Date Completed: _____
Mentor Initials: _____

A mentor can change everything.

ESM PREP

"Advanced" Problems Part One

Please take under timed pressure: ***30 minutes.***

This quiz is comprised of questions you will encounter in the final 10-15 questions of the ACT Math test. As you complete each question, please consider if, on an actual test, you would:

- **ATTACK** the question: you can answer it quickly and accurately, possibly using a strategy or a formula from the ACT Math Formula Sheet.
- **SKIP** the question: you don't know how to answer it quickly and do not have a strategy or formula to use *(Remember: try to eliminate as many answers as possible before guessing)*.

1. For all real numbers a, b, c such that $a < b$ and $c < 0$, which of the following inequalities must be true?

A. $a > b + c$

B. $ac < bc$

C. $a + c > b + c$

D. $a < b - c$

E. $\frac{a}{c} < \frac{b}{c}$

2. For a positive integer n, the ones digit of $n^2 + 2$ is 3. The ones digit of $(n-1)^2$ is 4. What is the ones digit of n?

A. 1
B. 3
C. 5
D. 8
E. 9

3. For all $x > 0$, which of the following expressions is NOT equivalent to $\sqrt[3]{\sqrt{x^3}}$?

A. $\sqrt{\sqrt{x^2}}$

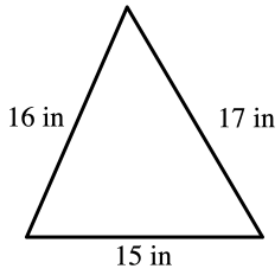
B. $x^{\frac{1}{2}}$

C. $\left(\sqrt[12]{x}\right)^6$

D. $\sqrt[6]{x^3}$

E. $\sqrt[3]{x^2}$

4. The triangle shown below has side lengths 15, 16, and 17 inches. Which of the following expressions gives the measure of the largest angle of the triangle?



A. $\cos^{-1}(-\frac{15^2-16^2-17^2}{2(16)(17)})$

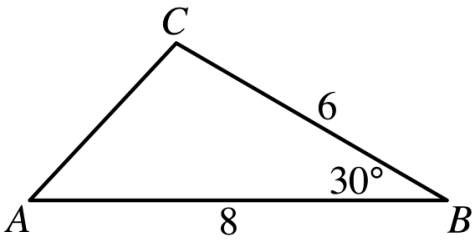
B. $\cos^{-1}(-\frac{17^2-15^2-16^2}{2(15)(16)})$

C. $\cos^{-1}(15^2-16^2-17^2+2(16)(17))$

D. $\cos^{-1}(16^2-15^2-17^2+2(15)(17))$

E. $\cos^{-1}(17^2-15^2-16^2+2(15)(16))$

5. What is the area of triangle *ABC*?



A. 9
B. 12
C. $6\sqrt{7}$
D. $12\sqrt{3}$
E. 24

6. A jar contains 9 marbles: 2 blue, 3 red, 4 green. Each marble has a single number on it. Blues are numbered 1 and 2, reds are numbered 1 through 3, and greens are numbered 1 through 4. What is the probability that a marble is drawn that is red or has an even number on it?

A. $\frac{1}{9}$

B. $\frac{4}{9}$

C. $\frac{5}{9}$

D. $\frac{2}{3}$

E. $\frac{7}{9}$

7. Five balls—numbered $1, 2, 3, 4$ and 5—are placed in a bin. Two balls are drawn at random without replacement. What is the probability that the sum of the numbers on the balls drawn is 6?

A. $\frac{1}{20}$

B. $\frac{3}{25}$

C. $\frac{3}{20}$

D. $\frac{1}{5}$

E. $\frac{1}{4}$

8. Given that y varies directly as the square of x, if $y = 36$ when $x = 3$, what is y when $x = 2$?

A. 6

B. 8

C. 9

D. 12

E. 16

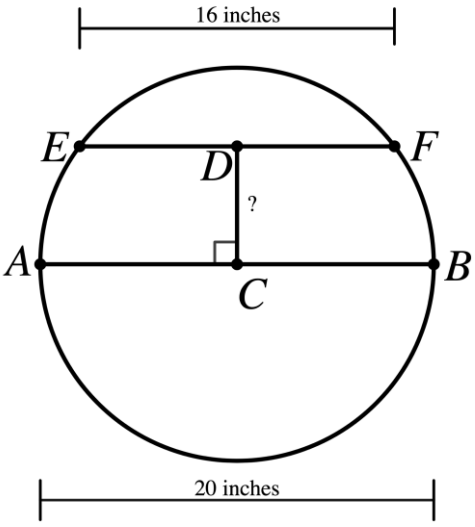
9. If a and b are real numbers such that $a > 0$ and $b < 0$, then which of the following quantities is not equivalent to the others?

A. $a - b$
B. $|a + b|$
C. $|-a + b|$
D. $|a| + |b|$
E. $a + |b|$

10. The ones digit of 2^{101} is 2. What is the ones digit of 2^{106}?

A. 0
B. 2
C. 4
D. 6
E. 8

11. In the diagram below, chord EF is parallel to diameter AB. The length of EF is 16 inches and the length of AB is 20 inches. C is the center of the circle, and D is the midpoint of EF. What is the distance, in inches, from C to D?



A. 4
B. 6
C. $6\sqrt{2}$
D. $6\sqrt{3}$
E. 8

12. Adam and Bella are meeting four of their friends for lunch. All 6 friends will be seated randomly around a circular table. What is the probability that Adam and Bella will NOT sit next to each other?

