WISCONSIN SCHOOL of BUSINESS

JAMES A. GRAASKAMP CENTER for REAL ESTATE

Financial Basics: Six Functions of a Dollar (or a Euro, Peso, Yen, Renminbi…)

Stephen Malpezzi
stephen.malpezzi@wisc.edu
http://smalpezzi.marginalq.com
http://reudviewpoint.blogspot.com/

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Finally: The Lecture You've Been Waiting For.

- The Key to Understanding the Universe:
  - (well, the key to understanding the ***financial*** universe):
- A dollar today is worth more than a dollar tomorrow.
- How much more? That depends on your *discount rate*.

More Universal Truth

- “Eighty percent of life is showing up.” (W. Allen)
- Eighty percent of understanding business/financial “life” is:
  - Estimating one or more cash flows;
  - Taking their weighted sum (weighted because cash flows in the future are worth less than cash flows today), and;
  - Figuring out what weights to use when adding up those cash flows.

The Miracle of Compound Interest

- Everyone knows this one. If the interest rate is, say, 10% *per annum*, deposit $1.00, in a year you have $1.10.
- In, say, three years you have:

  (($1*1.1)*1.1)*1.1

  = $1*(1+.1)^3

  = $1.33
- The extra 3¢ is due to compounding.
- General formula: $A_t = A_0(1+i)^t$

Present Value is Compound Interest Running in Reverse

If we want to know how much an amount received at time t is worth today (time 0), we know that:

$A_t = A_0(1+i)^t$, so

$A_t/A_0 = (1+i)^t$, and therefore

$A_0 = A_t / (1+i)^t$

which is the *present value* of A_t

NB: cash flows can be positive or negative.

Present Value of Several Cash Flows

- To calculate the present value of several cash flows, we simply add them, ***after*** discounting:

$$A_0 = \frac{A_1}{(1+i)} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3}$$

- To represent this more generally, use the following notation, where the Greek letter Sigma means "sum" (add over all periods, from the initial period 0 to the final period T):

$$A_0 = \sum_{t=0}^{T} \frac{A_t}{(1+i)^t}$$

A Handy Shortcut

- Recall our formula for present value of discrete payments:

$$A_0 = \sum_{t=0}^{T} \frac{A_t}{(1+i)^t}$$

- It can be shown that, in the limit, ***if each and every*** $\boldsymbol{A_t}$ ***is the same***, as $t \rightarrow \infty$ this formula becomes:

$$A_0 = \frac{A}{i}$$

Example: an apartment building

- Suppose our building throws off NOI of $64,000, starting next year, and NOI grows at 2 percent per year.
- Suppose we hold the building for three years, then sell at the end of three years for the fourth year's NOI "capped" at 8 percent.
- Suppose the market discounts cash at this level of risk at 15 percent. What's the value of the property?

Example: An Apartment building

$$\text{Present Value} = \frac{64{,}000}{1.15} + \frac{65{,}280}{1.15^2} + \frac{66{,}586}{1.15^3} + \frac{865{,}946}{1.15^3}$$

- So this unit is worth $718,168. Congratulations! You’ve just valued your first income property. (Unless Sharon and you beat us to it).
- (Why do we ignore income taxes and mortgage financing when valuing a building?)


TIME
IS
MONEY

Discount Rates

- The discount rate tells us how much we value a dollar today, versus a dollar a year from now. It's the rate at which we're indifferent between a dollar today, and a dollar tomorrow.
- Sometimes the relevant discount rate is determined by the market (the rate of interest, or the "cap rate," to give two examples). Sometimes it's personal.
- What is *your* discount rate?

Internal Rate of Return

- IRR is the discount rate that yields a present value of zero. That is, solve:

$$0 = \sum_{t=0}^{T} \frac{A_t}{(1+i)^t}$$

- for i. Generally this must be done iteratively. For IRR to make sense, some cash flows must be negative, some positive; and for IRR to be "well behaved" (unique), we prefer n negative cash flows followed by T-n positive cash flows. Situations with switching positive/negative cash flows will have two solutions.

