the function and meaning for the entire work.

Why does the woman try to dream the narrator's dreams? To give herself the ability to dream. She thinks that practicing might help her do it on her own.

Looking at the answers, A would not be right. It is too strong, plus we know trying to dream others' dreams is not very effective. B is not right either. She doesn't do this out of excitement, but rather disappointment that she can't do it on her own. The same applies to C. It is not a confident approach.

The correct answer is D. The author describes the woman in the passage as self conscious of her inability to dream. We also learn that she tries to dream his dream "in the privacy of her room, with the light out, hiding..." (lines 53-54). This question exemplifies the author's voice, as the author's attitude of the woman's character is presented in this question.

Before we look at the final types of questions, let's practice reading and annotating the Humanities passage on the next page.

READING TEST

Passage III

HUMANITIES: This passage is adapted from the article "Wher-ever He Went, Joy Was Sure to Follow" by Stanley Crouch (©2000 by The New York Times Company). *Tin Pan Alley* is a district famous for its composers and publishers of popular music.

As a jazz trumpeter and a singer, Louis Armstrong asserted a level of individuality in musical interpretation, recomposition and embellishment far more radical than any that had preceded it in Western music. When faced with a musical theme, Armstrong improvised an arrangement that boldly rephrased it, dropping notes he didn't want to play and adding others. His featured improvisations brought the role of the jazz soloist to the fore. The immaculate logic of his improvised melodies, full of rhythmic surprises and virtuosic turns, influenced showtune writers, jazz composers, big band arrangers and tap dancers. His harmonic innovations, as fellow trumpeter Wynton Marsalis has noted, were the most brilliant in the history of jazz: Armstrong figured out how to articulate the sound of the blues through Tin Pan Alley popular music tunes without abandoning their harmonic underpinnings. "Louis Armstrong took two different musics and fused them so that they sounded perfectly compatible," Mr. Marsalis says.

It was during the 1920's and 30's that Armstrong's reputation took off. He set the music scene in his home town of New Orleans on fire before traveling to Chicago in 1921 to join his mentor, the cornetist King Oliver. For a year he went to New York, where he joined Fletcher Henderson's jazz orchestra and turned the rhythm of the music around with his conception of playing with a swinging beat. Now almost a national musical terror, Armstrong returned to Chicago, then finally settled in New York in 1929.

From 1925 through the early 1930's, he recorded dozens of masterpieces with large and small bands, popularized scat singing (jazz singing that uses non-sense syllables) and took on Tin Pan Alley, introducing one tune after another into jazz, where they became part of his repertory. His tone could be broad, soft and luminous or vocal or comical, or suddenly and indelibly noble, and when his music conquered Europe in the 30's, it carried the tragic optimism of the American sensibility into the world at large. Wherever he went, swing was sure to follow. He almost single-handedly began a new spirit of freewheeling but perfectly controlled improvisation, tinged with playfulness, sorrow and sardonic irony.

Like all innovators, Armstrong was also called upon to perform superhuman feats. Armstrong had endless energy and could play and play and play with the evangelical fire and charisma that brings a new art into being. He extended the range of his instrument, asserted unprecedented rhythmic fluidity and had the greatest endurance of any trumpet player who ever lived. As a young man, he could play five shows in a theater a day, be the featured soloist on virtually every piece and end each show with 100 high C notes. His glissandos—rapid slides up or down a musical scale—were so pronounced that trumpeters of the London Philharmonic Orchestra had to inspect his horn to be convinced that it was

not made differently from theirs.

By his death in 1971, Armstrong had influenced the entirety of American music, instrumentally and vocally, inspiring his own generation and successive ones. I can recall some 30 years ago talking with a concert percussionist who knew Armstrong and the rest of the people who were rising to the top during the middle and late 20's. Referring to a certain concert piece, which had a more extensive drum part than usual, he said, "When I get that going, I can put my Louis Armstrong influence in and, without them even knowing it, the orchestra starts to swing for a bit." On a more recent occasion, unless I was imagining it, I even heard rapper Heavy D slip a phrase over the mechanical hip-hop beat that had an Armstrong arch to it.

To get right down to it, no one in jazz ever played with greater emotional range than Armstrong, whose New Orleans experiences meant that he worked every-thing from christenings to funerals. In the streets, he picked up all the folk chants and songs. While traveling around town, he heard traces of French and Italian opera that suffused his sensibility and his memory. But beyond all that, what Armstrong wanted to give his listeners was the kind of pleasure music gave him, which is what most artists are after. When he wrote or talked of New Orleans, of being out there with his horn or following the parades or listening to mentors like Joe Oliver, Armstrong never failed to project a joy so profound that it became an antidote to the blues of daily living. He had a determination to swallow experience whole and taste it all and only then to spit out the bitter parts.

EXAMPLES OF QUESTION TYPES (CONT'D)

Main Ideas

Which of the following statements best expresses the main idea of the passage?

A. Armstrong was an exceedingly gifted musician whose emotional range was nonetheless somewhat narrow.
B. One of the greatest jazz trumpeters of all time, Armstrong is best known for his soft and luminous tone.
C. Armstrong has had a profound effect on music, one that has been both wide ranging and long lasting.
D. A pioneering jazz trumpeter and singer, Armstrong recorded numerous masterpieces in the mid to late 1920s.

The focus of the passage was on Louis Armstrong and his contributions to music. Looking at the answers we can easily rule out several. First option A- Armstrong was exceedingly gifted but most of the essay is about the expansiveness of his range, not the narrowness. This is out. Option B seems slightly off too. While Armstrong was one of the greatest jazz trumpeters of all time, "His tone could be broad, soft and luminous, or vocal or comical, or suddenly and indelibly noble…" (lines 35-37). That is one broad tone, so this doesn't seem right. Option D is mostly right (he was a pioneering jazz trumpeter and singer who recorded numerous masterpieces) but the main idea of the passage is not just about those masterpieces he recorded in the 20s—It is about his legacy to music. For that reason, the correct answer is C. The author notes musical contributions such as, "interpretation, individuality, recomposition, improvisation, and melodies, in order to show the wide-ranging effects on music (Paragraph 1). The author also says, "By his death in 1971, Armstrong had influenced the entirety of American music…" (lines 58-60). The author believes Armstrong's effect is long lasting.

Generalizations

The passage suggests that Armstrong's most important contribution to jazz was his:

A. musical conquest of Europe.
B. emphasis on improvisation.
C. work with King Oliver.
D. invention of the blues sound.

Generalization questions asks you to boil down a lot of information into a more concise form. In this case, you need to determine Louis Armstrong's **most important** contribution, in a passage full of his contributions. This one is especially tough because most of these answer choices are mentioned. This is where our annotations might help us. Though A and C are mentioned, they are briefly mentioned. D is not not mentioned.

The correct answer is B. Go back to the first paragraph and notice the repetition of instances of improvisation. Next, notice the adjectives used to describe his improvisation, these are strong describing words that emphasize the grandiosity his improvisation had on jazz music. Note that while generalization questions are the most tempting to look for outside information, you never need to do so.

