MATH CURRICULUM

ACT

ESM PREP

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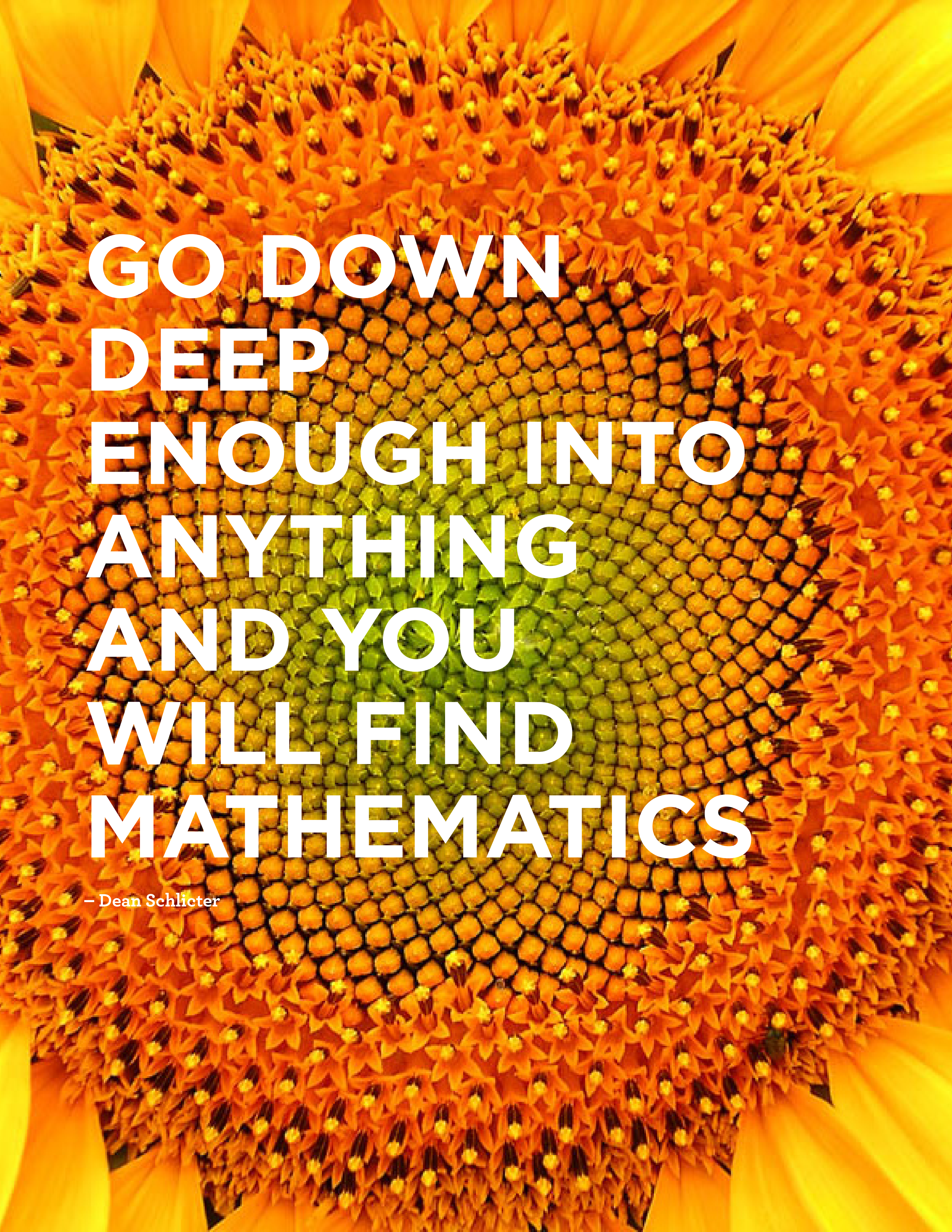

GO DOWN DEEP ENOUGH INTO ANYTHING AND YOU WILL FIND MATHEMATICS
– Dean Schlicter

INTRODUCTION

What's on the Test?

The ACT Math Test consists of 60 multiple-choice questions that test your knowledge in six different subject areas. The subject area breakdown is the same for every test:

Subject Area	Number of Questions	Percent of Test
Pre-Algebra	14	23%
Elementary Algebra	10	17%
Intermediate Algebra	9	15%
Coordinate Geometry	9	15%
Plane Geometry	14	23%
Trigonometry	4	7%
Total	**60**	**100%**

The six main subject areas cover the following topics:

1. **Pre-Algebra:** order of operations; basics of exponents and roots; ratios and proportions; percentages; factors and multiples of integers; absolute values; linear equations; simple probability; interpreting charts, tables, and graphs; basics of mean, median, and mode.

2. **Elementary Algebra:** expressing relationships by using variables; substitution; basic operations of polynomials; factoring polynomials; solving simple quadratics; solving linear inequalities; properties of exponents and roots.

3. **Intermediate Algebra:** using the quadratic formula; radical and rational expressions; inequalities and absolute value equations; sequences; systems of equations; inequalities involving quadratics; functions; matrices; roots of polynomials; and complex numbers.

4. **Coordinate Geometry:** using graphs of number lines as well as points, lines, polynomials, circles, and other curves in the standard coordinate plane; relationships between equations and graphs; slope; parallel and perpendicular lines; distance; midpoints; transformations; and conics.

5. **Plane Geometry:** properties of triangles, rectangles, parallelograms, trapezoids, circles; angles; parallel and perpendicular lines; geometric translations, rotations and reflections; simple 3-D geometry; measurement of perimeter, area, and volume.

6. **Trigonometry:** trigonometric ratios for right triangles (i.e. SOHCAHTOA); values, properties and graphs of trigonometric functions; trigonometric identities; trigonometric equations; modeling with trigonometric functions.

Why is this breakdown important?

Knowing the breakdown by subject allows you and your academic mentor to effectively focus your prep time on the areas that will help increase your score the most. For example, it does not make sense for you to begin your preparation for the Math Test by reviewing trigonometry questions if you have significant room to improve in algebra or geometry, as those questions make up 93% of the test!

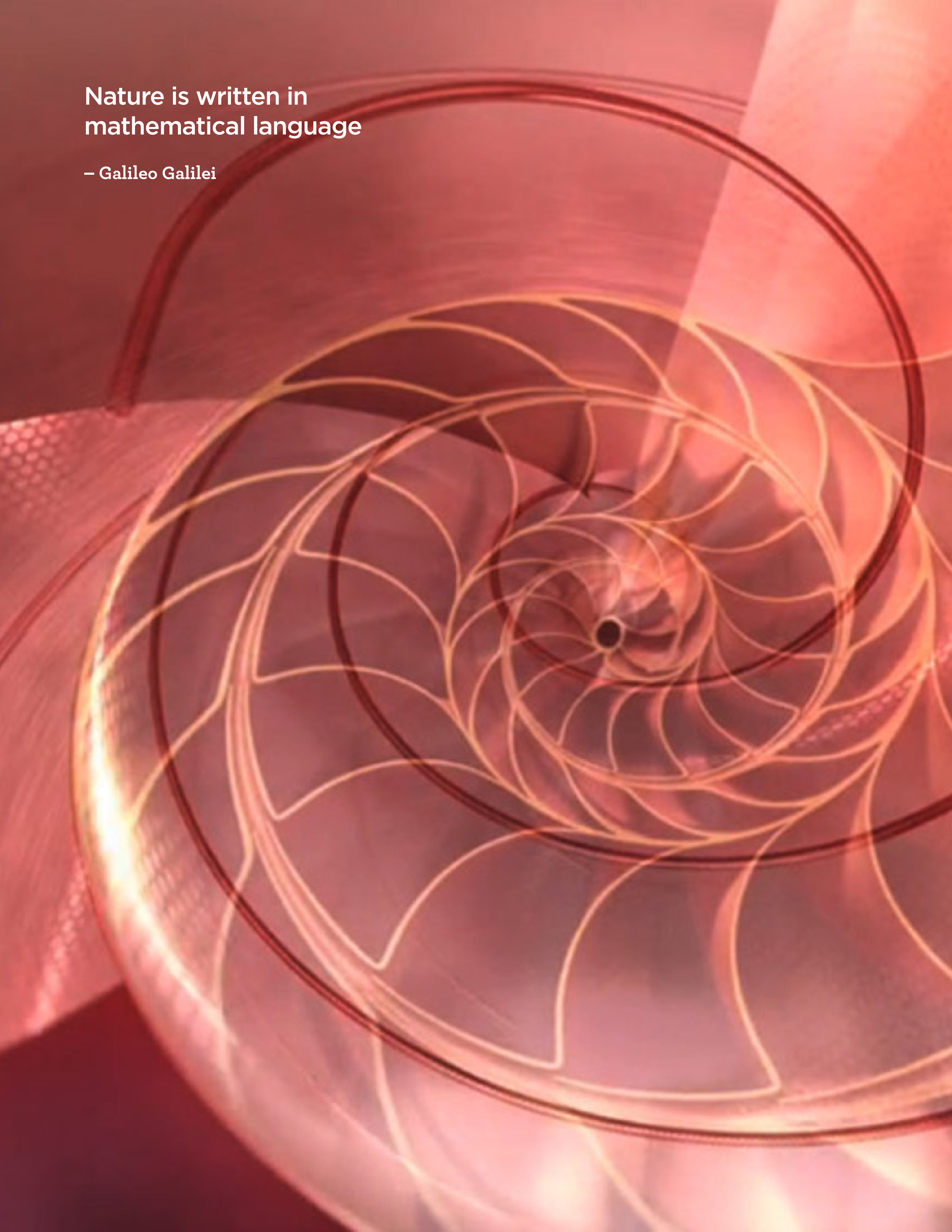
General Subject Area	Number of Questions	Percent of Test
Algebra	33	55%
Geometry	23	38%
Trigonometry	Just 4!	7%

How long is the ACT Math Test?

The test is sixty minutes long, which means that students have an average time of exactly one minute for each question. This may not seem like a lot of time, and for most students it isn't, but if you prep effectively and efficiently for the test, it can be more than enough.

Nature is written in mathematical language

– Galileo Galilei



THE GENERAL APPROACH

Because there is a set breakdown of questions for each test, and time is such an issue, students should answer the questions they know right away, and then come back to questions they are not sure how to solve to at least arrive at an educated guess:

THE BASIC STEPS

Below is a general outline of the steps to finishing the ACT Math Test. While the smaller details of these steps vary depending on skill level and pacing, the basics should generally be followed by all students.

Step 1. Read the question CAREFULLY to determine what it is asking you to find.

This step may seem like common sense, but do not underestimate its importance! After all, you cannot answer questions if you don't know what to solve for. If you are unsure what a question is asking for after you read it the first time, skip it and come back to it later. Your time is best used answering all the questions you immediately understand, and then coming back to the ones that were unclear to you. MAKE SURE YOU MARK ON YOUR ANSWER SHEET ANY QUESTIONS SKIPPED.

Step 2. What are you given?

You want to extract as much information from every question as you can and jot it down in your workspace. While you may not be able to see the importance of all of the information immediately, writing down EVERY piece of information can help you see the relationships much more clearly.

Does an algebra question give you an equation to work with or the values of any variables? Write them down!

Writing down given information is especially important in geometry problems. If a problem provides you with a figure, such as a triangle, and information that is relevant to the figure, such as the length of sides or angle measures, write this information directly on the figure! Doing so can point you towards other info like side lengths or angle measures. If a geometry problem does not give you a figure, draw one yourself and add all of the information you can.

If a geometry problem involves geometric keywords such as area, perimeter, circumference, midpoint, or distance, write down the formulas that define those words (which you will have memorized by test day!) Doing so can help you to easily determine what variables you need.

Step 3. What do the answers look like?

After reading the question and writing down all relevant information, take a peek at the answer choices. Knowing what form the answers are in can point you in the right direction. Answer choices can also offer hints about the problem if you look closely. For example, if a geometry question involving triangles has answer choices containing $\sqrt{2}$ or $\sqrt{3}$, you might be able to realize that using the properties of special right triangles is needed.

Step 4. Can you solve it?

If you read a problem, know what it's asking for, and immediately know how to solve for the answer, jump right in and solve it! These are the questions that we want to complete FIRST, no matter where they are in the test.

If you do not how to solve the problem right away, check to see if you can use any of the more unorthodox strategies we will practice in the next lesson.

If you do not know how to solve a problem, do not waste time trying to figure out how to do so! Just circle the question in your test booklet, make an educated guess using one or more of the guessing strategies we have practiced, and move on!

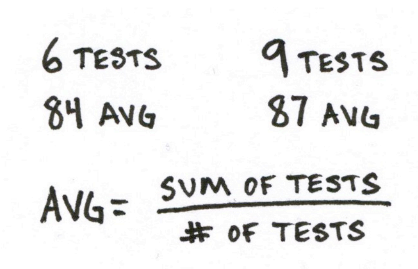
Once you have gone through each of the 60 questions and completed the questions you knew how to do immediately, return to the ones you were unsure of and try to solve them using other methods. The perplexities you initially faced when reading the question the first time often disappear after you have gotten into the flow of the section and answered most of the questions!

EXAMPLES

Now that we have reviewed the basic approach to any math problem on the ACT, let's practice putting it into action by looking at some examples. We'll start with a tough algebra question:

> John has taken 6 of the 9 equally weighted tests in his Algebra class this semester, and he has an average score of exactly 84.0 points. How many points does he need to average on the final 3 tests to raise his average score to exactly 87.0 points?
>
> **A.** 99
> **B.** 93
> **C.** 90
> **D.** 87
> **E.** 84

After carefully reading the question, we must first determine for what it is asking. Here, we are asked to find the average score for the 7th, 8th, and 9th tests that will bring Tom's average score up to exactly points. Next, go back into the question to extract all of the important information from the question, and jot down any formulas that might be useful.



The phrase "average score" is used twice in the question, so let's write down the equation for average below the answer choices. Even if we don't know how to solve the problem right away, we can use this equation to help point us in the right direction. We are also given that John has an average score of 84.0 point after 6 tests, and needs an average score of 87.0 point on 9 tests. We can plug these values into the equation we wrote down to see what information we can derive:

6 TESTS 9 TESTS

84 AVG 87 AVG

$$\text{AVG} = \frac{\text{SUM OF TESTS}}{\text{\# OF TESTS}}$$

$$84 = \frac{\text{SUM OF 6 TESTS}}{6} \qquad 87 = \frac{\text{SUM OF 9 TESTS}}{9}$$

$$\text{SUM} = 504 \qquad \text{SUM} = 783$$

$$783 - 504 = 279$$

$$\text{AVG} = \frac{279}{3} = \boxed{93}$$

Just by using the one equation we wrote down, we were able to find the sum of the first six tests and what the sum needs to be for the 9 tests. We can subtract the sum of the first 6 test from the desired sum of the 9 tests to find the total points needed from the remaining three tests. By using the average equation one more time, and dividing the points needed by the three tests, we arrive at the correct answer of B!