Present Values

- What have we established so far?
  - The value of a dollar depends on when we receive it. The timing of cash flows is important.
  - The discount rate is a measure of the time value of money, i.e. how we trade off present versus future cash flows.
  - We can relate this more precisely with formula based on $A_t = A_0(1+i)^t$, which tells us how much an amount A_0 is worth at the end of t periods.

The Five Way Path to (Financial) Enlightenment

- Write down cash flows, figure out the weights for each period, and add them up. Preferably in a spreadsheet.
- Shortcut formulas (see handouts, DS&R p. 600).
- Punch data into your financial calculator. Make sure you know what goes on behind the scenes.
- Rules of thumb. Used before spreadsheets and financial calculators.
- Financial tables (DS&R, Appendix, pp. 601 ff.)

Great Moments in Present Value History (I)

- In 1626, Peter Minuit bought Manhattan Island for 60 guilders (more or less).
- According to folklore, 60 guilders was worth $24. It wasn’t, but let’s honor the myth, and assume it was so.
- (1) What would be your personal discount rate for a risky investment: raw undeveloped land in the middle of (then) nowhere?

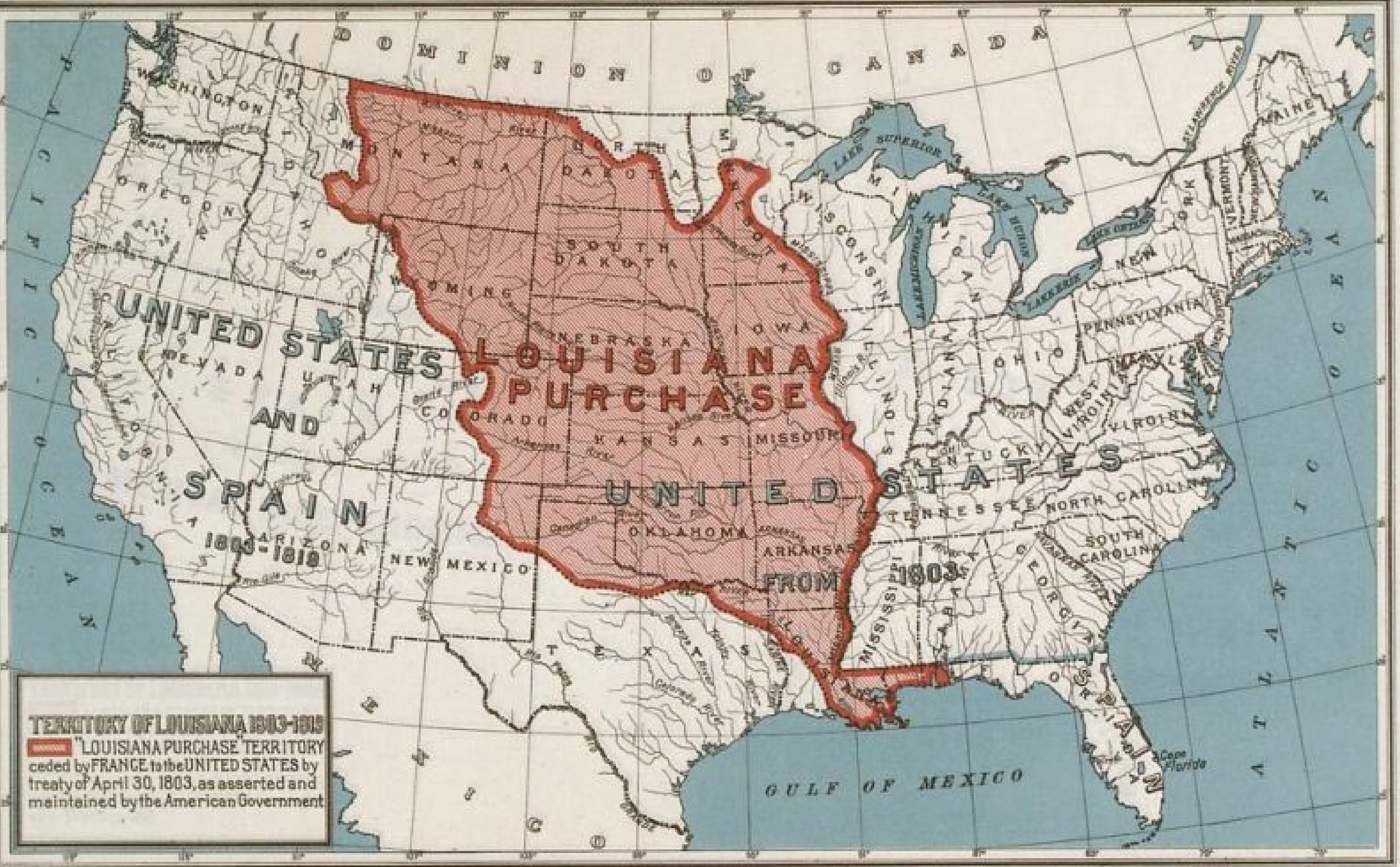
"THE BIG THING."

