ESSENTIAL FORMULAS FOR ACT MATH

Linear Equations

Slope-Intercept Form: $\boldsymbol{y = mx + b}$

- Slope = m
- Y-intercept = b

Point-Slope Form: $\boldsymbol{y - y_1 = m(x - x_1)}$

Slope Formula: $\boldsymbol{m = \frac{y_2 - y_1}{x_2 - x_1}}$

Midpoint Formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Distance Formula: $\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

Quadratic Equations / Parabolas

Standard/Quadratic Form: $\boldsymbol{f(x) = ax^2 + bx + c}$

- Vertex: $\left(-\frac{b}{2a}, f(-\frac{b}{2a})\right)$
- Discriminant: $b^2 - 4ac$ $\begin{cases} < 0 & \text{2 imaginary roots} \\ = 0 & \text{1 double root} \\ > 0 & \text{2 real roots} \end{cases}$

Vertex Form: $\boldsymbol{f(x) = a(x - h)^2 + k}$ Vertex: (h, k)

Factored Form: $\boldsymbol{f(x) = a(x - s)(x - t)}$

- Vertex: $\left(\frac{s+t}{2}, f\left(\frac{s+t}{2}\right)\right)$

Quadratic Identities

$(x + a)(x + b) = x^2 + (b + a)x + ab$

$a^2 - b^2 = (a + b)(a - b)$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Percentages

$\% = \frac{part}{whole} \times 100$ $\%\ change = \frac{new-old}{old} \times 100$

Miscellaneous

Mean or Average $= \frac{sum\ of\ terms}{\#\ of\ terms}$

$Distance = Rate \times Time$

Complex Numbers

$i^1 = i$ $i^2 = -1$ $i^3 = -i$ $i^4 = 1$

$i^5 = i$ $i^6 = -1$ $i^7 = -i$ $i^8 = 1$

Logarithms

$\log_b y = x \rightarrow b^x = y$

$\log_b x + \log_b y \leftrightarrow \log_b xy$ $\log_b x^a \leftrightarrow a * \log_b x$

$\log_b x - \log_b y \leftrightarrow \log_b \frac{x}{y}$ $\log_b b^x \leftrightarrow x$

Powers/Exponents/Roots

$x^a \times x^b = x^{a+b}$ $x^{-a} = \frac{1}{x^a}$

$\frac{x^a}{x^b} = x^{a-b}$ $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

$(x^a)^b = x^{ab}$ $(xy)^a = x^a y^a$

$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$ $x^0 = 1$

$(-1)^n = \begin{cases} 1, if\ n\ is\ even \\ -1, if\ n\ is\ odd \end{cases}$

Exponential Equations

Growth/Decay Formula: $\boldsymbol{A(t) = P\left(1 + \frac{r}{100}\right)^t}$

- P = Principle (initial amount)
- r = % increase/decrease
- t = time interval (in any unit)

Direct/Inverse Variation

Direct Variation: $y = kx$

Inverse Variation: $y = \frac{k}{x}$ k = constant of variation

Arithmetic Sequences

$a_n = a_1 + (n - 1)d$ $S_n = \frac{n}{2}(a_1 + a_n)$

Geometric Sequences

$a_n = a_1 * r^{n-1}$ $S_n = \frac{a_1(1-r^n)}{1-r}$

Expected Value Theorem (Weighted Average)

$EV = x_1 * P(x_1) + x_2 * P(x_2) \ldots x_n * P(x_n)$

x = values $P(x)$ = probability that x occurs

Empirical Rule (68 – 95 – 99.7 Rule)

For a normal distribution:

- **68%** of data falls within 1 standard deviation from the mean.
- **95%** of data falls within 2 standard deviations.
- **99.7%** of data falls within 3 standard deviations.

Basics of Probability

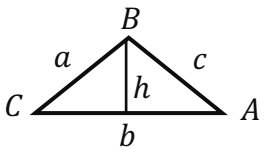
$P(A)$ **or** $P(B) = P(A) + P(B)$

$P(A)$ **and** $P(B) = P(A) \times P(B)$

ESSENTIAL FORMULAS FOR ACT MATH

Triangles

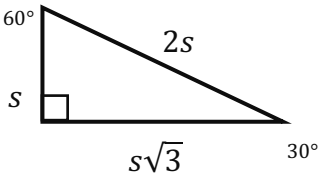
Area $= \frac{1}{2}bh = \frac{1}{2}ab\sin C$

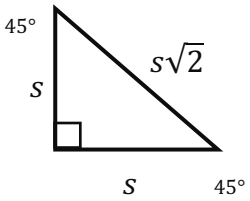


Area of Equilateral Triangle: $\frac{\sqrt{3}}{4}s^2$

Pythagorean Triples: (3,4,5) , (5,12,13) , (7,24,25)

Special Right Triangles





Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Circles

Area $= \pi r^2$ Circumference $= 2\pi r = \pi d$

Arc Length $= \left(\frac{n}{360}\right)\mathbf{2\pi r}$ Sector Area $= \left(\frac{n}{360}\right)\boldsymbol{\pi r^2}$

- n = central angle of arc/sector

Center-Radius Equation: $\boldsymbol{(x-h)^2 + (y-k)^2 = r^2}$

- Center: (h,k) Radius = r

Ellipses

Horizontal Major Axis: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = \mathbf{1}$

Vertical Major Axis: $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = \mathbf{1}$ $\Big\rangle\ a > b$

Center: (h,k) $2a$ = length of major axis

Foci: $c = \sqrt{a^2 - b^2}$ $2b$ = length of minor axis

Hyperbolas

Horizontal Major Axis: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = \mathbf{1}$

Vertical Major Axis: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = \mathbf{1}$

Center: (h,k) $2a$ = length of major axis

Foci: $c = \sqrt{a^2 + b^2}$

Vectors

Component Form: $ai + bj$

- Vector starts at (0,0) and ends at (a,b)
- Magnitude (length) = $\sqrt{a^2 + b^2}$

Trigonometry

$sine = \frac{opp}{hyp}$ $cosine = \frac{adj}{hyp}$ $tangent = \frac{opp}{adj}$

$\sin^2\theta + \cos^2\theta = 1$ π radians = 180 degrees

Graphing Trigonometric Functions

$$y = A\sin B(x-C) + D$$
$$y = A\cos B(x-C) + D$$
$$y = A\tan B(x-C) + D$$

A = amplitude period $= \frac{2\pi}{B}$ for sin/cos

C = horizontal shift period $= \frac{\pi}{B}$ for tan/cot

D = vertical shift

Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

Miscellaneous Shapes

Area of Parallelogram $= base * height$

Area of Trapezoid $= \left(\frac{b_1+b_2}{2}\right)h$

Volume of Rectangular Prism $= l * w * h$

Volume of Cube $= s^3$ Diagonal of Cube $= s\sqrt{3}$

Volume of Right Cylinder $= \pi r^2 h$

Surface Area of Right Cylinder $= 2\pi r^2 + 2\pi rh$

Sum of interior angles in polygon $= 180(n-2)$

Euler's Formula: $Vertices + Faces - Edges = 2$

Fundamental Theorem of Algebra

Highest Degree = Total Zeros = # of Real + Complex Zeros

Matrices

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix}$$

$$X\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = \begin{bmatrix} X * a_1 & X * b_1 \\ X * c_1 & X * d_1 \end{bmatrix}$$

Rules for Multiplying Matrices

$[rows_1 \times columns_1] \times [rows_2 \times columns_2]$

Must be equal

Dimensions of product matrix