Analysis of Factors that Influence Stellar Formation Using Smooth Particle Hydrodynamics

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The process by which protostars form from clouds of molecular hydrogen relies on multiple interconnected factors. It is generally difficult to determine the precise factors that influence the stellar formation process due to the large distances and long timescales, which requires the use of models and simulations. There is not much literature surrounding the major factors in this process. Therefore, this paper studies the effect of five of the characteristics that have the most influence – the number of particles, the molecular cloud mass, the molecular cloud radius, the end time for the simulation, and the smoothing length – on the density and pressure of the final protostar. For this analysis, a Python simulation based on smooth particle hydrodynamics was utilized to represent the spherical model of accretion during the stellar formation process. The data collected in this study confirmed the linear relationship between density and number of particles as well as the square root relationship between density and mass. Apart from these results, it was found that radius and end time had no tangible impact. Finally, the smoothing length had a complicated effect that was difficult to quantify, but after the first few values, it did not change much. Overall, it was concluded that the major factors that affect protostars are the number of particles and total mass. However, due to the smooth particle nature of the simulation and the presence of other untested factors, number of particles and mass are likely not the only factors in the stellar formation process.

Keywords: Stellar formation, protostar, smooth particle hydrodynamics, density, pressure, spherical accretion

Introduction

Background

Star formation is the process through which dense regions of interstellar matter collapse into a star. Throughout the entire process of star formation, the main cause is due to gravitational effects on different scales [1]. Within the depths of the interstellar medium, certain regions become more dense than their surroundings, generating a larger self-gravity. While this gravity is originally counteracted by gravitational force that galaxies exert on the region, the gas eventually becomes sufficiently dense such that the region's self-gravity overcomes the galactic gravitational force [1]. The final result of this process is the creation of a molecular cloud. These cold, dense clouds are defined by their abundance of molecular hydrogen [2]. The structure of molecular clouds plays a significant role in the formation of stars. Within a molecular cloud, there are numerous substructures of different sizes and masses. These substructures can range from one solar mass to thousands of solar masses, with sizes between 0.1 parsecs to multiple parsecs across. A parsec, which is the distance a star would have to be to be viewed as one arcsecond using the parallax method, is approximately 31 trillion kilometers. To put it into perspective, Proxima Centauri, which is the nearest star excluding the Sun, is 1.3 parsecs, or 4.2 light-years from Earth [1].The end result of the collapse of a molecular cloud is a young star known as a protostar.

A protostar, which is a young star that continues to gather mass from its parent molecular cloud, is the earliest phase of stellar evolution. This phase begins when the molecular cloud collapses due to self-gravity and ends when the protostar evolves into a pre-main-sequence star [2]. The evolution of a protostar proceeds with the accretion process, in which an object gathers matter and mass through gravitational effects. The simplest type of accretion is spherical accretion; this involves matter forming a singular isothermal sphere due to a lack of rotation and magnetic fields [1].

Over time, a protostar under spherical accretion passes through four main phases: gradual expansion, when the mass is less than six solar masses; swelling, when the mass is between six and ten solar masses; contraction, when the mass is between ten and thirty solar masses; and gradual expansion, when the mass is greater than thirty solar masses [3].

While the initial conditions of the molecular cloud and its collapse are important to the final star's physical properties, it is not the principal factor that determines the outcome of star formation. The accretion of the protostar and the resulting mass almost entirely dictates the properties of the protostar [1]. Because of this, the major factors that influence the stellar formation process arise during the accretion phase.

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This research paper explores some of the factors that influence the process through which a cloud of gas becomes a young protostar. Currently, research on the formation of stars is difficult due to the scale of the process. This includes the large distance from Earth and the long time required for the process. Additionally, there is generally obscuration by dust and gas and variability in the environments. Because of this, it is important to use simulations to model the accretion phase of stellar formation in order to determine the process. Specifically, this research will contribute to the understanding of the formation of stars by investigating various potential factors.

The methodology used in this study was a Python simulation to test the effects of independent variables on the density and pressure of the resulting protostar. These independent variables were the number of particles, the molecular cloud mass, the molecular cloud radius, the end time for the simulation, and the smoothing length. Because of this, the experimental hypothesis was that each of the independent variables have a significant impact on the density and pressure. On the other hand, the null hypothesis was that the independent variables do not have a significant impact on the density and pressure. Thus, the purpose of this research is to determine the influencing factors during stellar formation.

Theoretical Framework

The simulation that was used in this study involved smooth particle hydrodynamics, which is a theoretical framework that is necessary to understand the results. Smooth particle hydrodynamics is a system of physics based on the assumption that considers the clumps of gas to be smooth particles, as shown in Figure 1, in order to make it easier for hydrodynamic equations to be applied4. The region of interest that is highlighted in the figure demonstrates how the various variables are related to each other and the particle. For example, it shows that the velocity, the gradient of velocity, the viscous stress tensor, and the external force are all characteristics of a single particle whereas the support radius is related to the interaction between particles. The reason why the particles are considered to be spheres rather than other shapes is due to the simplicity of the spherical assumption; most hydrodynamic equations become complicated when there is no radial symmetry.