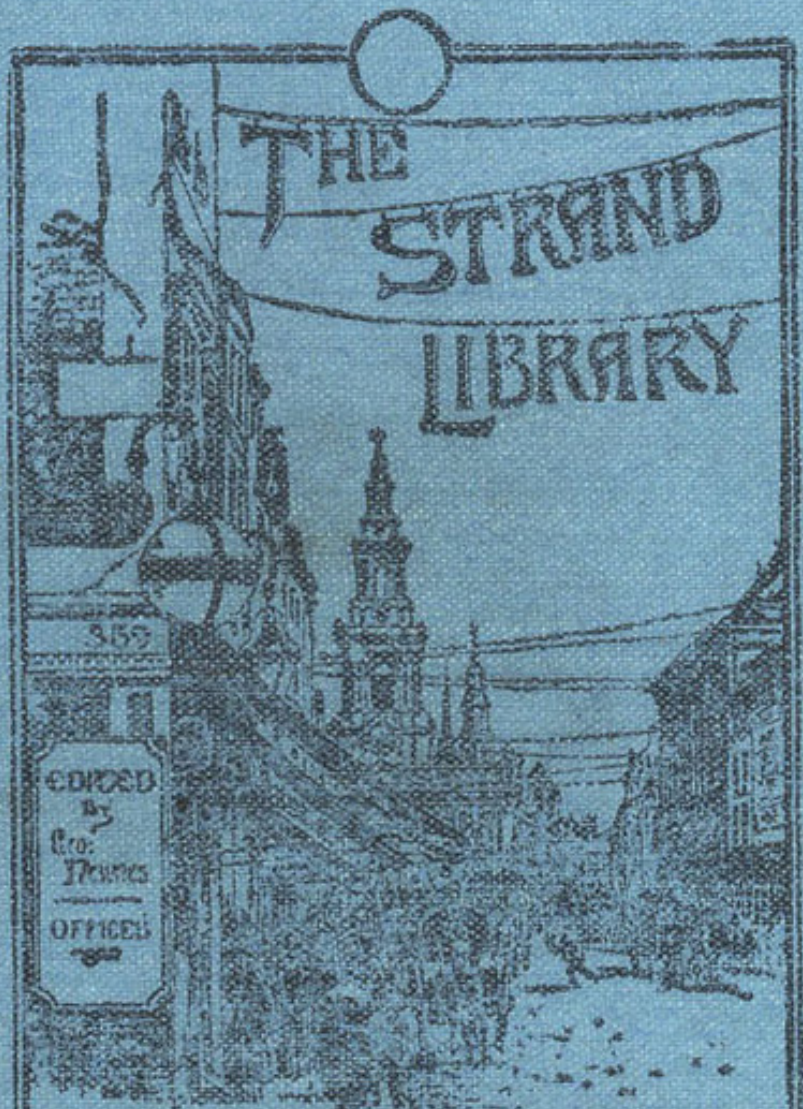
Voice

Which of the following words best describes how the orchestra referred to in the fifth paragraph (lines 58-71) is said to have started to swing?

A. Reluctantly
B. Intentionally
C. Unconsciously
D. Optimistically

It says the orchestra started to swing a bit "without them even knowing it" (line 67-68). We can rule out answers here or we can jump right to the correct one. The correct answer is C. This question is an example of author's method.

The Adventures of Sherlock Holmes



Books are a uniquely portable magic

– Stephen King

THE TIMING CONUNDRUM

Timing is by far the biggest roadblock to scoring highly on the ACT Reading Test. Reading and annotating a passage and then answering ten tough questions in 8:45 is an extremely tall task! With dedicated practice, you should be able to get your timing close to the 9-minute mark. But what happens if you accidentally spend too much time on one passage on test day and need to make up for it elsewhere? What should you do if you just can't your timing down no matter hard you try?

The Inside Out Strategy

The goal of the Inside Out strategy is to answer the majority of a passage's questions correctly in a relatively short amount of time, so that you can avoid blindly guessing on a lot of questions.

When to use: When you reach the Natural Science passage with only a few minutes left, or have an average of 5 minutes or less left for each passage on the test (i.e. 10 minutes remaining after Social Science passage).

How to use: This strategy is best used on Natural Science passages, which are usually chocked full of dense information that either supports or refutes an author's main argument. Our strategy focuses on knowing what the dense information is going to support rather than the information itself, and skipping over it almost entirely. Below are the main steps in detail:

1. **Read the Introduction** – this step does not change from our previous strategy.

2. **Breeze through the passage to circle keywords**– take 90-120 seconds to skim through the passage with the main goal of circling every proper noun and number you see. Ideally, you should briefly read the introduction and the first sentence of each sentence to get a vague idea of what the passage is about, but you may not have enough time to do so if you only have a few minutes remaining. Remember, DON'T worry about memorizing any details.

3. **Strategically attack the questions** – Unlike the rest of the passages, you should not attempt to do these questions in order. Instead, try to answer questions in the following order:

    1. Vocabulary in Context Questions
    2. Questions with line references
    3. Refer Questions with proper nouns in the question or answer choices
    4. Main Idea of Paragraph or Passage/Generalization Questions
    5. Refer Questions without proper nouns

    These questions are in order of how long they should take you to answer correctly. In addition, by answering more local questions first, you will also gain a greater knowledge of the passage as a whole, which should give you a greater chance at answering the later global questions correctly!

    Note: When answering any question, circle the correct answer in your book. Do not put them on your answer sheet immediately, as you are prone to make a bubbling error when you do questions out of order.

4. **Bubble in your answers** – With a minute remaining, you should record the answers to the questions you were able to answer in your booklet, and guess on any questions you were unable to get to.

Now that we've reviewed the steps, let's take a look at this strategy in action on a Natural Science passage.

Passage IV

NATURAL SCIENCE: This passage is adapted from Consider the Eel by Richard Schweid (©2002 by Richard Schweid)

The known facts, as they are pretty much universally accepted among biologists and naturalists today, are that all the eels in all the rivers of eastern North America and the Caribbean countries, and all the eels in all the rivers of eastern and western Europe, are born in the same area of the Sargasso Sea, a huge area within the Atlantic Ocean, between Bermuda and the Azores, the surface of which is frequently covered with sargassum seaweed. In fact, the word "Sargasso" comes from the Portuguese sargaço, meaning seaweed. The sea is about 2,000 miles long and 1,000 miles wide, set off from the surrounding waters of the Atlantic by strong currents. It includes the area known in popular legend as the Bermuda Triangle.

Eels hatch in the Sargasso as larvae and are carried by the ocean currents to either Europe or the United States, a journey that can cover thousands of miles and take years. Where they end up depends on which of two similar species they belong to. Those that are Anguilla anguilla invariably wind up in European rivers, and those that enter North American rivers always belong to the species Anguilla rostrata. The first person to find eel larvae in the Sargasso Sea was Danish researcher Johannes Schmidt, who published his findings in 1924, after spending 18 years hauling nets in search of eels.

The larvae of both species are shaped like small oval leaves and are called leptocephali. Each lepto-cephalus begins to assume the form of a tiny eel, called an elver or glass eel, when it gets close to the coasts of either Europe or the Americas. By the time it reaches brackish water, where fresh and salt water mix, it is thin and transparent, hardly bigger than a hair, with a pair of eyes like black dots at one end.

From the estuaries and mouths of rivers, the tiny eels frequently continue upstream, particularly the females, who sometimes go great distances inland. American eels have been found as far up the Mississippi River system as the rivers of Iowa. They keep going upriver until something tells them they've reached home, and then they stop. Whatever it is that signals to eels that they are home is definitive—they settle in and live there for as long as 20 years, growing up to a yard long before beginning their journey back to the Sargasso Sea. Scientists determine an eel's age using a microscope to read the growth rings of its otolith—a small, hard calcium deposit at the base of its skull.