Old Mother Seward. "I'll rub some of this on his sore spot: it may soothe him a little."

Great Moments in Present Value History (II)

- In 1867 the U.S. bought Alaska from the Russians for $7.2 million. Alaska has 571,000 square miles.
- Assume that Alaska is worth an average of $500 per acre in 1992. Assume that the relevant discount rate is 6 percent per annum.
  - (A) Estimate the value of Alaska, and the present value of the money we paid for it.
  - (B) Who got the better deal?
    - a.The U.S.A.
    - b.The Russians. But maybe we could buy all their plutonium, cheap, and make up our losses.

Map No. 4.


DOMINION OF CANADA
WASHINGTON
OREGON
MONTANA
NORTH DAKOTA
SOUTH DAKOTA
WYOMING
NEBRASKA
IOWA
MINNESOTA
WISCONSIN
MICHIGAN
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
LAKE ERIE
LAKE ONTARIO
MAINE
VERMONT
NEW YORK
PENNSYLVANIA
OHIO
INDIANA
ILLINOIS
WEST VIRGINIA
VIRGINIA
KENTUCKY
TENNESSEE
NORTH CAROLINA
SOUTH CAROLINA
GEORGIA
ALABAMA
MISSISSIPPI
LOUISIANA
ARKANSAS
MISSOURI
KANSAS
OKLAHOMA
TEXAS
COLORADO
NEW MEXICO
ARIZONA
UTAH
NEVADA
UNITED STATES AND SPAIN 1803-1819
LOUISIANA PURCHASE
UNITED STATES FROM 1803
SPAIN
PACIFIC OCEAN
ATLANTIC OCEAN
GULF OF MEXICO
MEXICO
Cape Florida
TERRITORY OF LOUISIANA 1803-1819
"LOUISIANA PURCHASE" TERRITORY ceded by FRANCE to the UNITED STATES by treaty of April 30, 1803, as asserted and maintained by the American Government
THE COLUMBIA PLANOGRAPH CO., WASHINGTON, D. C.

Great Moments in Present Value History (III)

- In 1803 the U.S. doubled in size with the Louisiana purchase. That well-known real estate mogul Napoleon Bonaparte sold us 827,000 square miles for $15 million.
- Assume that part of the U.S. is worth an average of $3,000 per acre in 1993. Assume that the relevant discount rate is 3 percent per annum. Who got the better deal?
  - a.The U.S.A.
  - b.La belle France. Plus their cheese is better. But we have Jerry Lewis.

Six Functions of a Dollar

- Compute the future value of a one time deposit (compounding).
- Compute the future value of a ***series*** of periodic payments (an annuity).
- Compute the present value of a one time future receipt (discounting).
- Compute the present value of a ***series*** of future receipts.
- Determine how much must be set aside periodically to accumulate a certain value (sinking fund).
- Determine the constant periodic payment necessary to repay (amortize) a loan.

Sinking Fund

- The problem: how much do we have to set aside each period, if we want a given amount at the end?
  - EG How much do we need to set aside each month, if we're going to replace the roof in four years?
  - EG How much will you have to set aside each month to pay for *your* kid's college?

