Present Value Mathematics for Real Estate

This presentation introduces the foundational mathematics of present value, a core concept for real estate valuation. We'll explore the time value of money, including discounting, compounding, and growth rates, with practical applications for real estate investment analysis.

NM

The Time Value of Money

Money Has Time Value

Dollars today are worth more than dollars in the future, even without inflation.

Real Productivity of Capital

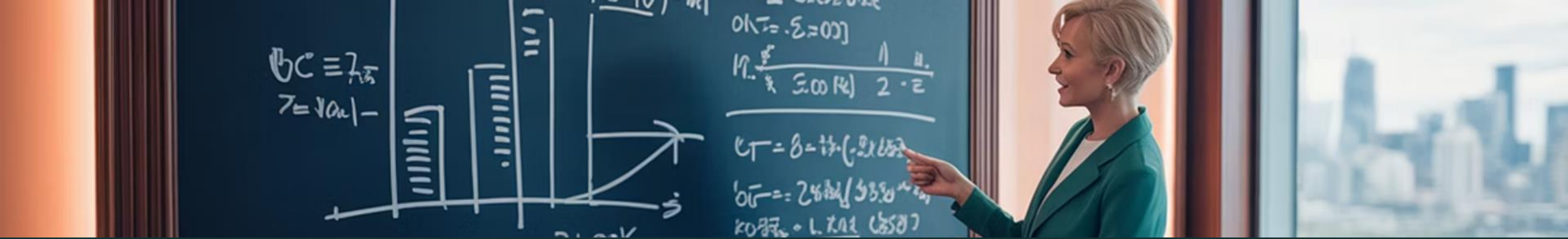
Capital can be invested to generate returns over time.

Risk Considerations

Future dollars are less certain than present dollars.

Comparison Tool

Present value mathematics helps compare dollars at different points in time.



Single-Sum Formulas

Single-Period Discounting

$PV = FV / (1 + r)$

Single-Period Growing

$FV = PV \times (1 + r)$

Multiple Periods

$PV = FV / (1 + r)^N$

Solving for Return

$r = (FV/PV)^{(1/N)} - 1$

Simple vs. Compound Interest

Simple Interest

Interest calculated only on the initial principal.

Example: $100 at 15% simple interest for 2 years = $130

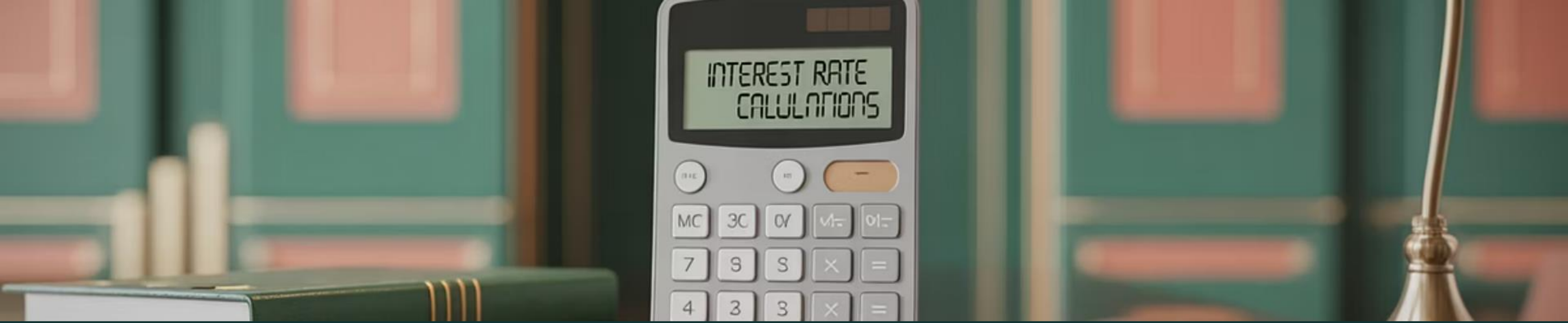
($100 + $15 + $15 = $130)

Compound Interest

Interest calculated on both principal and accumulated interest.