In preparation for the return journey to the Sargasso, sexually mature female eels feed voraciously and change color from the muddy-yellow/green of adult eels, often called yellow eels, to a darker green on top and snow-white on their bellies. At this stage, they are called silver eels. They swim downriver in the fall, on the first leg of their journey to the Sargasso, and when they reach estuarine waters, they rest, completing their final transformation as silver eels. They will have eaten heavily and will be about 28 percent body fat. They will never eat again, and their digestive systems will atrophy. Their pupils will expand and turn blue. They will need a new kind of sight adapted to the depths of the sea, where there is little light. They will also have to go through a drastic adjustment, via osmosis, in their blood chemistry, to prepare for the tremendous change in water pressure, going from some 14 pounds of fresh-water pressure per inch of their bodies to over a ton of ocean pressure per inch. Once they are back in the Sargasso Sea, the females produce eggs for the males to fertilize, and then the adults die.

At least that is what today's marine biologists and naturalists tell us, although adult eels have never been seen

swimming, reproducing, or dying in the Sargasso. In fact, live adult eels have never been seen there at all. The only two adult eels ever reported in the Sargasso Sea were dead, found in the stomachs of other fish. The eel's migration back to its birthplace and what it actually does when it gets there are assumed to take place far below the water's surface and, as of the year 2001, were still completely unobserved. However, the eel larvae—the leptocephali that Schmidt found in the Sargasso—were so small that it was certain they had been born recently, and nearby. Such small larvae have never been seen elsewhere, and while eels have never been observed reproducing in the Sargasso, they have never been seen doing so anyplace else either. Scientists believe the larvae hatch out of eggs at a depth of 100–300 yards and rise slowly toward the light at the sea's surface.

31. One of the main ideas established by the passage is that:

A. researchers have nearly exhausted their resources after spending decades investigating the Sargasso Sea.
B. significant gaps still remain in researchers' understanding of the life cycle of eels.
C. eels live their entire lives in the Sargasso Sea, but no one has ever seen them there.
D. female eels turn into silver eels toward the end of their lives.

32. Learning about which of the following had the largest impact on scientists' current understanding of where eels breed?

A. The direction in which ocean currents carry eel larvae
B. The relationship of the yellow eel stage to the silver eel stage
C. Schmidt's discovery of eel larvae in the Sargasso Sea
D. The adult eels found in the stomachs of other fish

33. The main purpose of the fourth paragraph (lines 34–47) is to describe the:

A. eels' transition from freshwater to the ocean.
B. method of determining the age of eels.
C. complexity of the Mississippi River system.
D. river stage of the eel life cycle.

34. The passage states that the Sargasso Sea is set off from the rest of the Atlantic Ocean by:

A. the Azores.
B. several Caribbean countries.
C. powerful winds.
D. strong currents

35. The passage notes that the Sargasso Sea includes:

A. the eastern North American shore.
B. the Bermuda Triangle.
C. certain coastal estuaries.
D. the mouth of the Mississippi River.

36. As it is used in line 13, the word popular most nearly means:

A. well liked.
B. commonly known.
C. scientifically accepted.
D. most admired.

37. As it is used in line 45, the word read most nearly means to:

A. learn from print.
B. observe.
C. think about.
D. predict.

38. The passage indicates that female eels' pupils expand and turn blue because the eels:

A. must adapt to see in an environment with much less light than they are used to.
B. are about to undergo a change in their blood chemistry.
C. no longer need to be able to recognize food sources since they have stopped eating.
D. need to be able to recognize the male eels that will fertilize their eggs.

39. The passage most strongly emphasizes that the process of osmosis is necessary for the eels' transition from:

A. shallower to deeper water.
B. feeding to nonfeeding.
C. immature to mature form.
D. elver to yellow eel.

40. According to the passage, which of the following characteristics of the eel larvae found by Schmidt provided the best evidence that the larvae were hatched in the Sargasso Sea?

A. Size
B. Shape
C. Color
D. Species

The WONDERFUL WIZARD of OZ

BY
L. FRANK BAUM
PICTURES BY
W. W. DENSLOW

To learn to read is to
light a fire

– Victor Hugo

THE INSIDE OUT STRATEGY – A WALKTHROUGH

Step 1: Read the Introduction

We don't learn too much from the introduction of the passage, but we do learn that it will be about eels. Exciting!

Step 2: Blaze Through the Passage

As you can see in the passage, we have circled a vast number of proper nouns to use for questions. It may seem like a lot of circled items at some places, but we want to make sure we are thorough in this step. Missing a proper noun and having it mentioned in a question turns a potentially easy question into an extremely time-consuming one!

As for the information we can gather while skimming quickly, we don't have to extract too much information. It looks like the passage is discussing the life cycle of eels. Again, don't worry about not having more information than that!

Step 3: Strategically Attack the Questions

Let's separate the questions into the levels we created above:

1. Vocabulary in Context Questions: 36 and 37

These are questions that you should be able to answer quickly and correctly with *extremely* little knowledge of the passage:

36. As it is used in line 13, the word *popular* most nearly means:

A. well liked.
B. commonly known.
C. scientifically accepted.
D. most admired.

In the sentence "It includes the area known in *popular* legend of the Bermuda Triangle", popular most nearly means commonly known. Answer choice B is correct.

37. As it is used in line 45, the word *read* most nearly means to:

A. learn from print.
B. observe.
C. think about.
D. predict.

In the sentence "Scientists determine an eel's age using a microscope to read the growth rings of its otolith", read most nearly means observe. Answer choice B is correct.

2. Questions with line references - NONE

This is an important distinction to make. Though question 33 involves a line reference, it is actually a Main Idea question! Don't fall for this trap.

3. Refer Questions with proper nouns in the question or answer choices: 32, 34, 35, 40

This is where circling keywords pays off! We should be able to cycle through our circled terms to answer these questions.

32. Learning about which of the following had the largest impact on scientists' current understanding of where eels breed?

A. The direction in which ocean currents carry eel larvae
B. The relationship of the yellow eel stage to the silver eel stage
C. Schmidt's discovery of eel larvae in the Sargasso Sea
D. The adult eels found in the stomachs of other fish

We don't have a lot to go off of here, but we do see that Schmidt and Sargasso Sea are mentioned in choice C. We circled Schmidt in line 29, which says that Schmidt found larvae in the Sargasso Sea after searching for 18 years. Good enough! Let's choose answer choice C and move on.

34. The passage states that the Sargasso Sea is set off from the rest of the Atlantic Ocean by:

A. the Azores.
B. several Caribbean countries.
C. powerful winds.
D. strong currents.

We need to find a place in the passage that mentions the Sargasso Sea and the Atlantic Ocean close to each other. This occurs in the first paragraph in multiple places. We see in line 7 that the Sargasso Sea is a "huge area within the Atlantic Ocean, between Bermuda and the Azores", but that does not answer the question. We also circled "Atlantic" in line 12, where the passage says that the sea is set off from surrounding waters by strong currents. There is our answer! Answer D is correct.

35. The passage notes that the Sargasso Sea includes:

A. the eastern North American shore.
B. the Bermuda Triangle.
C. certain coastal estuaries.
D. the mouth of the Mississippi River.

After answering question 36, question 35 becomes MUCH easier! The correct answer is located in the sentence we already interpreted. Answer choice B is correct. If you didn't recognize this, simply look for the proper nouns in the answer choices to determine whether each answer choice is correct or incorrect.

40. According to this passage, which of the following characteristics of the eel larvae found by Schmidt provided the best evidence that the larvae were hatched in the Sargasso Sea?