A. $\frac{1}{4}$

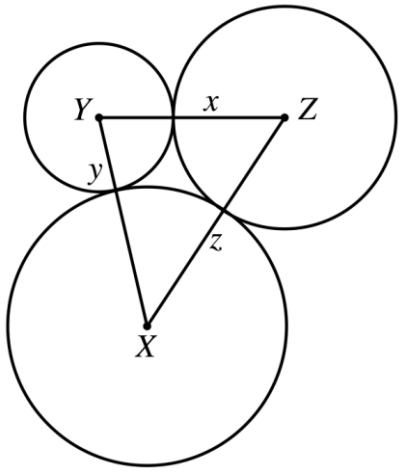
B. $\frac{2}{5}$

C. $\frac{1}{2}$

D. $\frac{3}{5}$

E. $\frac{3}{4}$

13. The circles with centers X, Y, and Z are mutually tangent, as shown below. They have radii of lengths z, y, and x, respectively. XY is 13 inches long, YZ is 10 inches, and XZ is 15 inches. What is the value of $x + y + z$?



A. 15
B. 17
C. 18
D. 19
E. 24

14. If the 4th term of an arithmetic sequence is 10 and the 9th term is 25, what is the sum of the first ten terms?

A. 125
B. 145
C. 150
D. 160
E. 175

15. If three members of a committee are selected from a group of 6 people, how many possible committees are there if there are no special roles? In a second scenario, if there is a president, vice president, and treasurer, how many committees are possible?

A. 20 and 60
B. 36 and 72
C. 40 and 80
D. 20 and 120
E. 60 and 120

16. Mike has 14 coins, all of which are nickels, dimes, and quarters. His number of dimes is three more than twice his number of nickels. He has two fewer quarters than dimes. In total, he has \$2.05. How many dimes does Mike have?

A. 5
B. 6
C. 7
D. 8
E. 9

17. For all real numbers x and imaginary number i, which of the following expressions is equivalent to $(x - 2i)^3$?

A. $x^3 + 4x^2i - 8x - 8i$
B. $x^3 - 6x^2i - 8x + 8i$
C. $x^3 - 4x^2i + 4x - 6xi + 8i$
D. $x^3 - 6x^2i - 12x + 8i$
E. $x^3 - 8i$

18. The art club designed and made red and green banners for their school fund-raiser. Each banner required $\frac{1}{5}$ yards of red material and $\frac{3}{8}$ yards of green material. The club originally planned to purchase exactly enough material to make 400 banners, but found the material to be cheaper if purchased in full scrolls—the red material in 12-yard scrolls and the green material in 20-yard scrolls. How many extra banners was the club able to make if they purchased enough full scrolls to make at least 400 banners?

A. 16
B. 20
C. 22
D. 24
E. 26

19. If $\log x = a$ and $\log y = b$, then $\log(xy)^3 =$?

A. $3ab$
B. $3a + b$
C. $4a$
D. $3a + 3b$
E. $(ab)^3$

20. For all $x > 0$, which of the following expressions is equivalent to $\frac{i}{\sqrt{x}+i}$, where $i = \sqrt{-1}$?

A. i
B. $\frac{\sqrt{x}}{x}$
C. $\frac{\sqrt{x}+1}{x-1}$
D. $\frac{i\sqrt{x}-1}{x+1}$
E. $\frac{i\sqrt{x}+1}{x+1}$

21. Bayside High School and Seabreeze High School have decided that some students will attend a matinee that costs $7 for a teacher and $4 for a student. One teacher and 8 students from Bayside will attend, and 3 teachers and 15 students from Seabreeze will attend. Which of the following matrix products represents the ticket cost, in dollars for each high school?

A. $[7 \quad 4]\begin{bmatrix} 1 & 15 \\ 3 & 8 \end{bmatrix}$

B. $[7 \quad 4]\begin{bmatrix} 1 & 8 \\ 15 & 3 \end{bmatrix}$

C. $[7 \quad 4]\begin{bmatrix} 1 & 3 \\ 8 & 15 \end{bmatrix}$

D. $\begin{bmatrix} 7 \\ 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 8 & 15 \end{bmatrix}$

E. $\begin{bmatrix} 7 \\ 4 \end{bmatrix}\begin{bmatrix} 1 & 8 \\ 3 & 15 \end{bmatrix}$

22. Billy is trying to figure out how many possible custom license plates he has to choose from for his new Chevy Astro. His license plate will be six characters long. The first character will be a letter from A to H. The second character will be an odd digit between 0 and 9, inclusive. The last four characters will be any alphabetical letter or single-digit number. If no characters can be used twice, what is the total number of possible license plates that Billy could choose?

A. $(8)(5)(36)(35)(34)(33)$

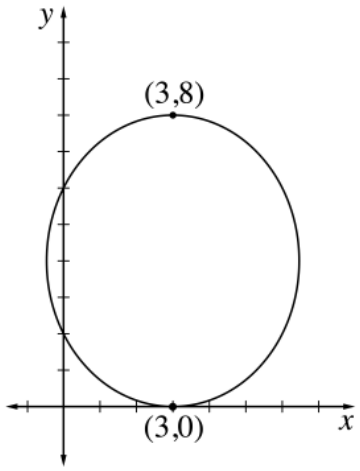
B. $(8)(5)(24)(23)(22)(21)$

C. $(36)(36)(36)(36)(8)(5)$

D. $(34)(33)(32)(31)(8)(5)$

E. $(36)(35)(34)(33)(32)(31)$

23. The ellipse shown in the (x, y) coordinate plane below has the equation $\frac{(x-3)^2}{12} + \frac{(y-4)^2}{16} = 1$. Which of the following sets of ordered pairs are the foci of the ellipse?



A. $(-1,4)$ and $(7,4)$

B. $(1,4)$ and $(5,4)$

C. $(3,0)$ and $(3,8)$

D. $(3,6)$ and $(3,2)$

E. $(3,1)$ and $(3,7)$