Next, lets take a look at a sample geometry question:

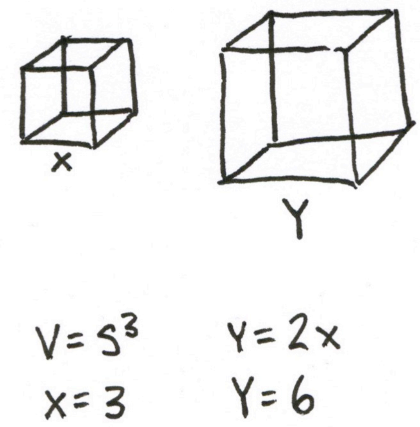
Cube X has an edge length of 3 inches. Cube Y has an edge length double that of Cube X. What is the volume, in cubic inches, of Cube Y?

A. 6
B. 9
C. 27
D. 36
E. 216

This plane geometry question is asking us to find the volume of Cube Y. Since we are asked to find the volume of a cube, let's jot down the equation under the equation. We can also draw a quick sketch of the two cubes if we are having trouble conceptualizing the problem. We are given that the length of the edges of Cube Y is double that of Cube X, which has an edge length of 3. Let's jot that down as well:

Cube X has an edge length of 3 inches. Cube Y has an edge length double that of Cube X. What is the volume, in cubic inches, of Cube Y?

A. 6
B. 9
C. 27
D. 36
E. 216



Next, check the answer choices to see if they tell us anything about how to approach the problems. Well, they are all multiples of 3, but there isn't anything that helps us to determine how to approach the problem. Finally, can we solve the problem? By looking at the equation for volume, we can see that the only piece of information we need is the length of the sides of a cube. We know that the length of the sides of Cube Y is double the length of the sides of Cube X, which the question tells us is equal to 3. Therefore, the length of the sides of Cube Y is 6, and the volume of Cube Y is equal to or 216. Answer choice E is the correct answer.

SPECIFIC ACT MATH STRATEGIES

As we mentioned, the ACT is fairly straightforward in how it asks questions, but the questions themselves can be complex. At times you will be asked to solve for a variable or understand the relationships between two quantities, but it will appear impossible to solve for these values. Don't worry. **There is no one correct way to solve any problem on the ACT.**

If you don't know how to solve a math problem on the ACT the "right" way, try using one of the test-taking strategies outlined below.

Use your own numbers to get rid of variables!

We can often simplify complex problems by plugging in our own numbers and then evaluating the problem. This works well for problem with variables in both the question and answer choices. Let's practice this by choosing numbers and plugging them in on several questions.

The sum of 3 consecutive odd integers is k. In terms of k, what is the sum of the 2 smaller of these integers?

A. $\frac{2k}{3} - 2$

B. $\frac{2k}{3}$

C. $\frac{2k}{3} + 2$

D. $k - 2$

E. $k - 3$

In the problem above, we can pick any three consecutive odd integers and evaluate their sum to find k. Let's choose 3, 5, and 7 as our three odd integers. The problem states that their sum is k, so we take 7 and find that . Now we just have to find out which answer choice equals 8, the sum of the 2 smaller of our three chosen integers.

A. $\frac{2(15)}{3} - 2 = 8$

B. $\frac{2(15)}{3} = 10$

C. $\frac{2(15)}{3} + 2 = 12$

D. $15 - 2 = 13$

E. $15 - 3 = 12$

The answer is A. While we used 3, 5, and 7 because they were easy odd numbers to use for this problem, it usually doesn't matter which numbers you choose. The exception there is that you want to avoid 0, 1, or 2 when picking numbers. These can be problematic. Otherwise, the correct answer will always equal the value you found when evaluating the problem.

The "If/Which of the Following" Situation

This type of ACT problem consistently shows up in medium and hard level questions, as they seem extremely abstract at first glance. These questions involve a parameter, such as an "if" or "for" statement ("if x is a positive integer", "for all x>0"), and the phrase "which of the following", which means that you will have to use the answer choices to find the correct answer. For questions like these, the plugging in values strategy works extremely well. Let's take a look at an example:

If n is a positive integer, which of the following expressions must be an odd integer?

A. 3^n

B. n^3

C. $3n$

D. $\frac{n}{3}$

E. $3 + n$

In this example, we have an if statement that tells us that our variable n must be a positive integer. All we have to do to evaluate this problem is to pick a number that is greater than 0, and then plug that into the answer choices to see which answer choice gives us our desired result. Let's use the number 3 for n, and evaluate each answer choice:

A. $3^3 = 27$; this is an odd number, but don't stop here! When you plug-in numbers, you must evaluate ALL answer choices!

B. $3^3 = 27$; we have a duplicate, but don't panic!

C. $3(3) = 9$; Another odd number. Still, don't panic.

D. $\frac{3}{3} = 1$; odd number

E. $3 + 3 = 6$; finally, we've narrowed it down.

We actually have four answer choices that give us an odd value here! If this happens to you on the test, do not panic. Simply use another number and go through the process again. In this case, the word must indicates that you are looking for a choice that will be odd no matter what! Let's set n equal to 4 instead.

A. $3^4 = 81$

B. $4^3 = 64$

C. $4(3) = 12$

D. $\frac{4}{3} = 1.33$

Answer choice A is the only answer that gives us an odd integer during our second try, so it must be the correct answer!

Tips and Tricks for choosing your values

- Be sure to test ALL of the answer choices. There is a possibility that two answer choices could give you the same result based on the value you chose to evaluate them. If this is the case, pick a second number with which to test them again.
- Don't make it hard on yourself! Choose small numbers that make doing arithmetic as easy as possible.
- Avoid using 0, 1, and 2 for questions involving exponents or multiplication/division. 2 creates problems because squaring it is the same as doubling it.
- For questions that involve variables but ask you to take percents in the question, use 100!

TRANSLATING FROM ENGLISH TO MATH

Word problems are everywhere on the ACT. Many students find these problems to be difficult and intimidating. However, these problems can be made easier by learning how to translate English into Math. Many of the words in these word problems have direct translations into mathematics. The following table will provide you with these translations and tell you how to use them on the test.

Key Words	**Meaning**	**Example**
Increased by, more than, combined, together, total of, sum, plus, added to	Addition	"Five more than thirty" 30 + 5
Decreased by, minus, less, difference between/of, less than, fewer than	Subtraction	"8 less than 12" 12 – 8
Of, times, multiplied by, product of, increased by a factor of	Multiplication	"Half of 12" ½*12
Per, a, out of, ratio of, quotient of, percent, decrease by a factor of	Division	"50 decreased by a factor of 10" $\frac{50}{10}$
Squared, cubed, square root, cube root, to the power of	Exponential	"The square of 8" 8^2
Is, are, was, were, will be, gives, yields, sold for, result is	Equality	"Five plus five is ten" 5 + 5 = 10
A number, what number, what percent, another number	Variable	A number is 3 less than 11 x = 11 - 3

*When you have a subtraction problem written in English, it can be confusing as to which number to put first. See the following examples of different ways to say 8 – 5:

"Eight **minus** five"

"Eight **is decreased by** five"

"Five **subtracted from** eight"

"Five **less than** eight"

Example

An integer, *n*, is added to 4. That sum is then multiplied by 8. This result is 10 less than twice the original integer. Which of the following equations represents this relationship?

A. $8(n + 4) = 2n - 10$
B. $8(n + 4) - 10 = 2n$
C. $8(n + 4) = 10 - 2n$
D. $n + 4 \times 8 = 2n - 10$
E. $4 + 8 = 2n - 10$

We start here with "An integer, n, is added to 4", which should tell us that we are going to simply write:

$n + 4$...

Next, the phrase "That sum is then multiplied by 8" tells us we have to group together and multiply the entire thing by 8. We do that with parentheses:

$8(n + 4)$...

The phrase "the result is" signifies that we have completed one side of our equation and are completing an operation that equals another:

$8(n + 4) =$ ...

and that is equivalent to "10 less than twice the original integer", which means that we double n and subtract 10. We write:

$8(n + 4) = 2n - 10$

Now check your answer choices! The correct answer is A.

THE “PLUG ‘N CHUG” METHOD

Plugging in answer choices can be a very effective strategy on the ACT, especially when a question asks you to find the value of a variable. This strategy is very similar to the “Guess and Check” strategy you have been told not to use in math class for years. The main difference is that, instead of having to “guess” a value to use, you are instead given five choices from which to choose.

Almost every question that asks you to find a value can be solved with this method, it can often take a long time. However, there is a way to find the correct answer by only plugging in two answer choices. Let’s look at an example.

For what value of x is th equation $2(x - 6) + x = 36$

A. 24
B. 16
C. 14
D. 10
E. 8

This is a rather simple example. The best-case scenario is that you’ll be able to solve this directly. For example’s sake, let’s pick the second smallest answer choice and plug it in for x.

$x = 10$
$2(10 - 6) + 10 = 18$

This is incorrect. Since this result is less than our desired result of 36, we know that we need to pick a larger answer choice. Since answer choice E. is smaller than answer choice D., we know in this case that it will give us a smaller result when plugged into the equation. That allows us to cross out both answer choices D and E.

Next, try answer choice B. It’s important to note that we are now choosing to plug in the second largest answer choice.

$x = 16$
$2(16 - 6) + 16 = 36$

In this case, answer choice B is correct! But even if it wasn’t, we would still be able to find the correct answer just by plugging in choice B. If it gave us a value that was bigger than 36, then answer choice C would have to be correct. If it gave us a smaller value, then answer choice A would have to be correct.

You should be careful using this method when plugging into absolute value functions or polynomials of even degree. Smaller integers, when plugged into these types of functions, will not always produce a smaller value for the function.

For example:

$f(x) = x^2$
$f(2) = 2^2 = 4$
$f(-3) = -3^2 = 9$

Notice that even though -3 is smaller than 2, it produces a bigger result when plugged into f(x). Be sure to test all answer choices when you are dealing with either type of function.

THE GRIP 'N RIP METHOD

Sometimes the easiest thing to do is to just use your calculator! If you can't figure out how to do a problem the right way, but know that you can find it "the long way" by entering a large number of equations into your calculator, go ahead and do it, but only after you have gone through the section and completed every problem that you know how to do quickly.

Among the following arthmetic operations, which could be the symbol ◊ represent given that the equation $(2 \lozenge 1)^4 + (6 \lozenge 3)^2 = 10$ is true?

I. Addition
II. Subtraction
III. Division

A. I only
B. II only
C. III only
D. I and II only
E. I, II, and III

This funky symbols question asks us to choose which operations, when plugged in for the diamond symbol, make the above equation true. We may know how to take a shortcut to arrive at the answer, but if not, we can always plug in each operation on our calculator and check to see if it works:

I. Addition: **Eliminate all answer choices that contain option I.**

II. Subtraction: **Answer II is correct. Notice that we can cross out all answer choices not containing option II, which means that our answer is G. We're done.**

III. Division: **No need to test that. But if we did, we would do it like that.**

Mathematics is the door and key to the sciences

– Roger Bacon

ESTIMATING METHOD

When To Estimate

1. When you don't know how to solve a problem.

Estimating is simply not as surefire a strategy as solving a problem correctly! Don't take shortcuts at the risk of answering questions incorrectly.

2. On geometry questions that include a figure.

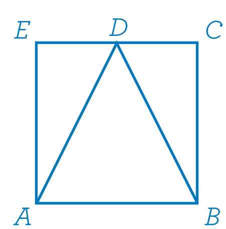
The directions at the beginning of the ACT Math Test state "Illustrative figures are NOT necessarily drawn to scale". This is NOT true! All of the figures on the ACT are drawn close enough to scale that you can estimate. You can also estimate on questions that require you to draw your own figure, but you must draw it as close to scale as possible.

Estimating is not as useful for algebra questions, as we can always use our calculators to find exact answers in a shorter amount of time.

Examples

In square $ABCE$ shown below, D is the midpoint of $\overline{CE}$. Which of the following is the ratio of the area of ΔADE to the area of ΔADB?

A. 1:1
B. 1:2
C. 1:3
D. 1:4
E. 1:8



This geometry question asks us to find the ratio of the areas between two triangles. If you don't know how to solve, eyeball the figure to estimate the relationship between the areas of the two triangles. Triangle *ADB* looks to be approximately twice as big as triangle *ADE,* so we can eliminate answer choices based on that estimation. They do not look like they are the same size, so answer choice F can be eliminated. Triangle *ADB* doesn't look four times or eight times bigger than triangle *ADE,* so eliminate answer choices J and K. That leaves us with two answer choices and a 50% chance of guessing correctly! Here, the correct answer is 1:2, or choice G.

In the figure below, ABCD is a square. Points are chosen on each pair of adjacent sides of ABCD to form 4 congruent right triangles, as shown below. Each of these has one leg that is twice as long as the other leg. What fraction of the area of the square ABCD is shaded?

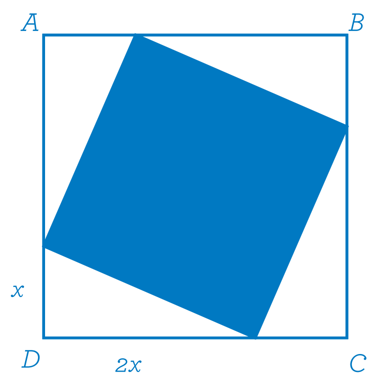
A. $\frac{1}{9}$

B. $\frac{2}{9}$

C. $\frac{4}{9}$

D. $\frac{5}{9}$

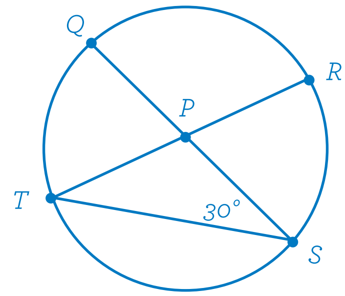
E. $\frac{8}{9}$



This question asks us to determine what fraction of the area of the larger square is made up of the shaded square. If we aren't sure how to find the exact answer, we can try to estimate. It looks like the shaded figure takes up more than half of the larger square, but not much more than that. Answer choice D is the only fraction that fits that description, so choose that answer and move on!