A. Size
B. Shape
C. Color
D. Species

In this question, the proper nouns "Schmidt" and "Sargasso Sea" are mentioned. Schmidt is mentioned in line 24 as the first person to find eel larvae, which doesn't address the question. He is also mentioned in line 79 along with "Sargasso", and the sentence mentions that larvae were so small that they had to have been born recently and nearby. Boom! Answer choice A is correct.

4. Main Idea/Generalization Questions – 33, 31

You will inevitably have to do some guessing on these questions, but you should try to eliminate some wrong answer choices based on the passage or your knowledge of common types of wrong answer choices. Note that we will attack question 33 first, because it only asks about a portion of the passage and has a line reference.

33. The main purpose of the fourth paragraph (lines 34-47) is to describe the:

A. eels' transition from freshwater to the ocean.
B. method of determining the age of eels.
C. complexity of the Mississippi River system.
D. river stage of the eel life cycle.

This main idea question only relates to a paragraph, so it should be pretty manageable. Answer choice C is not related to eels, so eliminate it. We know that the age of eels is mentioned because we read the final sentence of this paragraph in question 37, but it is only at the end of the paragraph. Similarly, we know that we mention the Mississippi River in the paragraph and rivers in Iowa, but we don't see any mentions of an ocean. Answer choice D mentions the life cycle that is part of the main idea of the whole passage, and mentions rivers, so let's go with that.

31. One of the main ideas established by the passage is that:

A. researchers have nearly exhausted their resources after spending decades investigating the Sargasso Sea.
B. significant gaps still remain in researchers' understanding of the life cycle of eels.
C. eels live their entire lives in the Sargasso Sea, but no one has ever seen them there.
D. female eels turn into silver eels toward the end of their lives.

This is a classic main idea question pertaining to the entire passage, so let's try and eliminate some answer choices based on our limited knowledge of the main idea: that we are discussing the life cycle of eels. Answer choice A is focused on the researchers' efforts. We know that we only have one researcher mentioned, and we never saw anything about not having any further resources, so let's eliminate it. Answer choice B makes a bit more sense: It mentions the lifestyle, and says that we have a lot more to learn. We can't eliminate B just yet. Answer choice C uses extreme language (entire) and we have a bunch of different places, such as Mississippi River, circled. Let's eliminate it. Finally, answer choice D is a rather small detail, and we have not seen anything to support it in our admittedly sparse review of the passage. B seems to be best, so let's circle it in our books and move on.

5. Refer Questions without proper nouns –38, 39

Chances are that, at this point, you will only have enough time to come up with a super quick guess to these questions. We have no idea where these answers are in the passage either. But that's okay! We've already answered 8 out of 10 questions correctly. Any correct guesses here are a bonus.

Step 4: Bubble in your Answers

Starting with question 31, bubble in your answer choices. Don't forget to put in your guesses to Questions 38 and 39!

ACT READING TEST SECTION OVERVIEW

TP

BASICS:

4 passages and 40 questions
Time: 35 minutes
Average time per passage: 8:45

Order of Passages:
1. Literary Narrative
2. Social Sciences
3. Humanities
4. Natural Sciences

KEY RESOURCES:

ACT Reading Curriculum

ACT Reading Quizzes

Question Types:

Refer Questions: require you to locate information that is clearly stated in the passage.

Reasoning Questions: require you to use your interpretation/understanding of the passage to answer complex questions.

GENERAL STRATEGY:

1. Read the introduction to extract information regarding the author, time period, and definitions of any obscure terms that are used in the passage.
2. Read the passage in 3-5 minutes and circle all proper nouns, dates, numbers, and other key terms.
    a. Your overall goal is to determine the main idea of a passage and how that main idea is developed throughout the passage.
3. While reading, annotate and make sure you understand:

Literary Narrative	*Social Studies, Humanities, Natural Science*
Character Traits and Plot	Main Idea of body paragraphs
Character opinions & feelings	Author's goal or intent
Relationships between characters	Author's feelings toward topics he/she discusses
Narrator's tone	

4. Read each question carefully and determine what the answer is in your own words.
5. Use your annotations, circled words or line references to locate necessary information.
6. Use the answer you came up with to eliminate wrong answer choices.
    a. If any part of an answer choice is incorrect, eliminate it.
7. If you cannot eliminate three answer choices, select the best answer choice from those remaining and move on to the next question. Do not fixate on any one question for longer than one minute!

EXTRA TIPS:

1. Embrace your inner cynic! Your goal is to find the least wrong answer choice. You are much more likely to find the correct answer when looking for what is wrong with each answer choice rather than what is right.
2. Make sure you fully understand the topic sentence of each body paragraph. When in doubt, refer to this sentence when answering a question that pertains to the paragraph.
3. Keep track of your timing, and have a backup plan! If you take too long on one passage and are short on time, try the Inside Out Strategy for the Natural Science Passage.
4. Don't give half of an effort when circling key words. Being thorough can change any difficult and time-consuming Refer question into a quick and easy one.
5. Beware of Time Wasters!
    a. Any Refer question for which you cannot quickly locate the answer in the passage.
    b. Any question that has an EXCEPT in it.
6. If you are faced with a Refer question without a key term that you would have circled, check if any answer choices contain key terms.
7. Beware of answer choices that contain extreme language (words like always, never, all, none, completely) or are outside the scope of the passage (these usually sound too good to be true).
8. Keep an eye out for predictive language and things listed in threes (very helpful for EXCEPT questions).

ESM
PREP

SCIENCE CURRICULUM

ACT

ESM PREP

Contents


SOMEWHERE, SOMETHING INCREDIBLE IS WAITING TO BE KNOWN
– Carl Sagan
©igor siwanowicz

INTRODUCTION

The ACT "Science" Test isn't meant to comprehensively analyze your knowledge of science. More than anything, it tests how *quickly and accurately* you can interpret charts and graphs. It should really be called the ACT "Charts & Graphs" Test. What this means, essentially, is that often the Science Test is not as hard as it looks. Understanding every little aspect of each experiment will not be necessary for achieving a great score, and is something we strongly discourage.

The hardest thing about the ACT Science Test is **timing**. You will need to exercise efficiency with your eyes, learning where to find crucial information quickly. If you take clear note of what the questions are asking for, and you methodically find that information in the charts & graphs presented, you will do great!

On the ACT Science Test, you are given 35 minutes to work through either six or seven passages and answer 40 questions. There are three passage types which can be distinguished based on the number of questions they have and the number of graphs and charts used:

1. ***Data Representation (5-6 questions each)*** - The easiest and simplest of the 3 types. Interpreting data quickly is the most important skill needed to score highly on these passages.

2. ***Research Summary (6-7 questions each)*** - Each passage usually includes two or three related experiments, each clearly marked (Experiment 1; Experiment 2; etc.). Interpreting data is still very important, but now you will have to compare different data sets.

3. ***Conflicting Viewpoints (7 questions each)*** - Usually two or more "experts" offering competing interpretations of a scientific event or phenomenon. Comparing the different viewpoints is most important, and little to no data will be included in the passage. These are essentially reading passages with scientific vocabulary. **Most students should save this passage for last.**

GENERAL STRATEGIES

Data Representation and Research Summary Passages

Data representation and research summary are very similar passage types. In both DR & RS, you should only read the text *as needed*. **Often, you won't have to read the text at all**. You should employ the following steps:

1. Take note of what is being measured on the x & y axes of graphs, and in the columns and rows of tables. Some students like to circle the information on the axes to help them visualize their work. In addition, pay attention to any trends between what is being measured, as knowing the basics trends of each graph and table can help you quickly answer questions later on. This entire step should take no more than ten to fifteen seconds.