Sinking Fund

- We already know how to compute the future value of a series of payments:

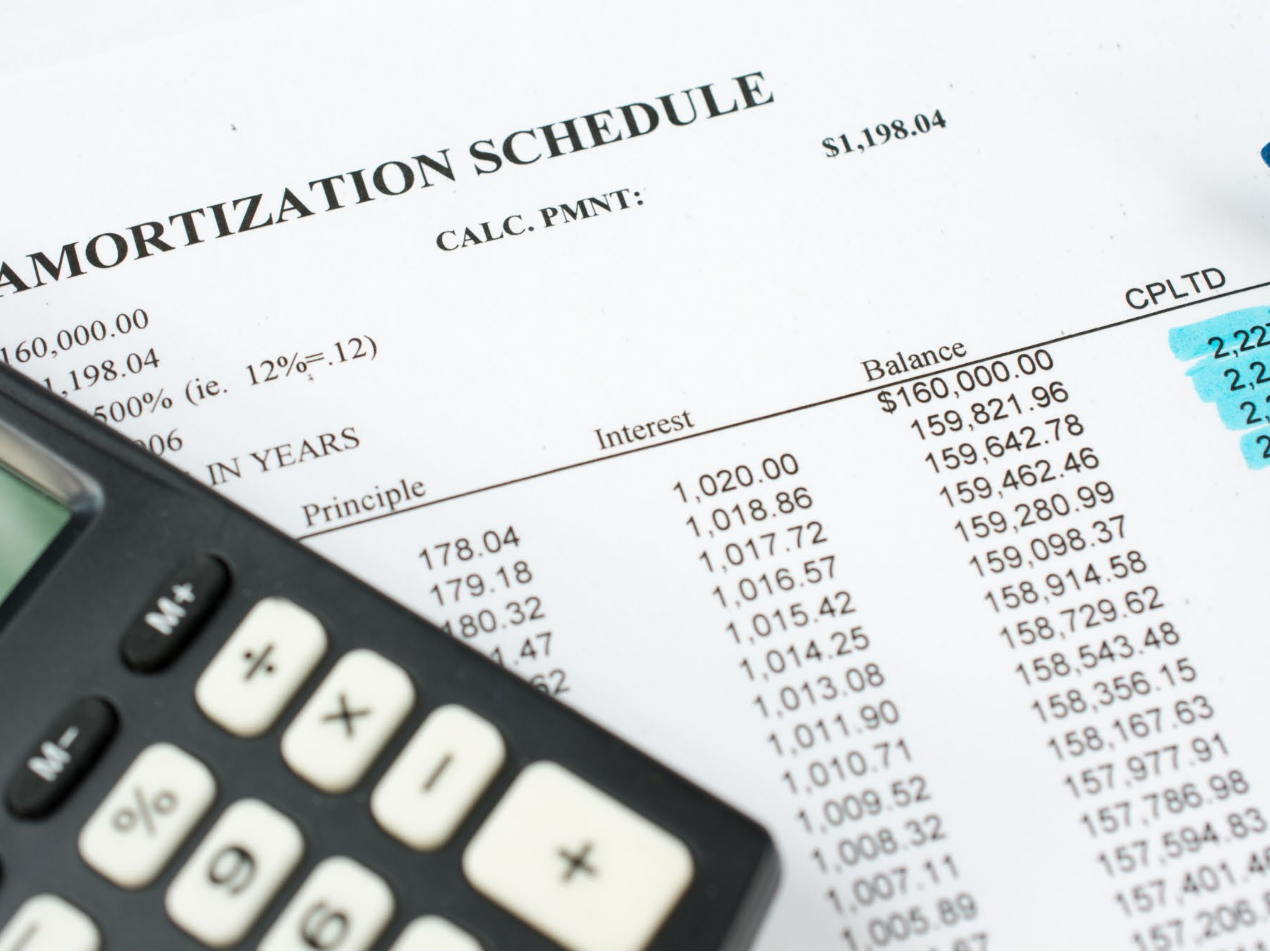
  $FV = A_1(1+i)^t + A_2(1+i)^{t-1} + \ldots + A_t$

- Let each A be the same, and solve for A, such that:

  A = FV * S, where S is the sinking fund factor:

$$S = \frac{1}{(1+i)^t + (1+i)^{t-1} + \ldots + 1} = \frac{i}{(1+i)^t - 1}$$

- S can be found using a table, or the =PMT function in Excel.