In the circle shown below, chords $\overline{TR}$ and $\overline{QS}$ intersect at P, which is the center of the circle, and the measure of $\angle PST$ is 30°. What is the degree measure of minor arc $\overset{\frown}{RS}$?

A. 30°
B. 45°
C. 60°
D. 90°
E. Cannot be determined from the given information



This question asks us to find the angle of an arc that is created by the intersection of two diameters of the circle. The only angle we are given is , which is equal to 30 degrees. The angle opposite arc RS looks to be bigger than , but smaller than 90 degrees. This only leaves answer choices G and H as potential answer choices, and gives us a 50% chance of guessing correctly! The correct answer here is 60 degrees, or answer choice H.

(Note: Cannot be determined from the given information is almost always a trap! Never guess this answer if you do not know how to solve a problem.)

ACT MATH TEST SECTION OVERVIEW

BASICS:

60 Questions in 60 minutes
Average time per question: 1 minute

Topic	*Approx. # of Questions*
Pre-Algebra	14
Elementary Algebra	10
Intermediate Algebra	9
Plane Geometry	14
Coordinate Geometry	9
Trigonometry	4

KEY RESOURCES:

ACT Math Curriculum
ACT Math Quizzes
ACT 'Need to Know' Formula Sheet

ACT Math Strategies to Use:

Choose Own Values – when there are variables in the question or the answer choices. Use small values!
Plug In Answers (B/D Method) – when you have to find the value of a variable.
Estimating – when you have a geometric figure, or when you have no other options!

GENERAL STRATEGY:

1. Read the question carefully to determine for what it is asking you to find.
2. Write down any information you are given in the question and any formulas that may be helpful.
3. Take a peek at the answer choices for fractions, variables, π or radicals (e.g. √2 or √3).
4. If you know how to solve the question, solve it immediately!
5. If you don't know how to solve a question, try one of ESM's ACT Math Strategies.
6. If you can't use a strategy, circle the question in your booklet, eliminate any answer choices you can, and make your best guess.
7. Once you have gone through every question, go back to the circled questions if you have time.

EXTRA TIPS:

1. Keep It Moving – Staring idly at a question is the biggest detriment to maximizing your score. If you don't know a question, circle it in your booklet and move on! There are easier points ahead.
2. Stay away of "Math Class Mode" – showing every step of a question in school is important to get an A on your math test. On the ACT, where efficiency is key, doing so can be a big inhibitor to your score!
3. Memorize your formulas – knowing all relevant math formulas saves time, turns random guesses into educated guesses, and changes geometry questions into simple algebra question. Spend time reviewing math formulas each week!
4. Be aware of your tendency to solve for x – the ACT has lots of questions that try to prey upon your natural tendencies by asking for a value other than the main variable (such as asking for $2x$ instead of x). The value of the variable itself will always be an answer choice, so be careful!
5. Use decimals – convert fractions and percentages into decimals to use in equations and on your calculator. Memorizing the easy conversions (1/2, 1/4, 1/5, etc.) can help you save time and minimize small mistakes.
6. Find the triangle – triangles are often the key to geometry questions that don't seem to test them. Drawing triangles in these figures can help you easily find angles and the lengths of lines or segments.
7. Learn to love word problems – they make up over 75% of the test, so your ability to work through them quickly and correctly is hugely important. Remember how certain key words translate into mathematical terms.
8. Be careful with your calculator – put all negative numbers in parentheses, and watch your order of operations. The ACT often creates their four incorrect answer choices by making a common arithmetic mistake, so it is hard to catch mistakes after the fact!

Now that you know the basic strategies for the ACT math section, it's time to improve your understanding of the content covered. Check out the Annotated Index or ACT Math quizzes later in the book.

MATH INDEX

ACT

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PRE-ALGEBRA AND ELEMENTARY ALGEBRA QUESTIONS

NUMBERS

Integer:	Any number that is not a decimal or a fraction.
Examples:	-99, -50, 0, 6, 15
Whole number:	Any number that is not negative and not a fraction.
Examples:	0, 2, 37, 455
Odd Integer:	Any integer that cannot be divided by 2 without a remainder.
Examples:	-111, -57, -1, 1, 67
Even Integer:	Any integer that can be divided by 2 without a remainder (including zero!)
Examples:	-34, -2, 0, 4, 10, 12
Consecutive Integers:	Numbers that directly follow each other on a number line.
Examples:	*-4, -3, -2, -1 ... or 3, 4, 5, 6 ...*
Variable Form: ...	*n, n + 1, n + 2, n + 3...*
Consecutive Odd Integers:	Odd numbers that follow each other on a number line.
Examples:	*-5, -3, -1, 1... or 3, 5, 7, 9...*
Variable Form:	*n, n + 2, n + 4, n + 6...*
Consecutive Even Integers:	Even numbers that follow each other on a number line.
Examples:	*-6, -4, -2, 0... or 2, 4, 6, 8...*
Variable Form:	*n, n + 2, n + 4, n + 6...*

Real Number: Any number that can be found on a number line. Excludes infinity and imaginary numbers.

Examples: *All integers, rational numbers, and irrational numbers*

Rational Number: Any number that can be written as a fraction (ratio of integers).

Examples: $\frac{2}{3}$, 4, $\frac{37}{99}$, -5, .20

Irrational Number: Any number that cannot be written as a fraction.

Examples: $\sqrt{3}$, π, *e* (Euler's Number)

Prime Number: A positive number that can only be divided by 1 AND itself.

Examples: *2, 3, 5, 7, 11, 13, 17, 19, 23, 2 9, 31, 37..*

Tips: *is NOT a prime number. is the smallest prime number, and prime numbers cannot be negative or even.*

Remainder: The amount left over when a quantity is divided by another number.

Examples:

$\frac{7}{7}$ = 1, remainder 0

$\frac{8}{7}$ = 1, remainder 1

Factors: The numbers that divide evenly *into* a given number without a remainder.

Examples: *The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.*

Multiples: The numbers that divide evenly by a given number without a remainder.

Examples: *Multiples of 30 are 30, 60, 90, 120...*

Digit Cycles Patters: A fraction that has a repeating set of numbers.

Example:

The fraction 3/7 is equivalent to $0.\overline{428571}$. What is the digit in the 103 rd place of $0.\overline{428571}$? (Note: The digit in the third place is 8.)

A) 1

B) 4

C) 2

D) 5

E) 8

To do this problem you want to note that there are 6 values being repeated, which means that the cycle starts over every 6 numbers. If you can find out how many times 6 goes into 103, you can see which number you will land on.

Example Problems

1. What is the least common multiple of 20, 40, and 60?

A. 20
B. 60
C. 80
D. 120
E. 240

2. What rational number is halfway between $\frac{1}{5}$ and $\frac{1}{3}$?

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{2}{15}$

D. $\frac{4}{15}$

E. $\frac{8}{15}$

3. Which of the following is a rational number?

A. $\sqrt{2}$

B. $\sqrt{\pi}$

C. $\sqrt{7}$

D. $\sqrt{\frac{5}{25}}$

E. $\sqrt{\frac{64}{49}}$

4. How many prime factors does the number 210 have?

A. 2
B. 3
C. 4
D. 8
E. 10

5. Five consecutive integers add up to 115. What is the smallest of these integers?

A. 15
B. 17
C. 19
D. 21
E. 23

6. What is the greatest integer smaller than $\sqrt{120}$?

A. 4
B. 8
C. 10
D. 11
E. 12

7. Which of the following is a rational number?

A. $\sqrt{3}$

B. π

C. $\sqrt{\frac{25}{2}}$

D. $\sqrt{\frac{36}{16}}$

E. $\sqrt{\frac{100}{80}}$

8. Each of the following is a factor of 80 EXCEPT:

A. 8
B. 10
C. 15
D. 20
E. 40

9. If x and y are positive integers such that the greatest common factor between x^3y and x^2y^2 is 63, then which of the following could be the value of x?

A. 9
B. 6
C. 4
D. 3
E. 2

ORDER OF OPERATIONS

Parentheses

Exponents

Multiplication

Division

Addition

Subtraction

Example: Simplify the following expression: $2(x + 3)^2 - (2x + 3^2)$

$2(x^2 + 6x + 9) - (2x + 9)$

$2x^2 + 12x + 18 - 2x - 9$

$2x^2 + 10x + 9$

Example Problems

10. What is the value of the expression when $2z - 3(2z - 1)^2$ when $z = 2$?

- **A.** -117
- **B.** -17
- **C.** -23
- **D.** -5
- **E.** 5

11. If, $2(x - 2)^2 + 4 = 4$, then $x =$?

- **A.** -4
- **B.** -2
- **C.** 2
- **D.** 3
- **E.** 4

PERCENTAGES

Percent: Means "divided by 100"

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}$$

Percent Change:

$$\frac{\text{(new value-origin value)}}{\text{original value}} \times 100$$

Example: If rent went from $1,380 to $1,630, it increased by 18 percent.

$$\frac{1630 - 1380}{1380} = \frac{250}{1380} = .18 \times 100 = 18\%$$

Percent Change Lesson

In everyday talk, we usually refer to the difference between two amounts in terms of addition or subtraction. For instance, if Timmy had 10 apples and Cindy only had 8, we might say that Timmy has 2 more apples than Cindy. Or if Sarah has lived in Scotland for 3 years and Manolo has lived there for 6 years, we might say that Sarah has lived there for 3 years less than Manolo.

However, another completely valid—and often much more informative—way of representing the difference between two amounts is by percent change. For example, instead of saying that Timmy has 2 more apples than Cindy, we might have said that the number of apples Timmy has is 25% more than Cindy. Or instead of saying that Sarah has lived in Scotland for 3 years less than Manolo, we might have said that the number of years Sarah has lived in Scotland is 50% less than the time Manolo has lived there.

This way of speaking might not be too popular, but it does appear in real life a lot. Taxes, sales, interest rates, stocks, and a bunch of other real life things are measured with percentages. So, it's extremely helpful to understand how to differ between amounts by percent change.

Luckily, percent change really isn't that hard to master! The idea is as follows:

To increase by a certain percentage, just multiply the original number by 1 plus the percent (in decimal form).

EX: What is a 20% increase of 140?

First, let's convert 20% to its decimal form: 0.20

Then, the math is easy!

$140 \times (1 + 0.20) = n$
$140 \times 1.2 = n$
$168 = n$

Now you might have your own way of doing this math, but trust me when I tell you that none will be faster or more useful when the numbers get bigger and scarier than this simple one step method.

Decreasing by a certain percentage is just as easy. The only change is instead of adding to 1, we will subtract our percent (in decimal form) from 1.

Ex: What is a 20% decrease from 140?

140 x (1 – 0.20) = n
140 x 0.8 = n
112 = n

It's as easy as that!

Now if someone were to ask you to go in the opposite direction, for instance asking, "140 is a 20% decrease from what number?", we already know how to set up our equation:
n x (1 – 0.20) = 140
n x 0.8 = 140
n = 140 / .08
n = 175

Similarly, for the question, 140 is a 20% increase from what number:
n x (1 + 0.20) = 140
n x 1.2 = 140
n = 140 / 1.2
n = 116.666...

These calculations above represent a single instance of percent change. However, in the real world, we are often interested in a series of percent changes that occur regularly over a period of time: for instance, the percent interest that accrues on a savings account year after year or the percent decrease of the population an endangered species each year.

Solving these problems might seem daunting at first, but it really just breaks down to doing the math we did above multiple times. We take our starting number and multiply by 1 plus/minus our percentage but instead of doing that just once, we do it as many times as the example dictates.

EX: Tom invests $140 with an expectation that his investment will increase by an incredible 20% every year. If his expectation is correct, how much money will he have after 8 years?

(We start the same way as before)

140 x (1 + 0.20) = n
140 x 1.2 = n

BUT HOLD ON!

We don't want to do this just once, we want to do it 8 times, right?

So, we add an exponent!

Exponents represent multiplying by the same number multiple times, and that's exactly what we want to do here. We want to multiply 140 by 1.2 not just once, but 8 times!

Therefore, by adding an exponent of 8 to the 1.2 in our equation, we have a simple calculation to find our answer:

140 x 1.2^8 = n
601.97 = n

So, Tom will have about $600 in his investment after 8 years. The same would hold true for a percent decrease over many years, but I'll leave that one up to you!

Example Problems

12. If 125% of a number is 530, then what is 50% of the number??

A. 662.5
B. 424
C. 331.5
D. 265
E. 212

13. If 60 percent of n is equal to v percent of 30, where v > 0, then what is the value of $\frac{n}{v}$?

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

E. 2

14. 30 percent of one-half of a number is 21. What is the number?

A. 7
B. 42
C. 63
D. 70
E. 140

15. If 22 is added to one-half of a certain number, the result is 64. What is the original number?

A. 42
B. 84
C. 106
D. 128
E. 150

16. The string group of a classical orchestra typically uses 22 violins, 8 violas, 8 violoncellos, and 6 double basses. Approximately what percent of the string group are violas or double basses?

A. 18%
B. 30%
C. 32%
D. 37%
E. 50%

17. The string group of a classical orchestra typically uses 22 violins, 8 violas, 8 violoncellos, and 6 double basses. If a circle graph were made of the data, what would be the central angle of the sector that represents the number of double basses?

A. 14°
B. 18°
C. 49°
D. 60°
E. 98°

18. An amusement park increased its revenue by 12% from 2010 to 2011 and by 18% from 2011 to 2012. By approximately what percent did revenue increase from 2010 to 2012?

A. 28%
B. 30%
C. 32%
D. 34%
E. 36%

SEQUENCES

Arithmetic Sequence: Each term is equal to the previous term plus d

Examples: t_1 , $t_1 + d$, $t_1 + 2d$, $t_1 + 3d$ ... OR t_1 , $t_1 + d$, $t_2 + d$, $t_2 + d$...

If $t_1 = 2$ and $d = 7$, then the sequence would be: 2, 9, 16, 23...

If $t_1 = 5$ and $d = -3$, then the sequence would be: 5, 2, –1, -4...

Geometric Sequence: Each term is equal to the previous term *multiplied* by r

Examples: t_1, $(t_1 * r)$, $(t_1 * r^2)$, $(t_1 * r^3)$ ...