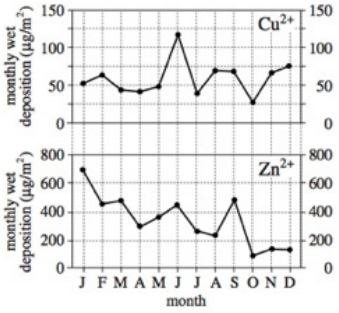


Figure 2

2. Next, proceed directly to the questions, referring back to the graphs and tables as needed. Most times, questions will tell you where to look ("In Figure 1..." ; "According to Study 3 ..." ; etc.). If they don't mention a figure or table by name, they will almost always mention one of the variables being measured on a graph or in a table.

3. Only read the text introductions to the experiments if you need to. Some of the harder questions (usually limited to the last one or two questions in research summary passages) might require this -- especially if they are asking questions about "process" or "methods" rather than data. If you are struggling to complete the section in time, NEVER go back into the passage to answer questions. You are better off making a quick, educated guess and moving on. There are much easier questions later in the section, and you want to be sure you get to them!

Conflicting Viewpoints Passages

For the conflicting viewpoints (CV) passage, you'll have no choice but to change your strategy -- here, there are rarely charts and graphs to be found. Instead, you'll find a block of text in which the views of two students or experts are compared. Because this section takes the longest amount of time and most students find it the hardest passage of the test, most students prefer to **save it for last**, regardless of where it shows up among the test's six passages. However, if you are a fast reader and score highly on the ACT Reading Test, you may want to seek out this passage and complete it first.

For CV, your approach should be:

1. Quickly read each student/expert's take on the subject at hand. Ignore the introductory text (before each viewpoint) until a question forces you to read it.

2. As you read, annotate the text.
    a. Mark key differences between the experts' takes if you can spot them.
    b. Note each expert's main idea.

3. Then, proceed to the questions:
    a. Be sure to notice what each question asks - in other words, ask yourself, "is this question about Scientist 1? Scientist 2? both?" and refer back to the text accordingly.
    b. If you annotated, you should be able to find key information quickly.
    c. Many questions will require you to make inferences. Some of these questions might stump you. Use what you know and remember to guess -- there is no penalty for a wrong answer on the ACT.

Again, **TIME IS OF THE ESSENCE**. You have only 35 minutes to answer 40 questions, which works out to about six minutes per passage and less than one minute per question. It is best to move through the DR passages in 4-5 minutes each, freeing up extra time for the CV & RS passages.

Ultimately, while the test is about "science" on the surface, it's actually more a test of whether or not students can read charts & graphs (DR & RS) and pick complicated information out of dense text (CV). **Don't worry if science isn't your best subject!** If you read the charts carefully and clearly identify what's being asked in each question, you can score highly, *even if you come in with **zero** knowledge about the sciences.* With practice, you will become increasingly confident in your ability to do this.

EXAMPLE 1: DATA REPRESENTATION

In a lab experiment, a student tested solubility of four compounds: sodium chloride, cerium sulfate, bisodium phosphate, and glucose.

The student controlled the amount of water at 100 mL, varying temperature at 0° C, 20° C, 60° C, and 100° C. The student determined how many grams of each compound would dissolve at each temperature.

As she calculated her results, the student was also interested in observing precipitation, which is when a compound's solubility decreases.

Temp	Sodium Chloride (NaCl) g	Caesium Bicarbonate () g	Bisodium Phosphate () g	Glucose () g
0°	42	19.4	14.7	140
20°	41.8	8.62	9.42	197
60°	37.9	3.97	22.6	311
100°	31	.0225	69	501

Figure 1: Solubility at various temperatures, shown in grams dissolvable per 100 ml water

Walkthrough of Questions

When you start, note your columns, rows, and what's measured.

- Columns: various compounds
- Rows: various temperatures
- Measurement: Grams soluble per 100 mL

When we look for trends for each compound, we can quickly see that, as temperature increases, solubility decreases for Sodium Chloride and Caesium Bicarbonate, and increases for Glucose. Bisodium phosphate's solubility decreases initially, and then increases.

Next, skip the text until you need it and head straight to the questions:

1. A student added 15 grams of the following compounds to 100 ml of 0°C water. Which of the following compounds will NOT be fully dissolved?

   A. Bisodium Phosphate
   B. Caesium Bicarbonate
   C. Sodium Chloride
   D. Glucose

The question asks which of the four compounds will NOT fully dissolve at 0°C if we put 15 grams into the sample. Looking at Figure 1, we hone in on the row corresponding to 0° C. We see that at that temperature, 42 grams Sodium Chloride, 19.4 grams Caesium Bicarbonate, 2.1 grams Bisodium Phosphate, and 140 grams Glucose dissolve. We therefore see that one answer choice is not like the others. Only Bisodium Phosphate dissolves less than 15 grams, which means that A is the correct answer.

2. At 20°C, which compound has the greatest solubility?

   A. Bisodium Phosphate
   B. Caesium Bicarbonate
   C. Sodium Chloride
   D. Glucose

This question is much like the first: looking to the row for 20° C, we should look for which compound will dissolve the most. Reading across we see Sodium Chloride will dissolve 41.8 grams, Caesium Bicarbonate 8.62 grams, Bisodium Phosphate 9.42 grams, and Glucose 197 grams. Therefore D is the correct answer, because far more Glucose will dissolve at 20° C than will any of the other compounds.

3. Saturated solutions of the following compounds were prepared at 20°C. Which of the following will precipitate upon cooling to 0°C?

A. Bisodium Phosphate
B. Caesium Bicarbonate
C. Sodium Chloride
D. Glucose

For this question, we can look back to the introductory paragraph to skim for the word "precipitate" - because a quick read through of our tables turns up nothing. When we do, we see that precipitation occurs "when a compound's solubility decreases." So, we need to chart solubility for each compound from 20°C to 0°C.

A. Bisodium Phosphate - 9.42 to 14.7 (increase)
B. Caesium Bicarbonate - 8.62 to 19.4 (increase)
C. Sodium Chloride - 41.8 to 42 (increase)
D. Glucose - 197 to 140 (decrease)

Only **D**, glucose, is less soluble at 0°C than at 20°C. Therefore it is the correct answer.

TIME-SENSITIVE STRATEGY: Again, we want to avoid looking back in the passage if we can. While we may not know what "precipitate" means, but we know that the question is asking us about the difference between compounds at 20°C and 0°C. Because glucose is the only compound that has a different relationship than the other three compounds, it is most likely the correct answer.

4. The labels on two bottles of chemicals from the table are illegible. Which of the following tests could conclusively differentiate between Sodium Chloride and Bisodium Phosphate?

   A. Mix 10 grams of each sample at 60°C. The one that dissolves is Sodium Chloride.
   B. Mix 30 grams of each sample at 100°C. The one that dissolves is Bisodium Phosphate.
   C. Mix 25 grams of each sample at 60°C. The one that dissolves is Sodium Chloride
   D. Mix each with Caesium Bicarbonate, and the one that creates a reaction is Sodium Chloride.

We need to find a clear way to prove which compound is which. Let's try each answer choice!

A. 10 grams of each sample at 60°C.
- 37.9 grams Sodium Chloride dissolves at 60°C.
- 22.6 grams Bisodium Phosphate dissolves at 60°C.
- Both will dissolve the test amount of 10°C.

B. 30 grams of each sample at 100°C.
- 31 grams Sodium Chloride dissolves at 100°C.
- 69 grams Bisodium Phosphate dissolves at 100°C.
- Both will dissolve the test amount of 30 g.