Example: $100 at 15% compound interest for 2 years = $132.25

($100 × 1.15 × 1.15 = $132.25)



Effective vs. Nominal Rates

Nominal Annual Rate (i)

The stated annual interest rate without considering compounding frequency.

Effective Annual Rate (EAR)

The actual annual yield after accounting for compounding frequency.

Relationship

$EAR = (1 + i/m)^m - 1$, where m is the number of compounding periods per year.

Bond vs. Mortgage Equivalent Rates

Mortgage Equivalent

Based on monthly compounding (m=12)

Common in real estate transactions

Bond Equivalent

Based on semiannual compounding (m=2)

Standard in bond markets

Comparison

Mortgage equivalent rates are lower than corresponding bond equivalent rates

Conversion

Convert by equating the effective annual rates

Continuously Compounded Interest

Definition

Interest compounded at every instant rather than at discrete intervals.

Formula

$PV = FV \times e^{(-kT)}$, where k is the continuously compounded rate and T is time in years.

Applications

Used in sophisticated financial analyses and certain banking products.

Multiperiod Cash Flows

Single Sums

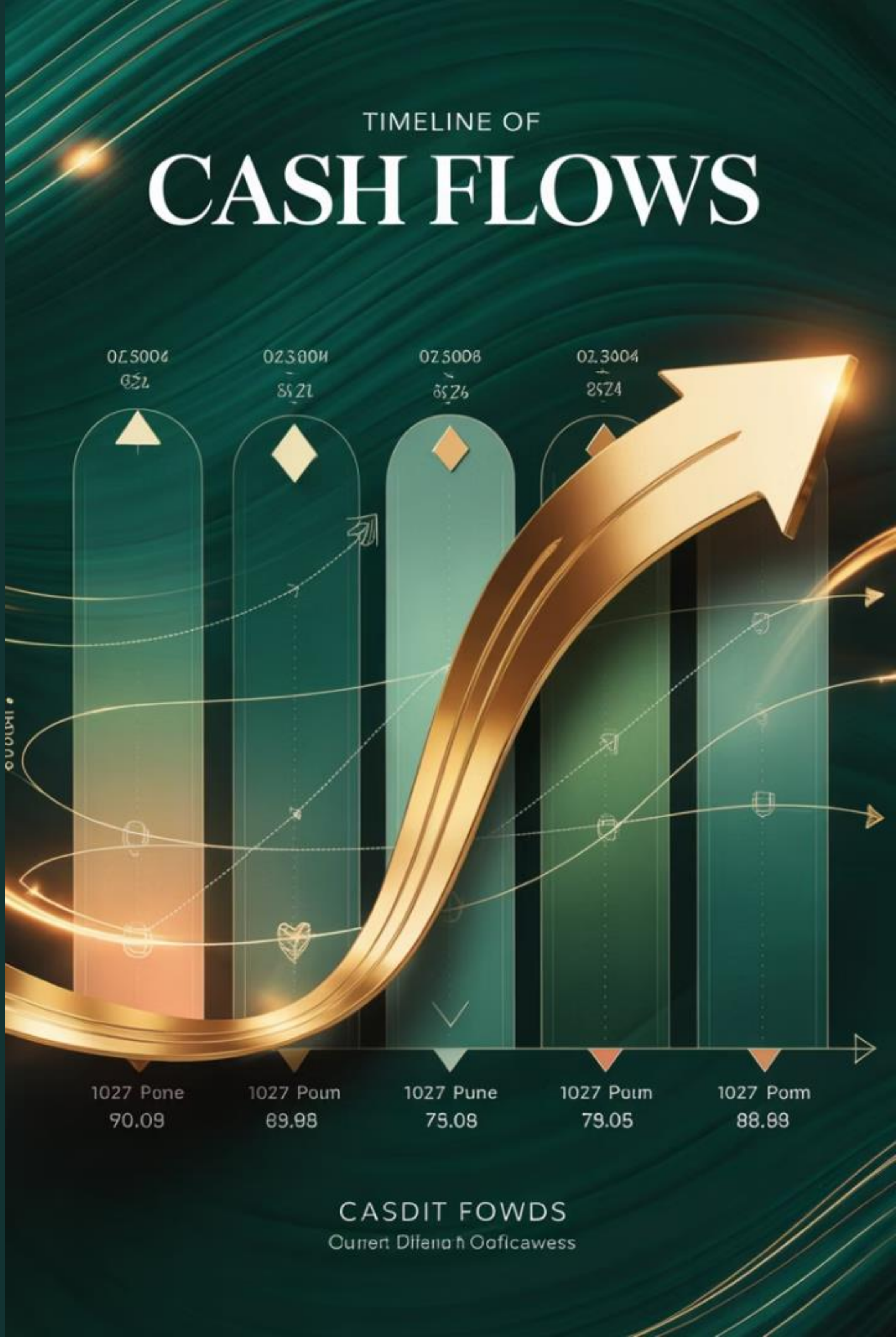
Basic building blocks for all present value calculations.