If $t_1 = 3$ and $r = 3$, then the sequence would be: 3, 9, 27, 81...

f $t_1 = 12$ and $r = \frac{1}{4}$, then the sequence would be: 12, 3, $\frac{3}{4}$, $\frac{3}{16}$

If this is your sequence: 3, 6, 12, 24, 48, _____, 192 and the question asks for the sixth number in the sequence, you simply want to:

- look at the multiplier: what do you multiply 3 by to get 6? 2
- multiply the fifth number by 2 to obtain the sixth: 48x2=96
- double check that this is correct by multiplying 96x2=192, which is the seventh number in the sequence, so you know that 96 is the correct answer.

It is important to note that geometric sequences can involve both multiplication of an integer, and multiplication of a fraction, which is the same as dividing. It can also involve multiplying by positive numbers or negative numbers.

Example Problems

19. What is the sum of the first 4 terms of the arithmetic sequence in which the 6th term is 8 and the 10th term is 13?

A. 10.5
B. 14.5
C. 18
D. 21.25
E. 39.5

20. What is the sum of the first 3 terms of the arithmetic sequence in which the 5th term is 8 and the 8th term is 20?

A. –12
B. –8
C. 0
D. 8
E. 28

TRANSLATING WORD PROBLEMS

Of: Multiply (x)

Examples: One half of 50 = $\frac{1}{2}$ * 50, A third of the circle's area = $\frac{1}{3}$ * πr^2

Per: Divide (÷)

Examples: 75 miles per hour = $\frac{\text{75 hours}}{\text{hour}}$, 3 apples per orange = $\frac{\text{3 apples}}{\text{orange}}$

Percent: Divide by 100 or ($\frac{}{100}$)

Examples: 35 percent = $\frac{35}{100}$, 700 percent = $\frac{700}{100}$

Is: Equals (=)

Examples: 20% of x is 4 means $\frac{20}{100}$ x $x = 4$

Diameter is twice the radius means d = 2r

A number: Variable (typically x or y)

Examples: Half of a number is twice another number = $\frac{1}{2}x = 2y$

Example Problems

21. 20 percent of one half of a number is 13 . What is the number?

A. 6.5
B. 13
C. 26
D. 65
E. 130

22. When the sum of a number and 5 is multiplied by 3, the result is the number divided by 2. What is the number?

A. $-\frac{15}{2}$
B. -6
C. -3
D. 3
E. 6

23. The total price for two slices of pizza and half of a salad is $8. The total price for one slice of pizza and one full salad is $5.50. How much does a salad cost?

A. $1.50
B. $2.00
C. $2.50
D. $3.25
E. $3.50

24. Monica wants to drive 300 miles in 6 hours to San Francisco. She is considering towing a trailer, but will be driving an average of 10 miles per hour slower if she does. How many hours longer will it take her to drive 300 miles with the trailer attached to her car?

A. 1
B. 1.5
C. 2
D. 4
E. 7.5

AVERAGES, COUNTING, STATISTICS

Average (Arithmetic Mean): $\frac{\text{sum of terms}}{\text{number of terms}}$

Average Speed: $\frac{\text{total distance}}{\text{total time}}$

Weighted Average:

$w_1a_1 + w_2a_2 + \ldots + w_{n-1}a_{n-1} + w_na_n$

w_1 = weight of object 1(decimal between 0 and 1)

a_1 = average of object 1

Example:

If your class grade was composed of 70 percent classwork and 30 percent final test grade, and you had a 92% going into the final, what do you need on the final for an A?

$.70(.92) + .3(x) = .90$

$x = .853$ or 85.3%

Mode: Value(s) that occurs most frequently

Example: Modes of [2, 5, 5, 6, 32, 32, 37] = 5, 32

Median:

Middle point of an ordered list

For set with odd number of elements: Middle number

For set with even number of elements: Average of two middle numbers

Examples:

median of [2,5,6,32,37] = 6

median of [2,5,6,32] = $\frac{5+6}{2} = 5.5$

Fundamental Counting Principle: If an event can happen *M* ways and another, independent event can happen *N* ways, then both events can happen in *N* * *M* ways.

Example: If you flip a coin and roll a die there are 2×6 = 12 possible results. (If three independent events, N*M*O)

Example Problems

25. In a town of 600 people, 200 females have an average age of 30 and 400 males have an average age of 45. To the nearest year, what is the average age of the town's entire population?

A. 33
B. 35
C. 37
D. 40
E. 42

26. Alisha's math class receives scores of 97 93, 78, 83, 76, 78, 84, 83, and 78 on their most recent test. What is the mode of the test scores for the entire class?

A. 73.3
B. 76
C. 78
D. 83
E. 97

27. Mario is bowling in a tournament and has the highest average after **5** games, with scores of **210, 225, 254, 231,** and **280**. In order to maintain this exact average, what must be Mario's score in his 6th game?

A. 200
B. 210
C. 231
D. 240
E. 245

28. A certain type of pizza costs $15 before sales tax is added. When you buy 5 pizzas you receive 1 additional pizza for free. What is the average cost per pizza for the 6 pizzas before sales tax is added?

A. 12
B. 12.5
C. 13
D. 15
E. 18

29. If Jimmy owns one suit, four shirts, and five ties, and two belts, how many outfits can he put together?

A. 11
B. 18
C. 20
D. 24
E. 40

30. A hiking group will go from a certain village to a certain town by bus on 1 of 4 roads, from town to a mountain by riding on 1 of 2 bicycle paths, and then from the mountain to their campsite by hiking on 1 of 6 trails. How many routes are possible for the hiking group to go from the village to the town to the mountain to their campsite?

A. 6
B. 12
C. 24
D. 48
E. 220

31. Any 2 points determine a line. If there are 7 points in a plane, no 3 of which lie on the same line, how many lines are determined by pairs of these 7 points?

A. 12
B. 14
C. 20
D. 21
E. 28

32. In a small town in Alaska, the low temperatures, in degrees Fahrenheit, for each of 7 recorded days in February were 2°F, 3°F, 1°F, 0°F, 3°F, –4°F, and 2°F. What was the median of these low temperatures?

A. –3.0°F
B. 0.0°F
C. 0.5°F
D. 1.0°F
E. 2.0°F

33. Eleanor and Juanita own a pizza shop: They offer 2 kinds of crust, 6 meat toppings, and 4 kinds of cheese. Each pizza is a combo of exactly: 1 crust, 1 meat, and 1 cheese. How many possible types of pizzas are there?

A. 14
B. 24
C. 30
D. 48
E. 60

34. Stephen has scored an average of 24.4 points per game in his last 10 games. How many points does he have to score in his next game to average 25 points per game overall?

A. 25
B. 29
C. 30
D. 31
E. 35

35. The local deli sells a fixed menu in which customers are allowed to pick one each of 4 sandwiches, 2 soups, 2 salads, and 3 drinks. How many different meal combinations are possible?

A. 2
B. 4
C. 12
D. 48
E. 72

PROBABILITY & RATIOS

Probability: $\frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$

Example:

Likelihood of rolling a six = $\frac{1}{6}$

One of Two Independent Events: Probability of A or B happening: P (A or B) = P(A) + P(B)

Likelihood of rolling a five or six = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Both of Two Independent Events: Probability of A and B both happening: P (A and B) = P(A) x P(B)

Example: Likelihood of rolling two sixes = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Ratios:

Tips: Represented as $\frac{\text{part}}{\text{part}}$ or $\frac{\text{part}}{\text{whole}}$ (part: part and part: whole will also be used)

Example: "part:part" questions are usually meant to confuse you. Turn them into part:whole by adding all parts together to form a denominator.

If a fruit bowl has only apples and oranges and the ratio of apples to oranges is 3: 2, then total number of fruit will be divisible by 5 (3 + 2)

Ratios Lesson:

Ratios are a way of expressing the relative amounts of two or more things. A common use for ratios is to express the relative make up of something that consists of various subgroups. For instance, a ratio could be used to express the relative number of boys and girls in a math class. One might say that the ratio of boys to girls in a class is 6:5, which simply means that for every 6 boys, there are 5 girls.

While it's not too hard to handle an example like the one above in your head, when the numbers are large or the ratio compares multiple subgroups instead of just two, similar problems can become quite tricky.

Luckily, there's an easy 3 step process that will allow you to solve any ratio problem that presents you with the subgroup ratio of a larger whole.

STEP 1: Add up the numbers in the ratio
STEP 2: Divide the total number of the group by this sum
STEP 3: Multiply each number from the ratio by this quotient.

EX 1:

Q: A class of 40 students has a 3:5 ratio of boys to girls. How many girls are in the class? How many boys?

STEP 1: 5 + 3 = 8

STEP 2: 40 students / 8 = 5

STEP 3: 5 x 3 boys = 15 boys

5 x 5 girls = 25 girls

EX 2:

Q: A tapestry is made up of 3 consecutive segments: one of silk, one of madras, and one of cashmere. The ratio of the lengths of these segments is 3:8:5. The total length of the tapestry is 304 inches.

What is the length of the longest segment of the tapestry? Which segment is it?

STEP 1: 3 + 8 + 5 = 16

STEP 2: 304 inches / 16 = 19

STEP 3: 19 x 3 silk = 57 inches of silk

19 x 8 madras = 152 inches of madras

19 x 5 cashmere = 95 inches of cashmere

OTHER RATIOS?

You should realize that ratios do not only compare subgroups within a larger whole. Ratios can also simply express the relative amounts of two independent things. For instance, in a cake there may be 1 candle for every 2 inches of cake. The ratio of candles to inches of cake is 1:2, but these are not subgroups of some larger whole. If you were asked, how many candles are on a 24 inch cake, you would not use the 3 step method above.

Instead, we should set up an equivalence:

Solving for x will give us our answer.

BUT WAIT!

It's important to understand that ratio problems are NOT the same as proportion, probability, or percent problems. These problems (the 3 P's) differ from ratios in that they do not relate the amounts of the individual subgroups that make up a larger whole, but rather relate a single subgroup to the whole.

Let's look at an example.

At a watering hole, there are 40 animals; 15 elk and 25 whitetail deer. If I were to give the ratio of elk to deer, I would simplify 15 elk to 25 deer to give my final ratio of 3:5.

However, one could also describe this group of animals using the 3 P's. The proportion of the animals that are deer is 25 deer / 40 total animals. Simplified, this gives 0.625. Similarly, the percent of the animals that are deer is 25 deer / 40 total animals x 100%. Simplified, this gives 62.5%. Finally, the probability that a randomly selected animal is a deer is also 25 deer / 40 total animals, which simplified gives 5/8.

It's important to understand the difference between ratios of subgroups, ratios of independent groups, and the 3 P's, so be sure to look out for them on the ACT!

Example Problems

36. A bag contains 12 red marbles, 5 yellow marbles, and 15 green marbles. How many additional red marbles must be added so that the probability of randomly drawing a red marble is $\frac{3}{5}$?

A. 13
B. 18
C. 28
D. 32
E. 40

37. An integer from 100 through 999, inclusive, is to be chosen at random. What is the probability that the number chosen will have **0** as at least one digit?

A. $\frac{19}{900}$

B. $\frac{81}{900}$

C. $\frac{1}{10}$

D. $\frac{171}{900}$

E. $\frac{271}{1,000}$

38. A deck of cards contains 13 hearts, 13 spades, 13 clubs, and 13 diamonds. In a certain game, players take turns, each drawing a card a card at random from the deck and putting the card on the table. When it is the 7th player's turn, there are 3 spades, 2 clubs, and 1 diamond on the table. What is the probability that the 7th player will draw a spade?

A. $\frac{5}{26}$

B. $\frac{5}{23}$

C. $\frac{11}{46}$

D. $\frac{13}{46}$

E. $\frac{13}{52}$

39. Janelle cut a board 30 feet long into 2 pieces. The ratio of the lengths of the 2 pieces is 2:3. What is the length, to the nearest foot, of the shorter piece?

A. 5
B. 6
C. 12
D. 15
E. 18

40. On a finch farm, 100 pounds of birdseed are required to feed 60 finches per month. To the nearest pound, how many pounds of birdseed are required to feed 25 finches for a month?

A. 15
B. 35
C. 41
D. 42
E. 45

41. Karen has 6 sweaters, 5 shirts, and 3 pairs of pants. Her pant selection includes one pair each of khakis, jeans, and dress pants. If Karen randomly selects an outfit consisting of 1 sweater, 1 shirt, and 1 pair of pants, what is the probability that the pants are khakis?

A. $\frac{1}{90}$

B. $\frac{1}{30}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

E. $\frac{1}{15}$

42. The ratio of the radii of two circles is 3:8. What is the ratio of their circumferences?

A. 9:64
B. 3:16π
C. 6:16
D. 3:16
E. 1:2π

43. Priya, Sheldon, and Bernadette shared a pizza. Priya ate $\frac{1}{4}$ of the pizza, Sheldon ate $\frac{2}{3}$ of the pizza, and Bernadette ate the rest. What is the ratio of Priya's share to Sheldon's share to Bernadette's share?

- 9:64:1
- 3:16:2
- 6:16:4
- 3:8:1
- 1:2:3

44. Rob and Jay cut a 45 ft. long board into two pieces. The ratio of the long piece to the short piece is 7:2. What is the length of the short piece?

A. 9
B. 10
C. 12
D. 14
E. 15

INTERMEDIATE ALGEBRA AND COORDINATE GEOMETRY QUESTIONS

ABSOLUTE VALUE & INEQUALITIES

Absolute Value: The distance from 0 (always positive)

Example: $|-7| = 7$

Note: *When working with an equation or inequality with an absolute value, remember that all negative values inside of that absolute value will become positive. That said, it's important to consider exactly what is allowed inside of the absolute value bars. Then find those values of x that will produce the values that are allowed.*

Example:

$|3x + 5| \leq 11$

$3x + 5 \leq 11$ and $3x + 5 \geq -11$

$x \leq 2$ and $x \geq \frac{-16}{3}$

Inequalities: The alligator is hungry and always eats the BIGGER value

Example: *1 is less than 2,* $1 < 2$

Tips: *When you divide an inequality by a negative number, reverse the inequality*

Example Problems

45. If $-8 \le x \le 8$, then which of the following must be true?

A. $x^2 \le 8$
B. $8x > 64$
C. $x^3 \ge 8$
D. $|x + 3| \le 11$
E. $x^2 \ge 8$

46. Which of the following is equivalent to the inequality $4x - 8 > 8x + 16$?

A. $x < -6$
B. $x > -6$
C. $x < -2$
D. $x > 2$
E. $x < 6$

47. Which of the following inequalities defines the solution set for the inequality $18 - 3x \le 10$?

A. $x \le \frac{8}{3}$
B. $x \ge -\frac{8}{3}$
C. $x \le -\frac{3}{8}$
D. $x \ge \frac{8}{3}$
E. $x \le -\frac{8}{3}$

48. If $| 3x - 5 | \leq 4$, what is the value of x?

A. $\frac{1}{3} \leq x \leq 3$

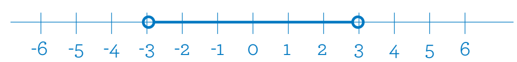
B. $x \leq 3$

C. $x \geq \frac{1}{3}$

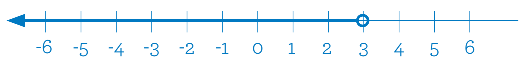
D. $x \leq \frac{1}{3}$

E. $x \geq 3$

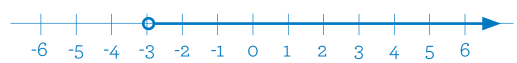
49. Which of the following graphs represents the solution set of the inequality $| x | > 3$ on the real number line?