C. 25 grams each sample at 60°C.
- 37.9 grams Sodium Chloride dissolves at 60°C.
- 22.6 grams Bisodium Phosphate dissolves at 60°C.
- Only Sodium Chloride will fully dissolve the test amount of 25 g.

D. Mix each with Caesium Bicarbonate, and the one that creates a reaction is Sodium Chloride.
- This is a red herring answer. We do not get any information about what will happen if we mix compounds.

The only answer to produce a clearly different result between Sodium Chloride and Bisodium Phosphate is **C**.

5. Suppose we were to test solubility at 200°C. How many grams of Caesium Bicarbonate would we expect to dissolve?

   A. Because Cerium (IV) precipitates with higher temperatures, large amounts of it would dissolve at 200°C.
   B. Because Cerium (IV) precipitates with higher temperatures, very little would dissolve at 200°C.
   C. Because Cerium (IV) precipitates with cooling temperatures, large amounts of it would dissolve at 200°C.
   D. Because Cerium (IV) precipitates with cooling temperatures, very little would dissolve at 200°C.

Looking to our column for Caesium Bicarbonate, we note that as temperature increases, less dissolves. In other words, it precipitates with higher temperatures, eliminating **A** and **C**. Since only .0225 dissolves at 100°C, we would expect even less than that to dissolve at 200°C. Only **B** reflects this.


The important thing is not to stop questioning
– Albert Einstein

EXAMPLE 2: RESEARCH SUMMARY

Methane (CH_4) is a colorless, odorless, tasteless, flammable gas produced whenever organic material is decomposed by bacterial action in the absence of oxygen. The atmosphere contains about 2.2 parts per million (ppm) by volume of Methane. This number is increasing as humans produce more organic waste.

Landfills emit a large quantity of methane as they break down the organic material in garbage. Because methane traps heat in the atmosphere, landfills thus contribute heavily to climate change. In fact, methane is about 9 times as strong as CO_2, and though it dissipates far more quickly than does carbon dioxide, it is more dangerous in the short term. Concerned about these emissions, a student wanted to study the effects of temperature on their quantity. He proposed that landfills in warmer climates would emit more methane.

Experiment 1

A student built four model landfills in 3ft x 5ft x 2ft boxes. He stored these in 3ft glass boxes with solar exposure and then measured the percent of methane in the air after three years. Each model contained different amounts of organic matter.

Temp	L1 - 35% organic material	L2 - 65% organic material	L3 - 75% organic material	L4 - 4% organic material
0°C	0.04%	0.07%	0.09%	0.00%
15°C	0.80%	2.50%	6.33%	0.03%
20°C	1.40%	4.92%	8.42%	0.65%
35°C	3.95%	6.25%	10.12%	0.67%

Figure 1: % methane in the air at various temperatures

Experiment 2

The student was also concerned with leachate, or the toxic liquid produced by the breakdown of organic material in landfills. He hypothesizes that a variety of grass, Chryspogon Zizanoides, may reduce leachate in landfills. He tests leachate waste in g/L of soil at 15°C with varying levels of Chryspogon Zizanoides

Time Elapsed	L1 - 1% Chryspogon Zizanoides by volume	L2 - 2% Chryspogon Zizanoides by volume	L3 - 5% Chryspogon Zizanoides by volume	L4 - 0% Chryspogon Zizanoides by volume
2 years	44.7	38	16.6	51.2
1 year	26.1	22.4	13.2	29.3
6 months	15.4	14.3	12.4	16.2
3 months	9.2	9.2	9.1	9.2

Figure 2: leachate in g/L with varying quantities of Chyspogon Zizanoides

Walkthrough of Questions

As with the Data Representation passage, skip the text and only refer back as necessary. First, however, you must note what your two experiments are measuring and jot down any trends you see:

Figure 1:

Columns: *four different landfills with different % organic material*
Rows: *temperatures*
Measurement: *% methane in the air*
Trends: *% methane increases with temperature, and decreases when % organic material decreases.*

Experiment 2:

Columns: *four different landfills with different % Chyspogon Zizanoides*
Rows: *time elapsed*
Measurement: *leachate in g/L*
Trends: *Leachate increases over time*

1. According to Experiment 1, which of the following produces the most methane?

   A. L1 at 20°C
   B. L3 at 15°C
   C. L4 at 0°C
   D. L2 at 35°C

First, note that we are told to look to Experiment 1, so look to Figure 1. Next, note what the question asks: which produces the most methane? This is as simple as reading the table:

A. L1 at 20°C produces 1.40% methane air content.
B. L3 at 15°C produces 6.33% methane air content.
C. L4 at 0°C produces 0.00% methane air content.
D. L2 at 35°C produces 6.25% methane air content.

Therefore, **B** is the correct answer because 6.33% is the highest number of the four.

2. Do the results from Experiment 1 support the hypothesis that landfills in warmer climates produce more methane?

   A. Yes, because methane production increased as temperature increased.
   B. Yes, because waste containing high volumes of organic material is clearly shown to produce higher quantities of methane.
   C. No, because relatively little methane is produced by the landfills at 0 C.
   D. No, because methane production decreased as temperature increased.

Again, we look to Experiment 1. When we do, we see that as temperature increases, each of the four landfills produces more methane. While **B** and **C** are true, they do not answer the question. **D** is the opposite of what we see. Therefore, the answer must be **A**.

3. The student notes landfills in the following locations:

City	Average Temp.
Rochester, NY	48
Phoenix, AZ	75
Kansas City, KS	53
Charleston, WV	55

If a company that hopes to MAXIMIZE methane emissions for capture and re-use wanted to build a landfill, which of the following would be the best location to do so?

A. Rochester, NY
B. Phoenix, AZ
C. Kansas City, KS
D. Charleston, WV

Note that we are asked to maximize methane emissions. We know from the last question - or from looking at the table again - that methane emissions are maximized at higher temperatures. Therefore, we would want to build a landfill in the hottest location to maximize methane emissions. This means choice **B**, Phoenix, is the correct answer. If we are unsure, we know that maximizing (and minimizing) usually involves a number at the extreme end of a data set. So we could confidently eliminate the two answer choices in the middle of our temperature range (C and D)

4. Which of the following is NOT strongly implied by the results of Experiment 1?

   A. Methane largely remains in the ground at temperatures at or below freezing.
   B. Plastics and other inorganic waste produce little methane.
   C. Landfill methane emissions have been exaggerated.
   D. Landfills with more organic content will produce more methane.

Again we look to Experiment 1. Note that we are looking for which answer choice is NOT implied by the data. Moving through the answer choices:

A. Methane largely remains in the ground at temperatures at or below freezing.
- Freezing = 0°C
- At this temperature, methane in the air is 0.00%.
- This answer choice is **true**.

B. Plastics and other inorganic waste produce little methane.
- We see that L4, which has very little organic content (4%), produces very little methane.
- This supports this answer choice, so it is **true**.

C. Landfill methane emissions have been exaggerated.
- We see no evidence of this answer choice.
- It may be true, but it is not in any way implied by the data.
- Therefore we cannot conclusive call this answer true, so let's call it **not verifiable**.

D. Landfills with more organic content will produce more methane.

- We see that landfill 3, with the most organic content (75%) produces the most methane at every temperature.
- This shows that this answer choice is **true**.

All answer choices are verifiably true except for **C**, making it the correct answer.

5. Which of the following correctly states the relationship between temperature and the production of leachate?

   A. As temperature increases, leachate production increases.
   B. As temperature increases, leachate production decreases.
   C. Net change in leachate production increases more quickly as temperature increases.
   D. The information provided is insufficient to determine the relationship between temperature and production of leachate.