Regular Patterns

Cash flows with consistent patterns allow for simplified formulas.

Combined Approach

Sum the present values of each future cash flow.



The Geometric Series Formula

The foundation for valuing annuities and perpetuities:

$$S = a\frac{1 - d^N}{1 - d}$$

Where:

- **S** — Sum of the series
- **a** — First term in the series
- **d** — Common ratio between terms
- **N** — Number of terms

Level Annuity in Arrears

Definition

Equal payments made at the end of each period.

Formula

$PV = PMT \times [1 - 1/(1+r)^{N}] / r$

Application

Classical mortgage payments structure.

Level Annuity in Advance

Definition

Equal payments made at the beginning of each period.

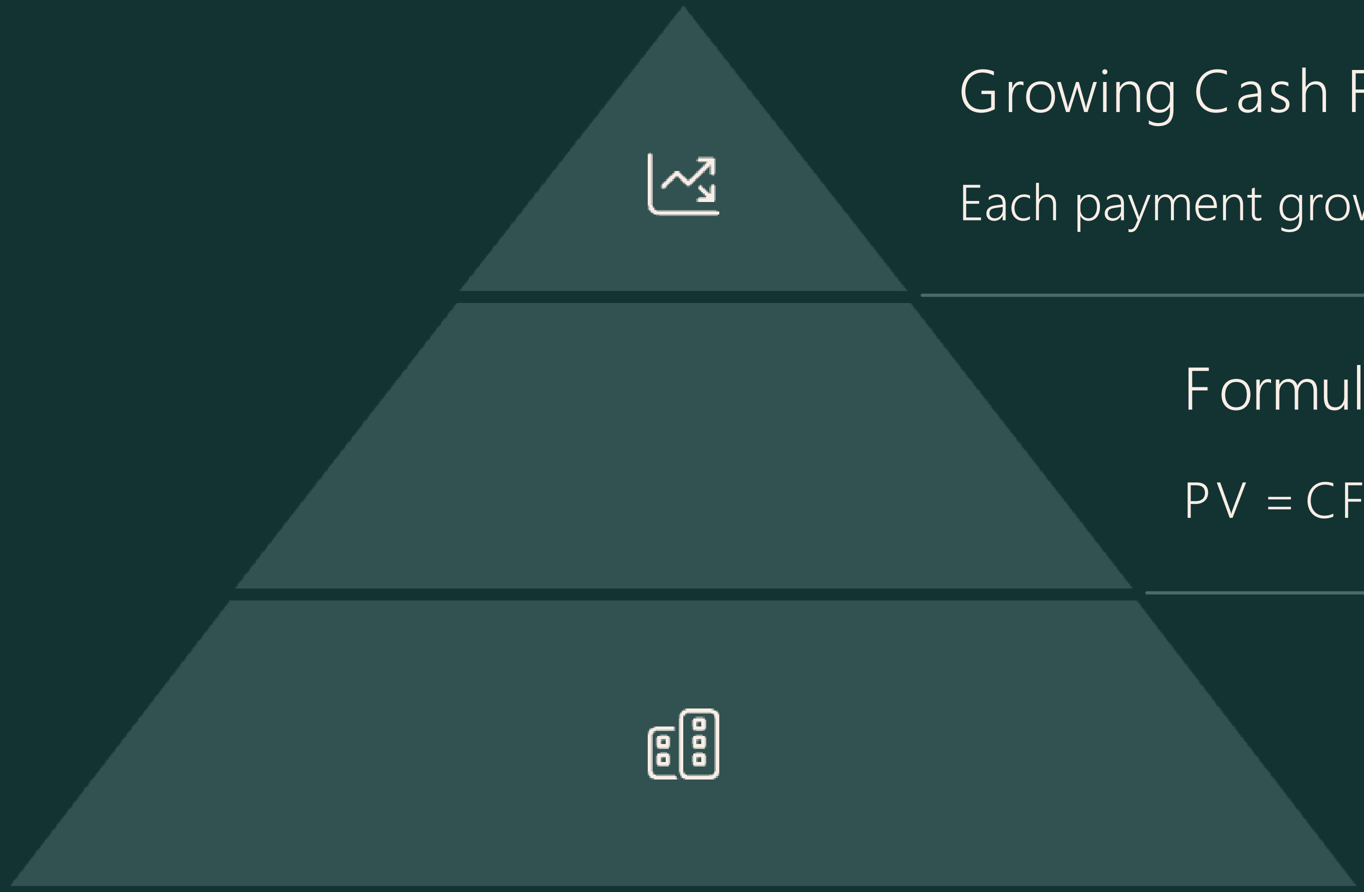
Formula

$PV = PMT \times (1+r) \times [1 - 1/(1+r)^N] / r$

Application

Common in rental payments and leases.

Constant-Growth Annuity



Growing Cash Flows

Each payment grows at a constant rate g

Formula

$PV = CF_1 \times [1-((1+g)/(1+r))^N] / (r-g)$

Real Estate Application

Leases with escalation clauses

Constant-Growth Perpetuity

∞

Infinite Time Horizon

Cash flows continue forever

CF_1

Initial Cash Flow

Starting point for growth

r-g

Denominator

Difference between discount and growth rates

The formula simplifies to: $PV = CF_1/(r-g)$

The Gordon Growth Model, GGM, aka Perpetuity Model Combines the current income yield and growth rate and can be used to generate a cap rate, R, where R = Discount Rate or Yield – Expected Growth Rate in Income

The total yield expected is approximately equal to the current income % plus the growth rate %.

GGM applied to Stocks

$$P = \frac{D_1}{r - g}$$

where:

P = Current stock price

g = Constant growth rate expected for dividends, in perpetuity

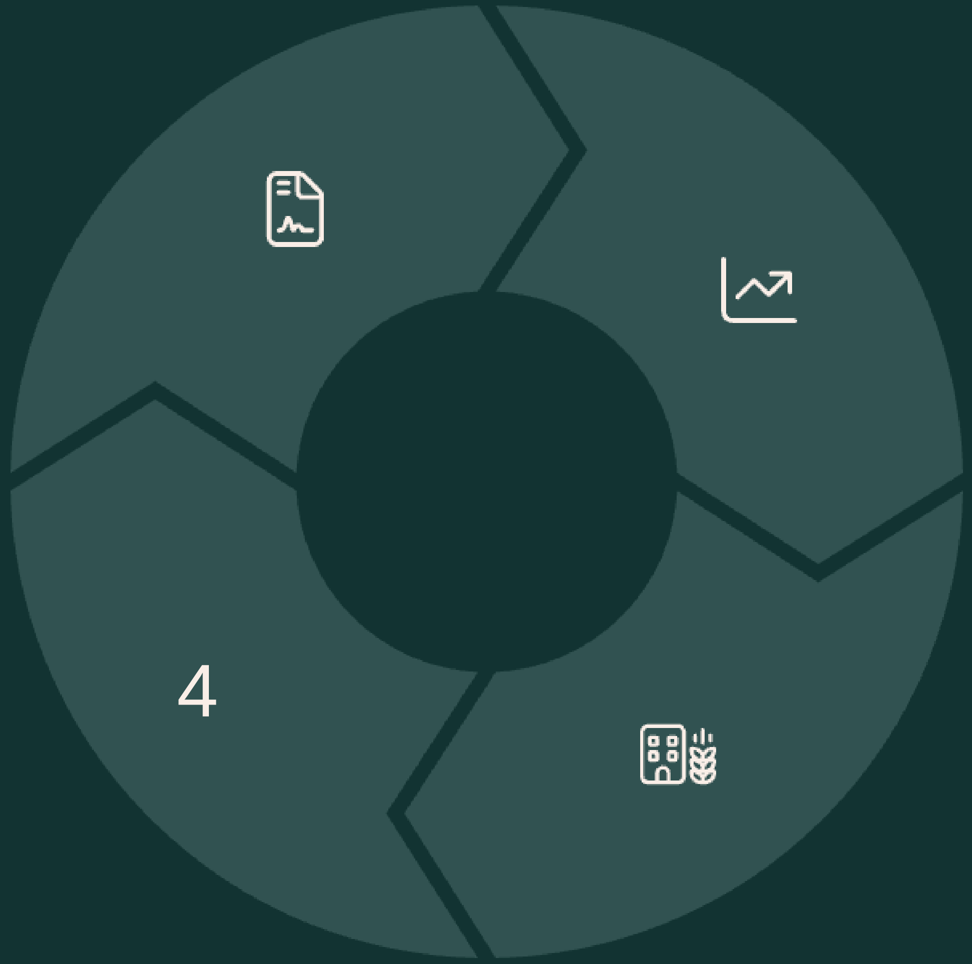
r = Constant cost of equity capital for the company (or rate of return)

D_1 = Value of next year's dividends

GGM applied to Real Estate

Value = NOI/(Discount Rate – Growth Rates of NOI)