A. 

B. 

C. 

D. 

E. 

50. Which of the following is an irrational number that is a solution to the equation $| x^2 - 13 | - 3 = 0$?

A. 2
B. $\sqrt{10}$
C. 4
D. $5\sqrt{2}$
E. $2\sqrt{3}$

POWERS, EXPONENTS & ROOTS

Multiplying Exponents: When two powers have the same base, add the exponents

Example:

$\boldsymbol{x}^a \times \boldsymbol{x}^b = \boldsymbol{x}^{a+b}$

$\boldsymbol{x}^5 \times \boldsymbol{x}^3 = \boldsymbol{x}^8$

$(\boldsymbol{x} \times \boldsymbol{x} \times \boldsymbol{x} \times \boldsymbol{x} \times \boldsymbol{x}) \times (\boldsymbol{x} \times \boldsymbol{x} \times \boldsymbol{x})$

$\boldsymbol{x}^{5+3} \times \boldsymbol{x}^8$

Dividing Exponents: When two powers have the same base, subtract the exponents

Example:

$\frac{x^a}{x^b} = x^{a-b}$

$\frac{5^5}{5^2} = 5^3 = 125$

$\frac{5x5x5x5x5}{5x5} = 5^3 = 125$

Negative Exponents: A negative exponent means that that the exponential term is on the wrong side of a fraction. When you see a negative sign in the exponent on the ACT, you will most likely just make the exponent positive and move the entire term into the denominator, leaving 1 in the numerator.

$x^{-b} = \frac{1}{x^b}$

Example:

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Two exponents, one base:

When one exponent is raised to another exponent, multiply the exponents

$(x^a)^b = x^{axb}$

Example:

$(a^3)^5 = a^{15}$

$(a^{-2})^5 = a^{-10} = \frac{1}{a^{10}}$

Distributive Property:

If a base has more than one element, apply the exponent to each element.

$(xy)^a = x^a \times y^a$

Example:

$6^5 = (3x2)^5 = 3^5 \text{ x } 2^5$

Base raised to power of 0:

Any base to 0 power always equals 1

$x^0 = 1$

Example:

$100^0 = 1$

Positives, negatives:

Any negative number to an even power is positive

Any negative number to an odd power is negative

$(-1)^n = 1$, if n is even

$(-1)^n = -1$, if n is odd

Examples:

$(-1)^8 = 1$

$(-1)^{-1} = -1$

Fractional Exponents:

$a^{\frac{1}{n}}$ is the same as taking the nth root of a

Examples:

$\sqrt{49}^{\frac{1}{2}} = \sqrt{49} = 7$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

Example Problems

51. $(3x^3)^3$ is equivalent to:

- **A.** x
- **B.** $9x^6$
- **C.** $9x^9$
- **D.** $27x^6$
- **E.** $27x^9$

52. For all $a > 1$ the expression $\frac{3a^4}{3a^6}$ is equivalent to what?

- **A.** 12
- **B.** $-a^2$
- **C.** a^2
- **D.** $-\frac{1}{a^2}$
- **E.** $\frac{1}{a^2}$

53. In the set of real numbers, what is the solution of the equation $8^{2x+1} = 4^{1-x}$?

- **A.** $-\frac{1}{3}$
- **B.** $-\frac{1}{4}$
- **C.** $-\frac{1}{8}$
- **D.** 0
- **E.** $\frac{1}{7}$

54. Which of the following equations expresses **c** in terms of ***a*** for all real numbers **a**, **b**, and **c** such that $a^4 = b$ and $b^3 = c$?

A. $c = a^6$
B. $c = a^{12}$
C. $c = a^7$
D. $c = \frac{2a}{3}$
E. $c = a$

55. Which of the following expressions is equivalent to $(2x^2)(x^2y^2 + 3xy)$?

A. $2x^3y^2 + 3x^3y$
B. $2x^4y^2 + 3x^3y$
C. $2x^4y + 6x^3y$
D. $2x^4y^2 + 6x^3y$
E. $2x^4y^2 + 2x^2y$

56. Which of the following expressions is equivalent to $(-x^3y^5)^7$?

A. $x^{10}y^{12}$
B. $7x^{21}y^{28}$
C. $-x^{10}y^{12}$
D. $-x^{21}y^{35}$
E. $x^{21}y^{35}$

57. The solution set of $\sqrt{x-3} > 7$ is the set of all real numbers **x** such that:

A. $x > 52$
B. $x > 49$
C. $x < 52$
D. $x > 8$
E. $x > 2$

FACTORING AND SOLVING QUADRATIC FUNCTIONS

"FOIL": $(x + a)(x + b) = x^2 + (b + a)x + ab$

Example: $(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$

Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$

Example: $(a - 3)(a + 3) = a^2 - 9$

Perfect Square Trinomials:

$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$

$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$

Examples:

$(x + 2)^2 = (x + 2)(x + 2) = x^2 + 4x + 4$

$(w - 3)^2 = (w - 3)(w - 3) = w^2 - 6w + 9$

Example Problems

58. When asked how many cats he had, Bashir said, "If you square the number of cats I have and then subtract 21 times the number of cats I have, the result is 46." How many cats does Bashir have?

A. 15
B. 23
C. 24
D. 30
E. 33

59. When $(3x-2)^2$ is written in the form $ax^2 + bx + c$, where **a**, **b**, and **c** are integers, $a + b + c =$?

A. −4
B. 1
C. 7
D. 11
E. 19

60. What is the sum of the 2 solutions of the equation $x^2 + x - 10 = 0$?

A. −8
B. −4
C. −1
D. 5
E. 8

61. What are the values for **x** that satisfy the equation $(2x + a)(x - b) = 0$?

A. $-\frac{a}{2}$ and b
B. $\frac{a}{2}$ and −b
C. −a and b
D. $-\frac{a}{2}$ b
E. a and −b

FUNCTIONS

Functions are operations in which you input one value (usually represented by x) into an equation to get an output (usually represented by f(x) or y) . They can tell you to do anything.

Let's do a couple examples with this function $f(x) = 2x^2 + 3$:

What is the value of f (3)?

Whatever is inside the parentheses always goes in for x

$f(3) = 2(3)^2 + 3$

$f(3) = 21$

In other words, if , then $x = 3$, then $y = 21$.

If $f(p) = 75$, what is the value of p?

This gives us the result (the y-value) and asks us to solve for a variable (the x-value)

$f(p) = 75$

$2p^2 + 3 = 75$

$p = 6$ and -6

In other words, if $y = 75$, then $x = 6$ or $x = -6$

What is the value of $f(x + 3)$?

Again, whatever is inside the parentheses always goes in for x

$f(x + 3) = 2(x + 3)^2 + 3$

$f(x + 3) = 2(x2 + 6x + 9) + 3$

$f(x + 3) = 2x^2 + 12x + 21$

In other words, if $x = x + 3$, then $y = 2x^2 + 12x + 21$.

What is the value of f (f (1))?

Now we have two functions to solve but the same rules apply. Start with the inside parentheses and we are going to have to solve it twice.

$f(1) = 2(1)^2 + 3 = 5$

Then $f(f(1)) = f(5)$

$f(5) = 2(5)^2 + 3 = 53$

In other words, if $x = 1$, then $y = 5$ and then if $x = 5$, then $y = 53$

Example Problems

62. A function $f(x)$ is defined as $f(x) = -8x^2$. What is the value of $f(-3)$?

A. −576
B. −72
C. 57
D. 72
E. 192

63. If $f(x) = x^2 - 2x + 1$ and $g(x) = 2\sqrt{x}$, then what is the value of $g(9) \bullet f(2)$?

A. 0
B. 1
C. 3
D. 6
E. 12

64. If $f(x) = x^2 + 5x - 6$, what is the value of $f(x + 2)$?

A. $7x + 8$
B. $x^2 + 9x + 8$
C. $x^2 + 9x + 4$
D. $x^2 + 5x - 8$
E. $x^2 + 4x - 6$

65. If $f(x) = 2x^2 + 3x - 5$, then $f(-3) = ?$

A. −32
B. −5
C. 0
D. 4
E. 22

"FUNKY" FUNCTIONS

Example: *If you are given the equation $x\ y = 2x - y$ and are asked to solve $3\ 2$, you would just substitute based on the symbols and position of the numbers.*

$3\ \ 2 = 2(3) - (2)$

$3\ \ 2 = 4$

Example Problems

66. Let the function **f (a,b)** be defined as **$f(a,b) = a^2 - b^2$**
For all x and y, **$f((x + y),(x - y))$** = ?

A. $4xy$
B. $2x^2 + 2y^2$
C. $2x^2 + 4xy + 2y^2$
D. $x^2 + 2x + y^2$
E. $2x^2 - 2y^2$

67. Let the operation a•b be defined as $a^2 + 2b + 3$.
What is the value of 2•4?

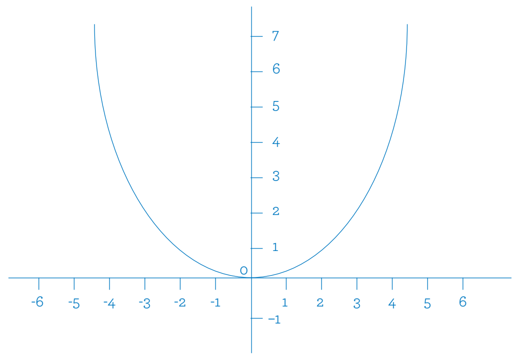
A. 3
B. 9
C. 11
D. 15
E. 23

GRAPHICAL TRANSFORMATIONS OF FUNCTIONS

Base Function:	y = f(x)
Amplitude Increase:	y = 3f(x)
(Vertical Stretch)	All y values multiplied by 3
Amplitude Decrease:	y = 1/2f(x)
(Horizontal Stretch)	All y values multiplied by ½
Horizontal Shift Right:	y = f(x – 2)
	All points shifted two to the right
Horizontal Shift Left:	y = f(x + 1)
	All points shifted one to the left
Vertical Shift Up:	y = f(x) + 2
	All points shift up two
Vertical Shift Down:	y = f(x) – 4
	All points shifted down four

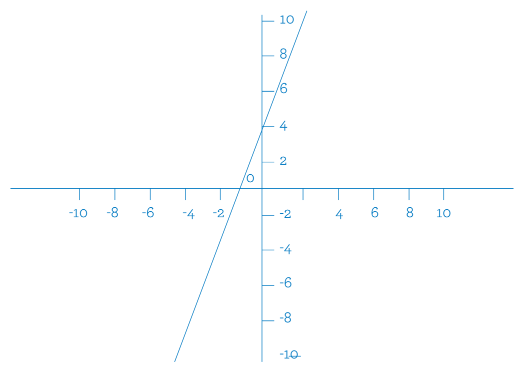
Example Problems

68. The graph of $\mathbf{y = x^2}$ is shown in the standard **(x,y)** coordinate plane below. For which of the following equations is the graph of the parabola shifted 2 units to the left and 5 units up?



A. $y = (x - 2)^2 + 5$
B. $y = (x + 2)^2 - 5$
C. $y = (x + 2)^2 + 5$
D. $y = (x + 5)^2 - 2$
E. $y = (x - 2)^2 - 5$

69. One of the following is an equation of the linear relation shown in the standard **(x,y)** coordinate plane below. Which equation is it?



A. $y = 4x + 3$
B. $y = -4x$
C. $y = 3x - 2$
D. $y = 3x + 2$
E. $y = 3x + 4$

MATRICES

Matrix Addition:

Matrices of the same dimensions can be added by adding the numbers that have the same position in the matrices.

Equation form:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$$

Matrix Multiplication:

In order to multiply two matrices, the number of rows of the left matrix must equal the number of columns in the matrix to the right.

For example, a 3x2 matrix can multiply a 2x3 matrix, but not a 3x2 matrix since there would be 3 rows in the left matrix and 2 columns in the right matrix.

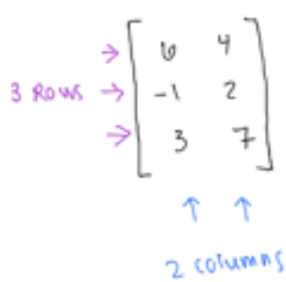
Matrix Multiplication is an operation completed as follows:

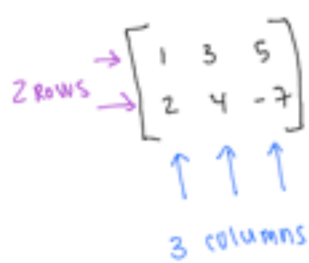
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} b_{11} + a_{12} b_{21} & a_{11} b_{12} + a_{12} b_{22} \\ a_{21} b_{11} + a_{22} b_{21} & a_{21} b_{12} + a_{22} b_{22} \end{pmatrix}$$

All other matrix operations will be defined on the test.

Matrix Multiplication Lesson:
The most important thing to remember about multiplying matrices is that humans made this up. It is simply a way of combining and organizing data, with a simple set of rules for construction.