We must realize here that we are now dealing with leachate production. This means we must look to Experiment 2 and Figure 2. However we do not see any information about temperature in experiment 2. This means we do not know anything about the relationship of temperature and leachate production. This rules out **A, B**, and **C.** Only answer choice **D** accounts for our lack of information, and it is therefore correct.

6. The results of Experiment 2 MOST strongly imply which of the following?

   A. Leachate production is immediately reduced by the presence of Chryspogon Zizanoides.
   B. Leachate production is unaffected by the presence of Chryspogon Zizanoides.
   C. Leachate production is significantly reduced by Chryspogon Zizanoides, but this relationship takes time to form.
   D. Leachate production is a function of the quantity of heavy metals in the soil.

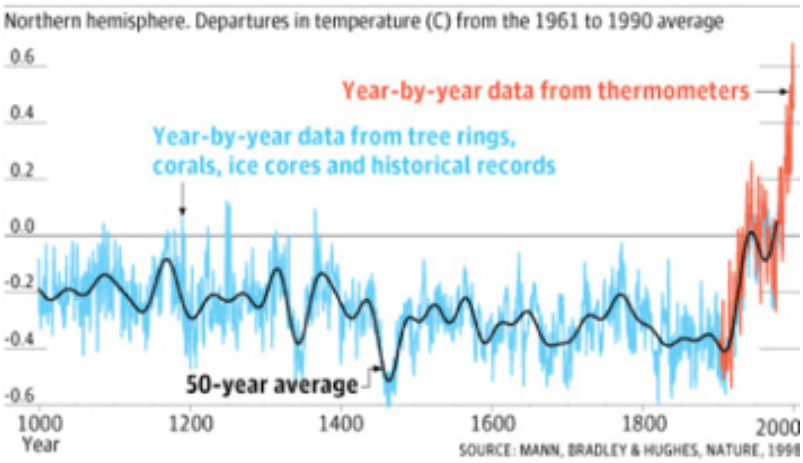
First we note to look to experiment 2. Next, we note that the first three answer choices trace the relationship between leachate production and the presence of Chryspogon Zizanoides in the sample. Answer choice D, which notes a relationship between leachate and heavy metals in the soil, has no basis in the data, so it can be eliminated. When we look to the figure to chart this relationship, we note that L3 (with highest percentage of Chryspogon Zizanoides) clearly produces the least methane after at least one year has elapsed. However, we note that after the shortest amount of time - at 3 months elapsed - methane production appears to be unaffected by Chrypsogon Zizanoides. There is therefore a lag time before it affects methane production. This means that answer choice **C** is correct.

MEN LOVE TO WONDER, AND THAT IS THE SEED OF SCIENCE

– Ralph Waldo Emerson

EXAMPLE 3: CONFLICTING VIEWPOINTS

Since the early 1990s scientists have come to a consensus on global warming. If we observe the global average temperatures from the beginning of the Industrial Revolution to today, we can see an increase of about 1 degree C. According to our models of past climates, this is the most rapid climate change in the history of planet Earth.



Student 1

Climate change is occurring, and the evidence overwhelmingly supports the conclusion that it is man-made. Today, carbon particles per million particles of air (parts per million, PPM) have broken 400 PPM, a level not seen since the Pliocene Era, when sea levels and temperatures were far higher than they are today. This carbon comes mostly from burning fossil fuels including petroleum and coal. 400 PPM is far beyond the "safe" equilibrium for life on Earth, and the problem is only getting worse.

Some studies have shown a pause in global warming since 1990, and this has caused some to question whether climate change is actually occurring. However, if a pause in the rise of surface temperatures has been observed, it is not because the Earth is not warming. Rather, it is because the oceans have acted as a heat sink, absorbing warmth which then

plummets to its depths. This heat trapping was precipitated by a sudden shift in northern Atlantic salinity, making water saltier and denser. Eventually this heat will distribute throughout the ocean, melting the ice caps.

Though we need to understand the Earth's various feedback loops more confidently, we can see that most of these feedback loops will exacerbate climate change. One such loop is the acidification of the ocean, which will have untold effects on currents and make the water less suitable for marine life. Another is the melting of the permafrost. When it melts, it releases thousands of years' worth of stored energy in the form of methane. This induces rapid climate change, more serious than pollution because methane traps heat about 9 times as effectively as does.

Student 2

Climate change is occurring, but we are still unsure about various feedback loops in the climate system. Until we understand these feedback loops further, we will not be able to accurately predict future temperature rise. The climate changes because of the "greenhouse effect," the process by which carbon dioxide in the atmosphere traps heat and warms the Earth. Though carbon PPM has recently surpassed 400, far above pre-Industrial Revolution levels of 350 ppm, we can't call any measurement "safe" or "unsafe" until we move from models to observations and learn how Earth responds.

While evidence suggests the climate has warmed over the last 150 years, warming has slowed dramatically over the last ten years. There are several explanations for this slowdown. The first is natural climate fluctuations, of which the weather phenomena El Niño and La Niña in the Pacific are the most important. The second is that radiation from the sun has been weaker than predicted in the past few years, counteracting global climate change.

Perhaps systems like El Niño and La Niña are permanently affected by changing climates. If they change to produce cooler weather, they constitute an example of what we could consider a positive feedback loop, a form of "self-regulation." We don't know for sure how many of these self-regulating systems Earth has.

Walkthrough Of Questions

1. According to Figure 1, the Earth experienced a "mini Ice Age" around which of the following years?

   A. 1250
   B. 1450
   C. 1750
   D. 1950

For this question, we are told to look directly toward Figure 1. We are asked about a "mini Ice Age" which sounds like cold weather. If we look to the graph, the minimum temperature was recorded between 1400 and 1500. Therefore B, 1450, is the best answer choice.

2. How do the two students most significantly differ in their view of what causes warming?

   A. Student 1 views warming over a longer time scale than does Student 2.
   B. Student 1 questions the efficacy of models, while Student 2 does not.
   C. Student 1 considers feedback loops as certain contributors to warming, while Student 2 believes that some of these feedback loops might counteract warming.
   D. Student 2 does not believe that human-caused climate change is occurring, while Student 1 does.

Now we get to the meat of this section: comparing viewpoints. Let's start with A. Student 1 mentions the Plioscene Era as a long-past reference point, but Student 2 doesn't mention an early boundary for his or her consideration of climate change. Let's move forward. For B, our descriptions are switched. If we look toward the text, it is Student 2 who actually questions models' efficacy, and it is Student 1 who does not. Watch out for answers that seem right but are actually logically backward, like this one. For C, we see that Student 1 considers only negative feedback loops, while Student

2 considers feedback loops that may slow climate change, such as the weakening of El Nino. Therefore this is the correct answer. D sounds right, but it states its case too strongly. While Student 2 wants to avoid jumping to climate conclusions, he or she still believes that human-caused climate change is occurring to some degree.

3. What can most reasonably be inferred about Student 1?

A. Student one is skeptical of PPM measurements.
B. Student one is skeptical of the global warming pause.
C. Student one believes in the promise of geoengineering solutions to global warming.
D. Student one is less alarmed by climate change than Student 2.