Long-Term Leases



Intralease Rates

Lower risk due to contractual obligations

Interlease Rates

Higher risk due to market uncertainty

Two-Step Valuation

Value each lease, then value the series of leases

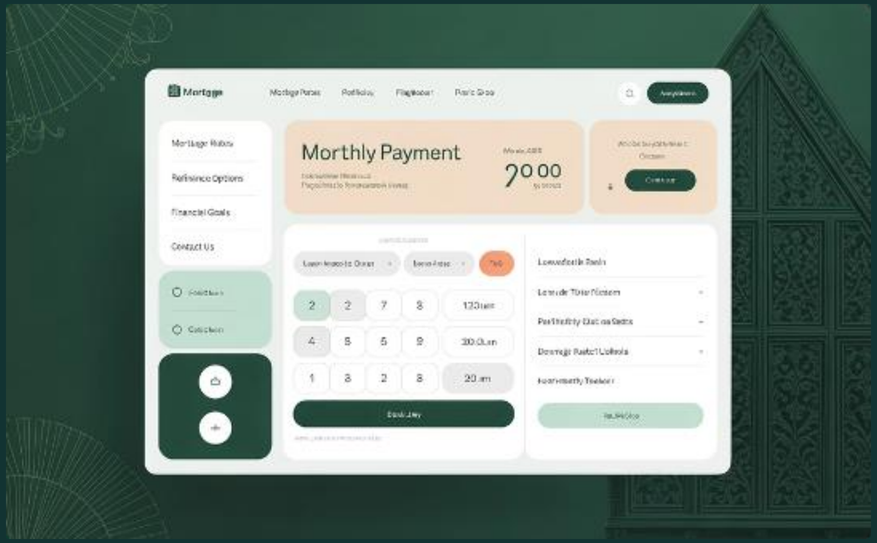
Building Value

Perpetuity of successive annuities

Future Value of Annuities

The future value of an annuity equals the present value multiplied by $(1+r)^N$. This calculation is useful for retirement planning, college savings, and other future financial goals.

Solving for Cash Flows

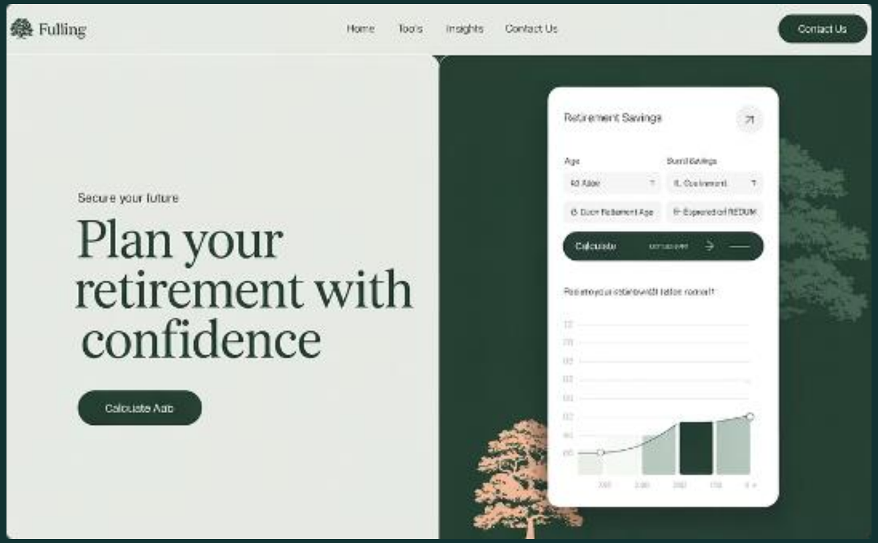


Mortgage Payments

$PMT = PV \times r \times (1+r)^N/[(1+r)^N-1]$

Loan Amortization

Calculate payments needed to fully repay a loan over time.



Savings Goals

Determine periodic contributions needed to reach a future value.