A matrix is constructed in a row by column fashion. To name a matrix, you state: number or rows, by number of columns. Examples: 3 rows x 2 columns= 3x2 matrix 2 rows x 3 columns=2x3 matrix

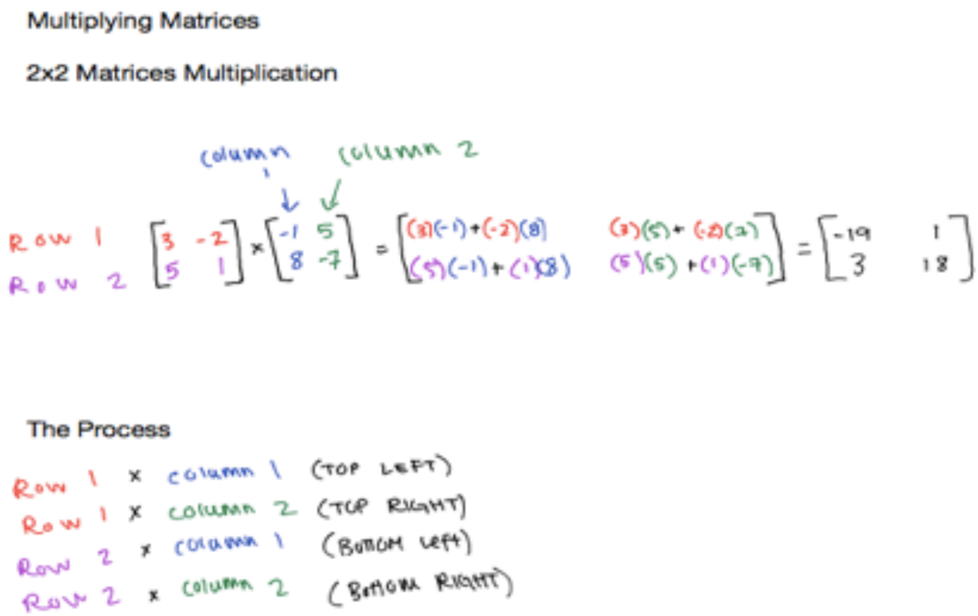




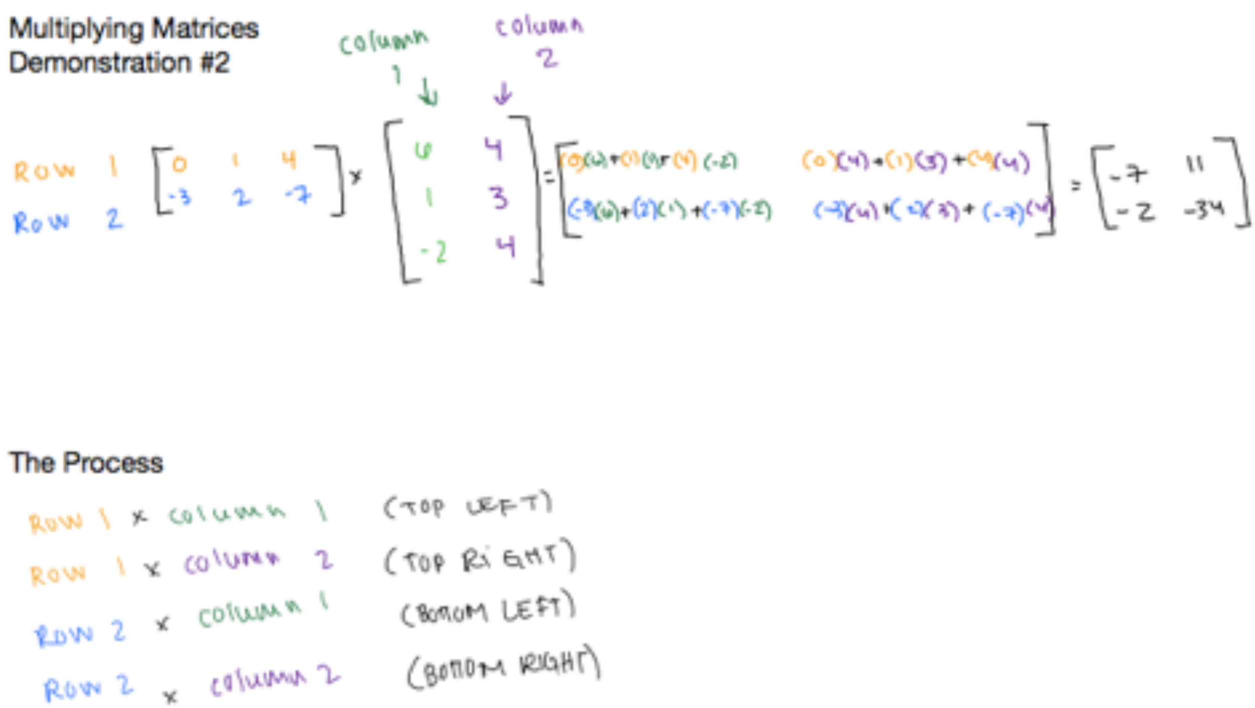
One can multiply two matrices if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.

As you can see by the highlighted "3s" above, you can multiply these two matrices.

If the matrices were 2x2 and 3x2, you could not multiply them. Example 1:



Example 2:



If using colors helps you master this process, practice a few of the examples with colored pencils or pens. Once you get the hang of it, practice without colors, since you won't be using color differentiation on the official exam.

Here's a good method to organize the process:

Draw circles around each row, and draw boxes around each column. As you go through the process, cover the row and column that you are not using, to focus on the row and column that you are using. When you are entirely done using row 1, cross it off and move on to row 2.

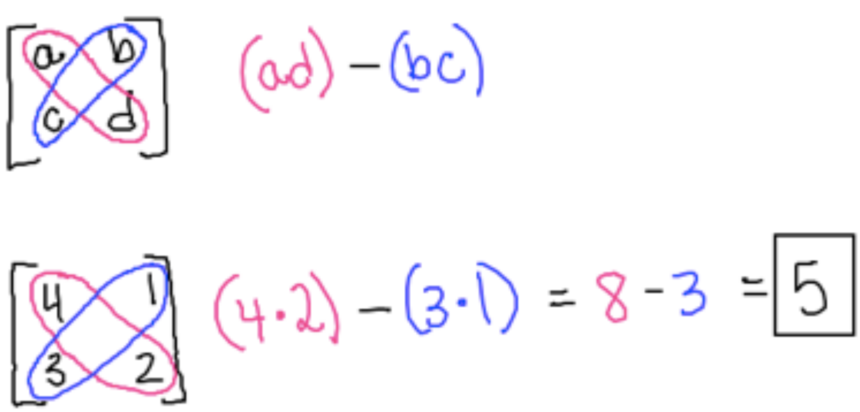
On the ACT, you may see a question that presents a matrix and asks you to calculate the determinant. It may also give you a matrix with a missing variable and the matrix's determinant, asking you to solve for the variable.

First of all: What is the determinant?

The determinant is a numerical value that proves useful in any field involving multiple interacting variables. People who work in data analytics, statistics, engineering, and other math-based professions, use determinants to help manage large sets of interacting data.

In our case, the determinant can be useful in either finding missing values in a matrix or simply answering an ACT question correctly!

To calculate the determinant of a 2x2 matrix, you cross multiply the top left term by the bottom right term and subtract the product of the top right term and the bottom left term. These variables are labeled a, b, c, and d in the example below. The first example uses variables, and the second example demonstrates the process with numbers.



In the case of the second example, you can conclude that the determinant of this 2x2 matrix is 5.

Example Problems

70. By definition, the determinant $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ equals **ad – bc**. What is the value of $\begin{pmatrix} 3x & 4y \\ 5x & 3y \end{pmatrix}$ when $\mathbf{x = -2}$ and $\mathbf{y = 1}$?

A. –58
B. 0
C. 6
D. 22
E. 32

71. What is the matrix product $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \end{pmatrix}$

A. $\begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 8 & 0 & -4 \end{pmatrix}$

B. $\begin{pmatrix} 2 & 0 & -4 \end{pmatrix}$

C. $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

D. $\begin{pmatrix} 0 \end{pmatrix}$

E. $\begin{pmatrix} 2 & 4 & 8 \\ 0 & 0 & 0 \\ -1 & -2 & -4 \end{pmatrix}$

LOGARITHMS

Logarithms are the inverses of exponential functions.
Put more simply, $\log_b a = c$ asks: b to what power c equals a?

So $\mathbf{\log_b a = c}$ can be rewritten as $\mathbf{b^c = a}$

Rules of Logarithms

Power Rule $\log_b a^c = c \cdot \log_b a$

Product Property $\log_b(ac) = \log_b a + \log_b c$

Quotient Property $\log_b \frac{a}{c} = \log_b a - \log_b c$

Logarithms Lesson

Having nothing to do with either logs or rhythm, logarithms can be a source of confusion for many students on the ACT. While you've likely seen them before, chances are you might not remember exactly how they work. Luckily, the log problems on the ACT are all relatively simple, and they only test a basic understanding of the log concept. So, while you might be initially put off by a problem containing the not-so-familiar "log" symbol, these questions are often some of the easiest on the test and should be considered easy improvements you can make to your score. By understanding a few simple facts about logs, you'll be able to get these questions right 100% of the time and help boost your math section score!

WOOHOO!
The most essential thing to remember about logarithms is the following idea:

$\log_3 9 = 2$

$3^2=9$

The two equations above represent the same idea, but one is in log form while the other is in exponent form. You can always switch between log and exponent form in the way shown above, and that's more or less all you need to know for the ACT.
For instance:

$\log_2 8 = 3$

$2^3=8$

$\log_4 16 = 2$

$4^2=16$

And so on and so forth. Note that you need not understand exactly what the log function does; just think of it as another way of expressing an exponent. On the test, you should almost always convert any logs you see into exponent form, as these should be more familiar to you and will make the problems easier.

Now let's see an example with a variable in it!

If $\log_x 81 = 2$, what is the value of x?

In log form this might look intimidating, so let's convert it to the more familiar exponent form:

$\log_x 81 = 2$

$x^2=81$

Now using some simple algebra, you can solve for x to find $x = \pm 9$.

NOTE: Log bases (the little subscript number next to the "log") can never be negative numbers. For this reason, the solution $x = -9$ for the above equation is not valid. Thus, the answer is only $x = 9$.

Sometimes the variable might have a coefficient or be part of a parenthetical term. Let's see examples.

$\log_{2x} 64 = 3$

$(2x)^3=64$

$8x^3=64$

$x^3=8$

$x=2$

OR

$\log_{x-4} 9=2$

$(x-4)^2=9$

$(x-4)(x-4)=9$

$x^2-8x+16=9$

$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x-7=0$ and $x-1=0$

$x=7$ and $x=1$

NOTE: Plugging in the solution $x = 1$ gives us a negative log base, which (as we discussed above) is a big no-no. Therefore, the only valid solution is $x = 7$. And that's the gist of it! That's how you solve log problems on the ACT!

Before you're off solving log questions left and right, there are a few other important things for you to know:

You might run into a log that doesn't have a base. It'll look like this:

$\log 100=2$

When the log has no base number, we assume it to be base "10". So the equation above actually says $\log_{10} 100=2$

Also, you can evaluate logs using your calculator. For instance, if a question asked you the following:

If $\log 27=x$, what is the value of x?

Use your graphing calculator: On a TI-84 plus, click the MATH button, scroll down to the function "logBASE" and input the number to easily solve the problem. Try it out on your calculator!

Example Problems

72. Which of the following is a value of b that satisfies **$\log_b 27 = 3$?**

A. 3
B. 9
C. 10
D. 27
E. 81

73. If **$x = 3$** and **$y = 9$**, what is the value of **$\log_3 \left(\frac{x}{y}\right)$**?

A. −1

B. $\frac{1}{2}$

C. 1

D. 2

E. 3

74. If **$\log_b x = z$** and **$\log_b y = w$**, then **$\log_b (xy)^2$**?

A. $z + w$
B. z^2w^2
C. $2(z + w)$
D. $2zw$
E. zw

COMPLEX NUMBERS

Complex numbers are expressions that involve the imaginary number i. They are usually expressed in the general form $a + bi$, where $i = \sqrt{(-1)}$. Imaginary numbers are essentially used most frequently to rewrite a radical when there is a negative number inside.

For example, $\sqrt{(-4)} = \sqrt{4} \cdot \sqrt{(-1)} = 2i$.

On the ACT, it's important to remember that $i^2 = -1$.
Powers of i repeat in groups of 4.

That is:

$i = \sqrt{(-1)}$ $\quad i^2 = -1$ $\quad i^3 = -\sqrt{(-1)}$ $\quad i^4 = 1$

$i^5 = \sqrt{(-1)}$ $\quad i^6 = -1$ $\quad i^7 = -\sqrt{(-1)}$ $\quad i^8 = 1$

The ACT will sometimes ask you to find large powers of i, for example i^{23}. We use the above pattern and recognize that 4 goes into 23 five times with a remainder of 3. There are then 3 more i terms that must be multiplied in to reach 23. We then know that the 23rd i is the same as the 3rd i in the repeating pattern.
Thus, $i^{23} = -\sqrt{(-1)}$.

Example Problems

75. Which expression is equivalent to $(3+2i)(4-3i)$?

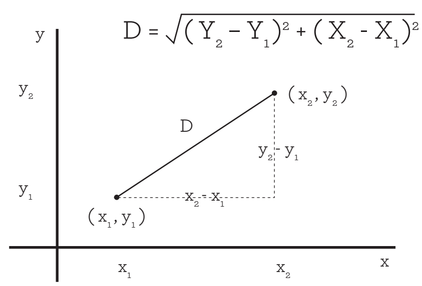
A. $18 - i$
B. $6 - i$
C. $6 + i$
D. $18 + i$
E. $12 - 1$

76. What is the value of $3i^{63}$?

A. $-3i$
B. $6 - i$
C. $6 + i$
D. $18 + i$
E. $12 - i$

COORDINATE GEOMETRY

Distance Formula: How to find the distance between two points (think of making a right triangle out of the two points and using the Pythagorean Theorem).

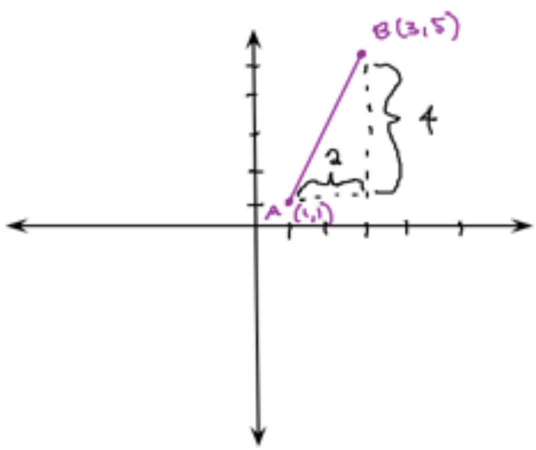


Common mistakes include:

1) Mixing up y1 and y2 or x1 and x2, which is why it is important to establish your coordinate points, as shown in the example above.