AMORTIZATION SCHEDULE
CALC. PMNT:
$1,198.04
CPLTD
160,000.00
,198.04
500% (ie. 12%=.12)
IN YEARS
Principle
Interest
Balance
$160,000.00
178.04
179.18
1,020.00
1,018.86
1,017.72
1,016.57
1,015.42
1,014.25
1,013.08
1,011.90
1,010.71
1,009.52
1,008.32
1,007.11
159,821.96
159,642.78
159,462.46
159,280.99
159,098.37
158,914.58
158,729.62
158,543.48
158,356.15
158,167.63
157,977.91
157,786.98
157,594.83
M+
M-

Payment to Amortize a Fixed Rate Loan

- What are the (constant) payments required to amortize a fixed rate loan?
- Think of the mortgage amount as a present value, and the interest rate as the discount rate of the lender. Given loan amount PV and an interest rate i at which to discount, and loan of term T, solve for payment A:

$$PV = \sum_{t=1}^{T} \frac{A}{(1+i)^t}$$

Payment to Amortize a Fixed Rate Loan

- Example: What annual payments are required to amortize an 8%, 3 year, $5,000 loan?

$$5000 = A/1.08 + A/1.08^2 + A/1.08^3$$

$$= A * 2.5771, \text{ therefore}$$

$$A = 5000/2.5771 = \$1940.17$$

Note: there's a similar example in the "More TVM" handout, with a (gasp) typo. Can you find the typo?

Payment to Amortize a Fixed Rate Loan

- One can use the =PMT function in Excel, a financial calculator, the Appendix tables, or the formula:

$$A = PV * \frac{i}{1 - \frac{1}{(1+i)^T}}$$

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$$A = PV * \frac{i}{1 - \frac{1}{(1+i)^T}}$$

The path to understanding

- There are some things you need to *internalize*, in particular:
  - The basic relation between present value, future value, and the discount rate:

$$A_t = A_0(1+i)^t$$

- Internalization is more than memorization.
- Calculators, formulas, and Excel help us calculate “large” problems, and variations on the theme.

Read this slide. Know it. Live it.

Discount rate

$$A_t = A_0(1+i)^t$$

Future value

Present value

- Which immediately implies:

$$A_0 = \frac{A_t}{(1+i)^t}$$

Know this too!

- Also, remember that present values are *additive*.
- The present value of a stream of cash flows is just the sum of each cash flow's present value. For multiple flows at times t, up to final time T,

$$A_0 = \sum_{t=0}^{T} \frac{A_t}{(1+i)^t}$$

Two more things you absolutely *must internalize*

- The internal rate of return is the discount rate that makes the net present value of multiple cash flows equal zero.
- IRR works only if you have some positive and negative cash flows.
- It’s also handy to remember the following:
  - If you have a set of cash flows that “flip signs” more than once, there will be multiple IRRs. We’ll deal with this later in the course.

Loan yields – Yield-to-Maturity (YTM)

- The Internal Rate of Return computed over the full remaining potential life of the loan, assuming the loan is not prepaid and is held until maturity

$$0 = -PV + \sum_{t=1}^{N} \frac{PMT}{\left(1 + YTM / m\right)^{t}} + \frac{FV}{\left(1 + YTM / m\right)^{N}}$$

Mortgage Mechanics

- Spreadsheet Fun!
  - PV function
  - FV function
  - NPV function
  - IRR function
  - PMT function

Know these terms

- Loan-to-value ratio (LTV)
- Loan principal
- Loan duration (a.k.a. loan term)
- Amortization
- Debt service

Mortgage Mechanics

- Review the payment to amortize a mortgage from "More on TVM, Six Functions of a Dollar" (esp. function 6).

$$PV = \frac{DS}{(1+i)} + \frac{DS}{(1+i)^2} + ... + \frac{DS}{(1+i)^n}$$

- where DS is the periodic debt service payment, i is the interest rate, and n is the term of the loan. This implies:

$$DS = PV \times \frac{1}{\text{Present Value of an Annuity Factor}}$$