Again we must move through the answers one-by-one, and that's why this section takes time. For most of its questions, there is no easy cross-reference to make with charts. Choice A suggests that Student 1 is skeptical of PPM measurements, but he/she never suggests as much. Choice B suggests that Student 1 is skeptical of the global warming pause, which we do see clearly when he/she states, "if a pause in the rise of surface temperatures has been observed, it is not because the Earth is not warming." This is the best answer. Choice C suggests that Student 1 favors geoengineering solutions, but we see no such evidence. Choice D suggests that Student 1 is less alarmed by climate change than Student 2, but in fact it is Student 2 who is less alarmed.

4. Which of the following conclusions, if true, would most likely undermine the argument of Student 2?

A. El Niño and La Niña were shown to be weak from 1995-2005.
B. 85% of observed feedback loops are likely to exacerbate warming.
C. The atmosphere of Venus is shown to be 95.6% .
D. Many scientists question the validity of solar radiation measurements.

Taking notes in the margins, we hopefully noted that Student 2 is more skeptical of climate modeling than Student 1, in particular because he/she questions the depth of our understanding of feedback loops.

The money quote: “we are still unsure about various "feedback loops" in the climate system. Until we understand these feedback loops further, we will not be able to accurately predict future temperature rise.” Near the end of the essay he/she considers the idea of “positive feedback loops,” which actually heal the Earth. So we are looking here for some finding that would undermine such confidence in positive feedback loops. Only **B** does that - if 85% of observed feedback loops make warming worse, we can’t put much faith in positive feedback loops winning out and smoothing the problem of warming over. Answers C is irrelevant and D has only passing relation to any of Student 2’s comments. Answer A seems appealing, perhaps, but Student 2 never argues for the opposite: that La Nina and El Nino were particularly strong in that timeframe.

5. According to Student 1, what accounts for the global warming pause?

   A. The gradual acidification of the ocean as it warms
   B. The melting permafrost
   C. The absorption of heat by the ocean
   D. Increased solar activity

Here we can scan Student 1’s essay for the word pause. It is found in the following line, “However, if a pause in the rise of surface temperatures has been observed, it is not because the Earth is not warming. Rather, it is because the oceans have acted as a heat sink, absorbing warmth which then plummets to its depths.“ This sounds exactly like choice **C**, which makes that the correct answer.

6. About which of the following do Student 1 and Student 2 most strongly agree?

   A. It is impossible to ever determine how much climate change will affect the Earth.
   B. Models can predict coming developments with near-perfect accuracy.
   C. Climate systems on Earth are linked in numerous, complex ways.
   D. A carbon prevalence of 450 PPM has clearly predictable implications.

Here we can use process of elimination. We know that Student 2 questions models and believes we cannot make accurate predictions. This is enough to rule out choices B and D. We know that Student 1 believes predictions are accurate. This is enough to rule out choice A. Therefore, we are left with choice C, which correctly posits that both students believe that Earth is linked in numerous, complex ways through various feedback loops.

7. While Earth has been shown to warm dramatically according to Figure 1, which of the following could call its results into question?

A. Historical evidence of a "Mini Ice Age"
B. Widespread disagreement over how to gauge temperature from coral and tree ring readings.
C. Temperature readings in C instead of K.
D. A five-year period of slight cooling.

We are looking for information that will undermine Figure 1, which shows dramatic warming since 1900. Choice A confirms what we see - a mini Ice Age around 1450. Choice C is irrelevant - regardless of what scale we use, temperature readings should be shown to increase over time. Choice D is also irrelevant - we know that climate may occasionally cool for a brief time as the Earth adjusts to its new realities, but that the trend is certainly toward warming. This leaves only answer choice B, and indeed, if scientists widely disagreed on how to gauge temperatures from coral and tree ring readings - in other words, if they widely disagreed on how to measure temperatures from past climates, their conclusions about long-term warming might be thrown into doubt.

ACT SCIENCE TEST SECTION OVERVIEW

TP

BASICS:

40 Questions in 35 minutes
Average time per passage:
If section has 6 Passages – 5:50
If section has 7 Passages – 5:00

Passage types:
Data Representation – Questions only require data analysis.
Research Summaries – Questions require data analysis or evaluating the setup of the experiment itself.
Conflicting Viewpoints – Questions require you to compare ideas and theories of various scientists or students.

KEY RESOURCES:

ACT Science Curriculum
ACT Science Quizzes

GENERAL STRATEGY (DATA REPRESENTATION AND RESEARCH SUMMARIES):

1. Quickly (less than 30 seconds) review each table and/or figure in the passage to:
    a. Locate keywords – usually found on axes, keys, units and column headers.
        i. If you don't know the meaning of a keyword, quickly find its definition in the text (usually italicized).
    b. Determine general trends of data – how does the y-axis change as the x-axis increases? How do columns in tables compare to each other?
2. Read each question carefully to determine, in your own words, what it's asking you to find.
    a. As you read a question, use the keywords mentioned in the question or answer choices to locate specific data in the appropriate table or figure.
3. If you know where to find the answer to a data interpretation question, then focus on finding the correct answer. If you are not sure, eliminate wrong answer choices based on the information you can locate and the relationships you can find in the data.
4. If you can't eliminate enough answer choices, circle the question in your booklet, and use your instincts and scientific knowledge to select an answer and move on.

GENERAL STRATEGY (CONFLICTING VIEWPOINTS):

1. Scan the introduction to define any key terms. Circle any proper nouns and/or numbers.
2. Read each viewpoint to determine the main idea(s). Circle any proper nouns and/or numbers.
    a. Once you read a viewpoint, try to keep its main idea in your head to compare to each successive viewpoint.
3. Read each question carefully to determine, in your own words, what it's asking you to find.
4. Use your knowledge of the passage and general scientific knowledge to eliminate wrong answer choices.
    a. Use proper nouns and numbers to locate specific information whenever possible.
    b. Do NOT go back into the passage if you don't know what you are looking for.
5. If you can't eliminate enough answer choices, circle the question in your booklet, and use your instincts and scientific knowledge to select an answer and move on.

EXTRA TIPS:

1. Don't read the text – it may be difficult to resist, but reading the text in Data Representation and Research Summaries passages is a waste of time. You should only look back in the text to define a key term on a table or graph, or to scan for an answer for questions about an experiment's procedure.
2. Embrace your uneasiness – when you don't read the text in passages, you may feel a bit insecure about your level of understanding. Don't! Trust your interpretation of the data, common sense, and outside scientific knowledge when answering even the toughest of questions.
3. Weed through the wordiness – the ACT will constantly ask rather simple questions in extremely complex and wordy language, or insert new information in a question to distract you from what they are really asking. Your ability to simplify questions to simple relationships is critical to your score.
4. Use both hands – use the index finger of your non-writing hand to zero-in on the appropriate data as you read questions in Data Representation and Research Summaries passages.
5. Use the answer choices – keywords in the answer choices are just as helpful as keywords in the question. If you don't see a keyword in the question, you are likely to find one in the answer choices. Units are especially common in answer choices, and can be extremely helpful!
6. Focus on reasoning – Many questions ask you to choose between two options (such as Yes/No) and give the correct reasoning behind your choice. Evaluate the reasoning first to eliminate wrong answers, as the reasoning is almost always based on the data.
7. Remember what you've learned in science class – the ACT will not try to trick you by including false scientific principles in passages. Therefore, you can use your previous scientific knowledge to help you in a pinch. Some of the most important concepts to know are photosynthesis, cellular respiration, parts of an atom, pH, and kinetic-molecular theory.
8. Look for shared keywords –a keyword present in more than one table or figure is extremely important, as it allows you to find even more relationships in the data. Many of the more difficult questions on the test are centered around shared keywords. correctly is hugely important. Remember how certain key words translate into mathematical terms.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