2) Making sign errors, so be sure to keep track of your positive and negative signs.

There is another easy way to look at these problems. Notice that the distance we are looking for makes the hypotenuse of a right triangle if you draw in the other lines. We can easily measure the change in the x direction and y direction, and then use the Pythagorean theorem to calculate our distance "c".



Pythagorean Theorem:

$a^2 + b^2 = c^2$

we are looking for the hypotenuse "C"

So...

$2^2 + 4^2 = c^2$

$4 + 16 = c^2$

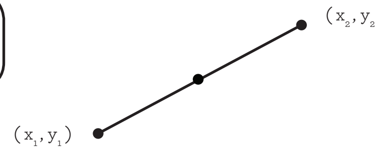
$20 = c^2$

$\sqrt{20} = c$

$2\sqrt{5} = c$

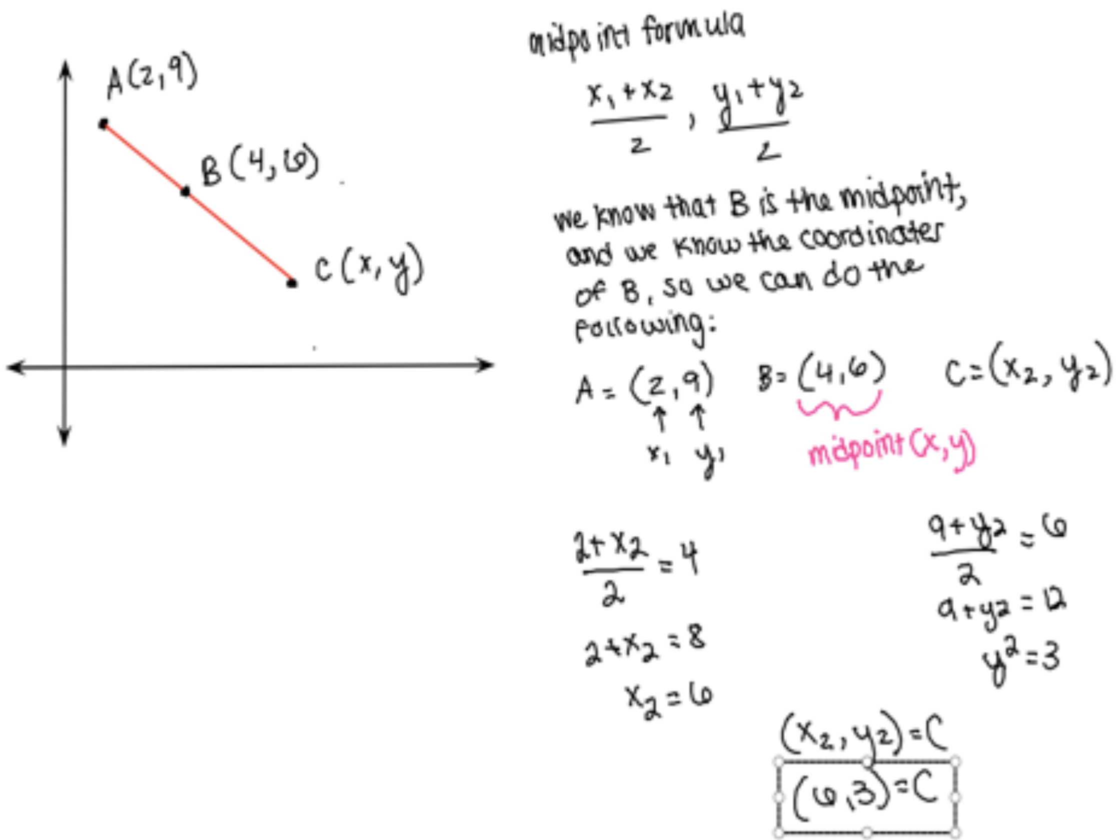
Midpoint Formula: How to find the midpoint of a line segment

$$\left(\frac{x_1 + x_2}{2} \quad \frac{y_1 + y_2}{2}\right)$$



Sometimes the ACT will give you an initial point and a midpoint, and ask you to find the end point. Your question could look like this:

Find the endpoint of line segment AC, given point A (2,9) and midpoint B (4,6)



Lines: Have a consistent slope - so the "rate of change" is constant. Therefore, a linear function will have the same slope between any two points.

Parallel lines have the same slope.

Perpendicular lines have opposite reciprocal slopes.

Horizontal lines have a slope of zero and are expressed as $y = a$, where a is a constant.

Vertical lines have an undefined slope and are expressed as $x = a$, where a is a constant.

Slope-Intercept Form: $y = mx +$, where m = slope and b = y-intercept.

Point-Slope Form: $y - y_1 = m(x - x_1)$, where m = slope

Standard Form of a Line: $Ax + By$, where A is a positive integer and B and C are integers.

Collinear Points: Points that lie on the same line.

Example Problems

77. In the standard (x,y) coordinate plane, point M with coordinates (5,4) is the midpoint of $\overline{AB}$, and B has coordinates (7,3) . What are the coordinates of A?

A. (17,11)
B. (9,2)
C. (6,3.5)
D. (3, 5)
E. (−3, −5)

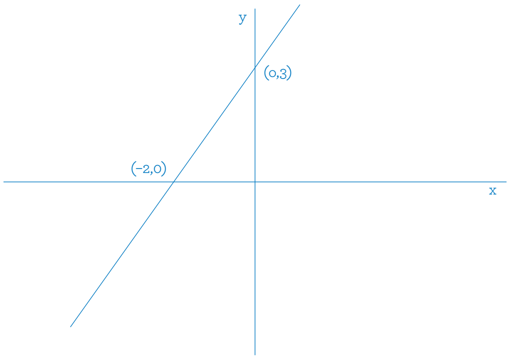
78. In the standard (x,y) coordinate plane, point A has coordinates (2,3) and point B has coordinates (5,7). What are the coordinates of the midpoint of AB?

A. (2, 5)
B. (3.5, 5)
C. (3.5, 7)
D. (5,7)
E. (7, 10)

79. In the xy-coordinate plane, lines l and q are perpendicular. If line l contains the points (0,0) and (1,2), and line q contains the points (1,2) and (0,t), what is the value of t?

A. −2
B. 0
C. 2.5
D. 3
E. 4

80. Line n (not shown) is perpendicular to line m and shares a y-intercept with line m. Which of the following points is on line n?



A. (-6, 8)
B. (-3, 5)
C. (2, 6)
D. (3, 0)
E. (6, -2)

81. The points (1,3) and (7,11) are endpoints of a diameter of circle O in the standard (x,y) coordinate plane. What is the length of the radius of circle O?

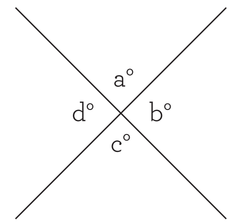
A. 3
B. 4
C. 5
D. 10
E. 14

PLANE GEOMETRY AND TRIGONOMETRY QUESTIONS

ANGLES

Vertical Angles:

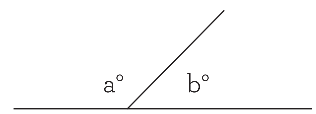
Vertical angles are formed by two intersecting lines or segments and are always congruent.



a = c and b = d

Linear Pair:
(Supplementary Angles)

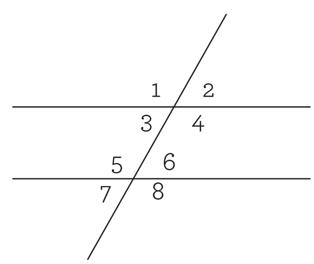
Two angles that form a line are equal to 180 degrees.



a + b = 180°

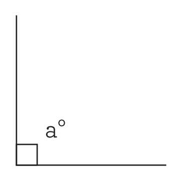
Angles formed by Parallel Lines:

Angles that are congruent:
Alternate Interior Angles (Ex. 3 & 6)
Alternate Exterior Angles (Ex.1 & 8)
Corresponding Angles (Ex. 1 & 5)
Angles that add up to 180 degrees:
Same Side Interior Angles (Ex. 3 & 5)



Right Angle:

A right angle measures 90 degrees.



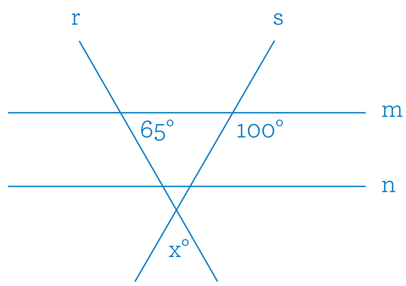
a = 90°

Complimentary Angles:

Two angles that add up to 90 degrees.

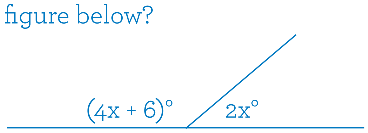
Example Problems

82. In the figure below, lines m and n are parallel, transversals r and s intersect to form an angle of measure x°, and 2 other angle measures are as marked. What is the value of x?



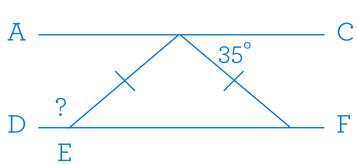
A. 15
B. 25
C. 35
D. 65
E. 80

83. What is the degree measure of the smaller of the 2 angles formed by the line and the ray shown in the figure below?



A. 14°
B. 28°
C. 29°
D. 58°
E. Cannot be determined from the given information

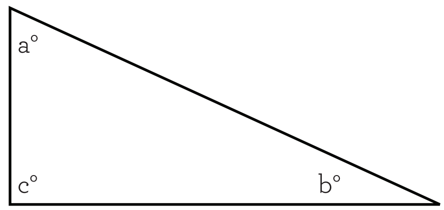
84. In the figure below, B is on AC, E is on DF, AC is parallel to DF, and BE is congruent to BF. What is the measure of ∠DEB?



A. 35°
B. 135°
C. 145°
D. 155°
E. 215°

TRIANGLES

Angles: The inside angles of a triangle always add up to 180 degrees.



$a + b + c = 180°$

Area: $A = \frac{1}{2}$ (base * height), where is the length of a side.

Area of Equilateral Triangle: $A = \frac{\sqrt{3}}{4} s^2$, where is the length of a side.

Triangle Inequality Theorem: The sum of the two shortest sides of a triangle is always greater than the length of the third side.

Equilateral Triangle: All three sides are equal and all three interior angles are 60 degrees.

Isosceles Triangle: Two equal sides. Base angles (angles across from the congruent sides) are equal.

Proportionality in Triangles: In every triangle, the longest side is opposite the largest angle and the shortest side is opposite the smallest angle.

Pythagorean Theorem: $a^2 + b^2 = c^2$, where a and b are legs of a right triangle, and is the hypotenuse.

Pythagorean Triples: Three integers that, as side lengths of a triangle, form a right triangle.

Examples: *3/4/5 or any multiple (6/8/10, 15/20/25, 30/40/50)*

5/12/13 or any multiple

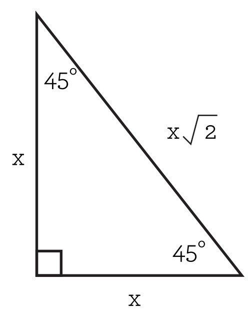
8/15/17or any multiple

7/24/25 or any multiple

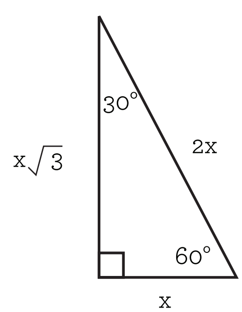
Similar Triangles: Triangles that have the same angle measures but different side lengths. If two corresponding angles are equal, the corresponding sides are proportional to each other.

Example: *A 3/4/5 and a 6/8/10 triangle must be similar because have the same ratios*

45-45-90 Triangles: (Isosceles Right Triangles) Have the following ratio of side lengths:



30-60-90 Triangles: Have the following ratio of side lengths:



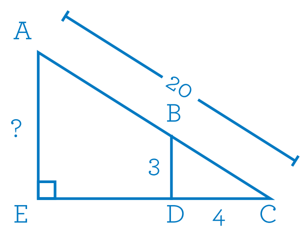
Example Problems

85. In ΔABC, the sum of the measures of $\angle A$ and $\angle B$ is 47 degrees. What is the measure of $\angle C$?

A. 43
B. 86
C. 94
D. 133
E. 223

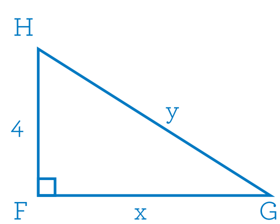
86. In the right triangle ΔACE below $\overline{BD}$, is parallel to $\overline{AE}$ and is perpendicular to $\overline{EC}$. What is the length, in feet, of $\overline{AE}$?

A. 10
B. 12
C. 15
D. 16
E. 18



87. For ΔFGH, shown below, which of the following is an expression for y in terms of x?

A. $x + 4$
B. $\sqrt{x^2 + 4}$
C. $\sqrt{x^2 + 8}$
D. $\sqrt{x^2 - 16}$
E. $\sqrt{x^2 + 16}$

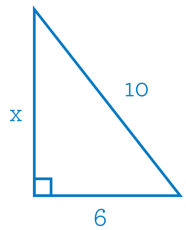


88. A triangle with a perimeter of 95 inches has one side that is 20 inches long. The lengths of the other two sides have a ratio of 2:3. What is the length, in inches, of the longest side of the triangle?

A. 20
B. 30
C. 40
D. 45
E. 50

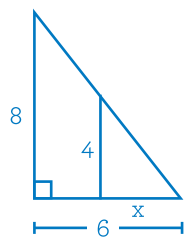
89. In the right triangle shown below, what is the value of x?

A. 4
B. 5
C. 7
D. 8
E. 9



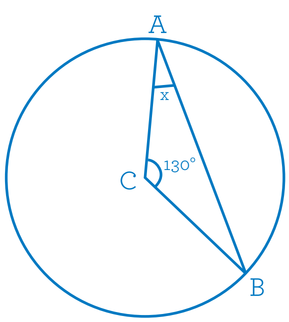
90. Two trees are planted next to one another on level ground such that at a certain time of the day, their shadows meet at the same place. One tree is 8 feet tall, the other is 4 feet tall. The length of the shadow of the taller tree is 6 feet, as depicted in the figure below. What is the length of the shadow of the shorter tree?