Mortgage Mathematics

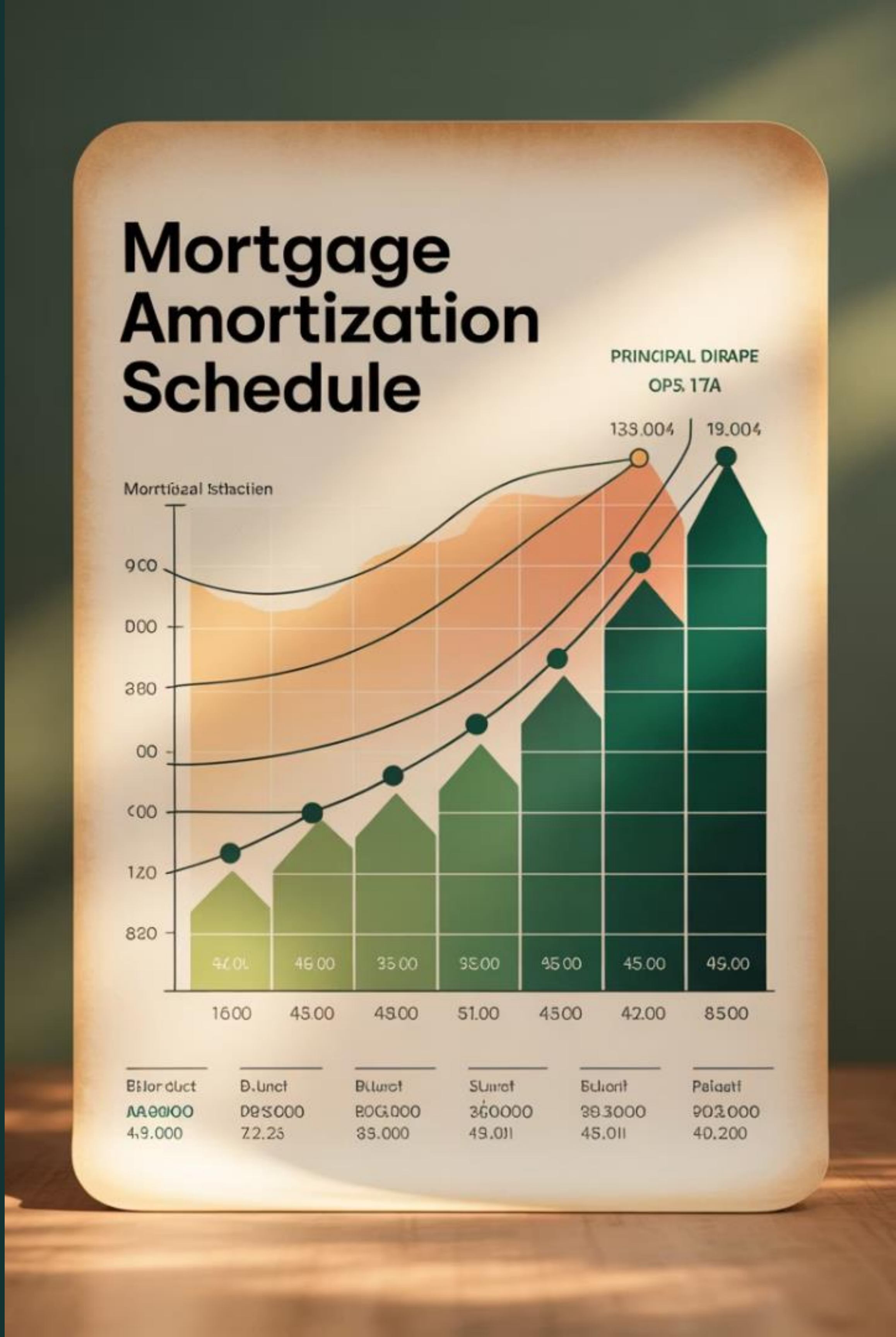
Loan Type	Cash Flow Pattern	Formula Components
Fully Amortizing	Level annuity only	$PV = PMT \times [1 - 1/(1+r)^N]/r$
Partially Amortizing	Annuity + balloon payment	$PV = PMT \times [1 - 1/(1+r)^N]/r + FV/(1+r)^N$
Interest-Only	Interest payments + principal at end	$PV = PMT \times [1 - 1/(1+r)^N]/r + PV/(1+r)^N$

Outstanding Loan Balance

After q payments on a loan with monthly payment PMT, the outstanding loan balance is:

$$OLB = PMT \times \frac{1 - 1/(1+r)^{N-q}}{r}$$

$80,000	$822.89	$74,734
Original Loan	Monthly Payment	Balance After 10 Years
30-year mortgage at 12%	Fixed for the life of the loan	Still 93% of original principal



Key Takeaways

Time Value Fundamentals

Money has time value due to productivity of capital and risk.

Formula Flexibility

Single-sum formulas are building blocks for complex cash flow patterns.

Real Estate Applications

Present value mathematics is essential for property valuation and investment analysis.

Cap Rate Insights

The perpetuity model reveals cap rate equals discount rate minus growth rate.