A. 1
B. 2
C. 3
D. 4
E. 5



91. Points A and B lie on the circle below, where central angle $\angle ACB$ measures 130°. What is the measure of $\angle BAC$?

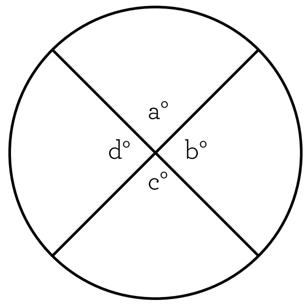
A. 15°
B. 25°
C. 45°
D. 50°
E. 60°



CIRCLES

Area of a Circle: $A = \pi r^2$

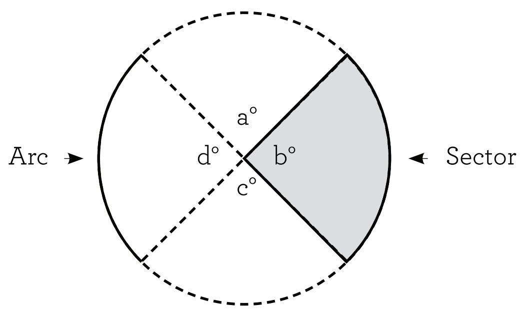
Central Angles: The central angles of a circle add up to 360 degrees.



Circumference of a Circle: $c = 2\pi r$ or $c = \pi d$

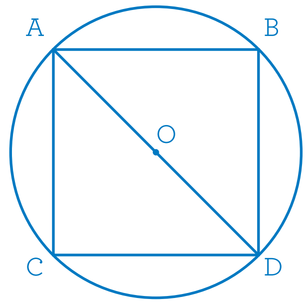
Area of a Sector: $\frac{n}{360} \times \pi r^2$, where n is the central angle

Arc Length: $\frac{n}{360} \times 2\pi r$ or $\frac{n}{360} \times \pi d$, where n is the central angle



Example Problems

92. Square ABCD has a perimeter of 8. What is the circumference of circle o?

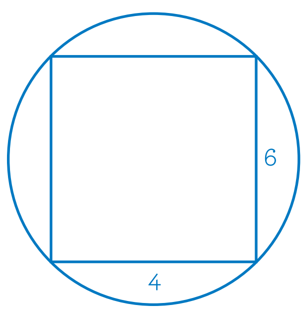


A. π
B. 2π
C. $2\pi\sqrt{2}$
D. 4π
E. Cannot be determined from the information given.

93. What is the arc length of a sector of a circle with a central angle of 45° and radius of 8?

A. π
B. 2π
C. 4π
D. 3π
E. 8π

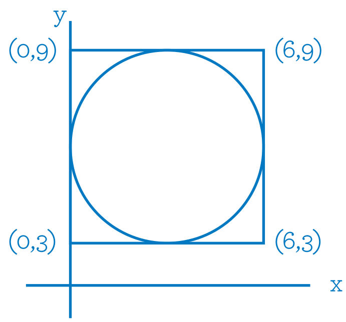
94. A 4-inch by 6-inch rectangle is inscribed in a circle as shown below. What is the circumference of the circle, in square inches?



A. $\pi\sqrt{10}$
B. $2\pi\sqrt{10}$
C. $2\pi\sqrt{13}$
D. 10π
E. 13π

95. In the standard (x,y) coordinate plane below, the vertices of the square have coordinates (0,3), (6,3) (6,9), and (0, 9). Which of the following is the area of the circle that is inscribed in the square?

A. 3π
B. 6π
C. 9π
D. 12π
E. 36π



96. Points A and B are the endpoints of the diameter of a circle with center C, as shown below. Point D is on the circle, and measures 30˚. The shortest distance along the circle from A to D is what fraction of the distance along the circle from A to B?

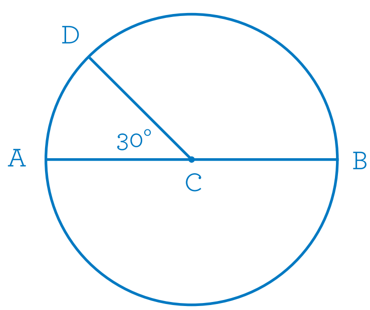
A. $\frac{1}{12}$

B. $\frac{1}{9}$

C. $\frac{1}{6}$

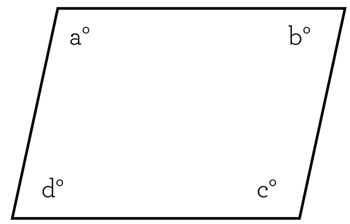
D. $\frac{5}{6}$

E. $\frac{11}{12}$



QUADRILATERALS

Interior Angles: Always add up to degrees



$a + b + c + d = 360°$

Area of Square: A = side x side, where is the length and is the width

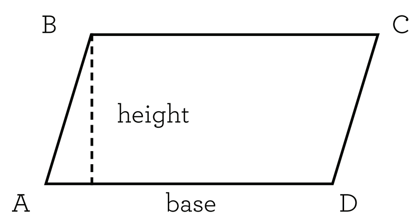
Diagonal of Square: side x $\sqrt{2}$

Area of Rectangle: A = length x width

Perimeter of a Rectangle: P = 2l + 2w, where l is the length and w is the width

Area of Parallelogram: A = base x height

(note: height is NOT equal to width, unless it is a rectangle, and the height must be perpendicular to the base)



Angles in Parallelogram: Opposite angles are equal

Area of a Trapezoid: Median x Height or $\frac{b_1 + b_2}{2}$ x h

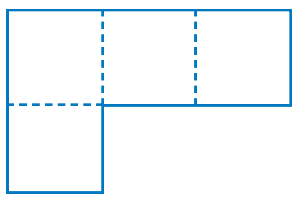
Example Problems

97. A rectangle has an area of 32 square feet and a perimeter of 24 feet. What is the shortest of the side lengths, in feet, of the rectangle?

A. 1
B. 2
C. 3
D. 4
E. 8

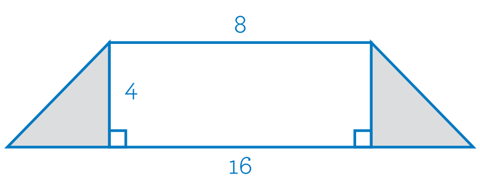
98. The 6-sided figure below is divided into 4 congruent squares. The total area of the 4 squares is 64 square inches. What is the perimeter, in inches, of the figure?

A. 20
B. 24
C. 36
D. 40
E. 64



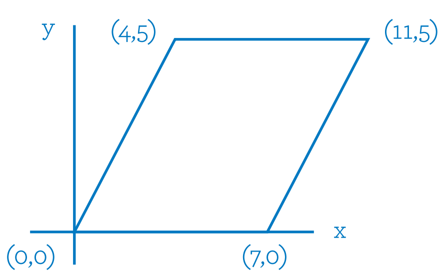
99. The trapezoid below is divided into 2 triangles and 1 rectangle. Lengths are given in inches. What is the combined area in square inches, of the 2 shaded triangles?

A. 8
B. 16
C. 24
D. 32
E. 64



100. In the standard (x,y) coordinate plane below, the points (0,0), (7,0), (4,5), and (11,5) are the vertices of a parallelogram. What is the area, in square coordinate units, of the parallelogram?

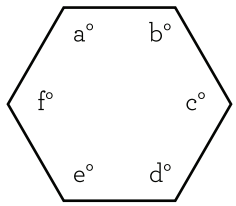
A. 15
B. 25
C. 30
D. 35
E. 55



POLYGONS

Area of a Polygon: $A = \frac{1}{2}aP$, where a is the apothem and P is the Perimeter.

Sum of Interior Angles of a Polygon: Sum = $(n - 2)180$, where n is the number of sides.



$a + b + c + d + e + f = (n - 2)180$

Interior Angle of a Polygon: Interior Angle = $\frac{(n-2)180}{n}$, where n is the number of sides

Example Problems

101. The measure of each interior angle of a regular polygon with n sides is $\frac{(n-2)180}{n}$ degrees. What is the measure of each interior angle of a regular polygon with n sides, in radians?

A. $\frac{(n-2)\pi}{4n}$

B. $\frac{(n-2)\pi}{2n}$

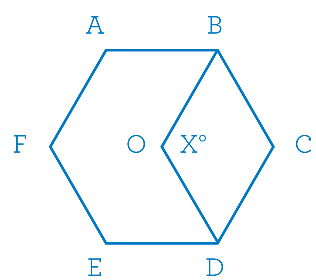
C. $\frac{(n-2)\pi}{n}$

D. $\frac{(n-2)2\pi}{n}$

E. $\frac{(n-2)4\pi}{n}$

102. In the figure below, ABCDEF is a regular hexagon. What is the value of x?

A. 60
B. 90
C. 100
D. 120
E. 160



103. The average of the measures of the interior angles of a polygon is 135°. Which of the following polygons has this property?

A. Triangle
B. Rectangle
C. Pentagon
D. Hexagon
E. Octagon

THREE-DIMENSIONAL SHAPES

Surface Area of a Sphere: SA = 4πr2

Surface Area of a Cylinder: SA = 2πr2 + 2πrh

Surface Area of a Prism: Sum of the areas of all sides

SA = 2(lw + hl + hw) or:

SA = 2B + Ph, where B is the area of the base, P is the perimeter of the base, and h is the height of the prism.

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Volume of a Cylinder: $V = \pi r^2 h$

Volume of a Prism: V = l x w x h

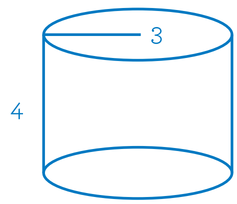
Volume of a Cube: $V = \text{side}^3$

Diagonal of a Cube: $\text{side}\sqrt{3}$

Example Problems

104. The height and radius of the right circular cylinder below are given in inches. What is the volume, in cubic inches, of the cylinder?

A. 9π
B. 12π
C. 18π
D. 36π
E. 72π



105. If the volume of a given sphere is equal to its surface area, what is the radius of the sphere?

A. 1
B. 3
C. 4
D. 6
E. 9

106. What is the radius of a sphere circumscribed around a cube, if the volume of the cube is 27?

A. $2\sqrt{2}$

B. $2\sqrt{3}$

C. $\frac{3\sqrt{3}}{2}$

D. $3\sqrt{2}$

E. $4\sqrt{3}$

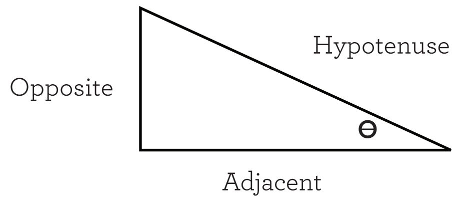
TRIGONOMETRY

Right Triangle Trigonometry (SOH CAH TOA)

SOH = sin Θ = $\frac{\text{opposite}}{\text{hypotenus}}$

CAH = cos Θ = $\frac{\text{adjacent}}{\text{hypotenus}}$

TOA = tan Θ = $\frac{\text{opposite}}{\text{adjacent}}$



Basic Trigonometric Identities

tan (Θ) = $\frac{\sin(\theta)}{\cos(\theta)}$

sec (Θ) = $\frac{1}{\cos(\theta)}$

csc (Θ) = $\frac{1}{\sin(\theta)}$

cot (Θ) = $\frac{1}{\tan(\theta)}$ or $\frac{\cos(\theta)}{\sin(\theta)}$

Pythagorean Identities

$\sin^2(\Theta) + \cos^2(\Theta) = 1$

$1 + \tan^2\Theta = \sec^2\Theta$

$1 + \cot^2\Theta = \csc^2\Theta$

Graphing Trigonometric Functions

$f(x) = A\sin(Bt - C) + D$

Amplitude = A

Period = $\frac{2\pi}{B}$ ($\frac{\pi}{B}$ for tangent)

Normal Period of Sine & Cosine: 2π

Normal Period of Tangent: π

Phase Shift = $\frac{C}{B}$

Vertical Shift = D

For our purposes sinusoidal functions are those that have the general form:

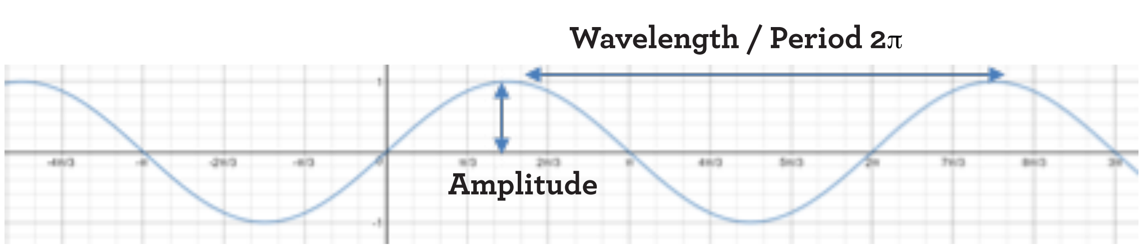
$f(x)=A\sin(Bt-C)+D$

When graphed on a coordinate plane, sinusoidal functions take the shape of a wave. There are only a few important things to remember about these functions.

First, the wavelength/period of the function can be measured as the distance from one peak of the wave to the next peak or from one trough to the next.

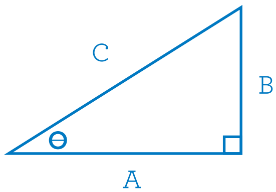
Second, the frequency of the function is the inverse of the wavelength: 1/wavelength.

Finally, the amplitude of the function is represented by the variable A, the number that appears directly in front of the sin function and is the height of the total function.



Example Problems

107. The dimensions of the right triangle shown below are given in meters. What is cos (Θ)?



A. $\frac{a}{b}$

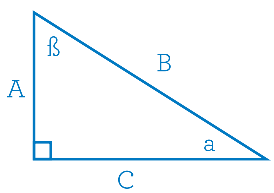
B. $\frac{a}{c}$

C. $\frac{b}{c}$

D. $\frac{b}{a}$

E. $\frac{c}{a}$

108. For the right triangle shown, sin (a) cos (ß) =



A. A

B. B

C. $\frac{a}{c}$

D. $\frac{A^2}{B^2}$

E. $\frac{A^2}{C^2}$

109. For the trigonometric function $f(x) = 8\tan(2\pi x \;\; 5) + 6$, what is the period?

A. $\frac{1}{2}$

B. $\frac{\pi}{2}$

C. $\frac{1}{4}$

D. $\frac{\pi}{4}$

E. 2π