Since smooth particle hydrodynamics treats fluids as collections of particles, there are hydrodynamic equations that can be applied to each particle. The major relationship is Euler's equation of continuity, which states that for a particle with a velocity $\mathbf{v}$ and position $\mathbf{r}$,

$$\frac{1}{\rho}\frac{d\mathbf{v}}{dt} = -\nabla P + \mathbf{f} \tag{1}$$

In this equation, ρ represents density, P represents the internal pressure of the star in Pascals, and $\mathbf{f}$ represents the external force.

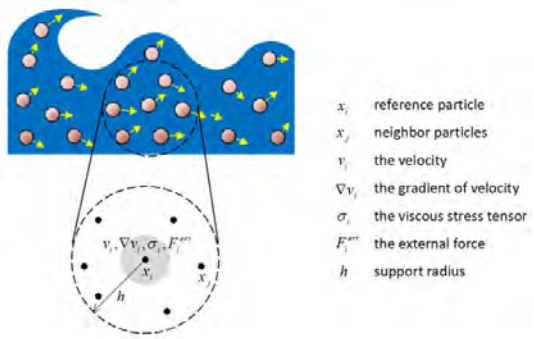

Fig. 1 Representation of gas as smooth particles in smooth particle hydrodynamics[4]

There are also specific equations for the pressure and external force, such as the following,

$$P = k\rho^{1+\frac{1}{n}} \tag{2}$$

$$\mathbf{f} = -\lambda\mathbf{r} - \nu\mathbf{v} \tag{3}$$

where k is the normalization constant, n is the polytropic index, $\lambda\mathbf{r}$ is the force due to gravity, and $\nu\mathbf{v}$ is the force due to viscosity[5].

In addition, the use of smooth particle hydrodynamics requires a smoothing function. Usually, a Gaussian kernel and its gradient are used, as follows,

$$W(\mathbf{r},h) = \frac{1}{h^3\pi^{1.5}} e^{-\frac{\|\mathbf{r}\|^2}{h^2}} \tag{4}$$

$$\nabla W(\mathbf{r},h) = \frac{-2}{h^3\pi^{1.5}} e^{-\frac{\|\mathbf{r}\|^2}{h^2}} \mathbf{r} \tag{5}$$

In this case, $\mathbf{r}$ is the position and h is the smoothing length. Both of these equations are vital due to the fact that discretization is useful when shifting the gradient onto the kernel[5]. Finally, the density and acceleration of the particles in the simulation are calculated using the following formulas,

$$\rho_i = \sum_j m W(\mathbf{r}_i - \mathbf{r}_j, h) \tag{6}$$

$$\frac{d\mathbf{v}_i}{dt} = -\sum_{i,j\neq 0}\left[m\left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right)\nabla W(\mathbf{r}_i - \mathbf{r}_j, h) + \mathbf{f}_i\right] \tag{7}$$

where m is the mass and $\frac{d\mathbf{v}_i}{dt}$ is the acceleration[5]. These equations were utilized in the Python simulation, ultimately causing the results to be influenced by the nature of smooth particle hydrodynamics.

Some key points regarding these issues revolve around the nature of the density value. In this case, the density calculated

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represents the proximity of each particle to the other particles rather than the standard mass per unit volume. This is due to the fact that the standard formula does not account for the discrete units of smooth particles that are used in this simulation. Therefore, while the units of this density are still the same as the units of standard density, these two values would not be the same for the star. This means that the pressure would also vary from the true internal pressure because the formula for pressure depends on the density. However, dimensional analysis of the equations indicate that the units of density and pressure are still kilograms per meters cubed and Pascals, respectively. Despite the fact that the density and pressure are not standard values, the density calculated in this paper is still related to the true density, which means that while the values may be different, the relationships still hold.

Methods

This research was an experimental study because it involved modifying the independent variables to determine their effects on the dependent variables. Specifically, it involved a simulation that modeled the formation of a star in order to determine the basic properties of the stellar formation process. This simulation utilized Python and matplotlib to display the animation. Since this simulation used smooth particle hydrodynamics to account for the physics of star formation, it was necessary to write functions that utilize the equations found in the previous section. Apart from this, the main code required for the simulation involved a matplotlib plot with the particles with given initial conditions but randomly-selected positions and velocities. This code can be found in Appendix A. Since the research only used a simulation, there were no ethical considerations associated with the study.

Data Collection

This simulation tested various independent variables, including the number of particles, the molecular cloud mass, the molecular cloud radius, the end time for the simulation, and the smoothing length, against the dependent variables of density and pressure. Table 1 displays the different tested values in the simulation. For each independent variable, there were ten trials of differing values; in addition to calculating the dependent variables in each trial, an image of the star formation process was produced. The analysis is detailed in the following section.

Data Analysis

After the data was collected, each set of data points was analyzed. If there was a clear mathematical relationship, then the data points were fit to a function to determine the precise relationship. This was performed through a series of steps. First, a new set of data points was created such that each value was the logarithm of the corresponding value in the original dataset. Then, the new data were plotted, forming a graph of the logarithm of the dependent variable versus the logarithm of the independent variable. Due to statistical methods, if the resulting graph was linear, then the original relationship was a power relationship with the exponent being the slope of the new line. This procedure was used to accurately determine the mathematical relationship between the independent and dependent variables. However, if there was no clear relationship, then the data was only described qualitatively.

Results

Number of Particles

In a smooth particle hydrodynamics simulation, the number of particles refers to the number of individual clumps of gas that is considered in the simulation. In other words, the total volume of the star is broken into N clumps of gas that interact with each other. As mentioned earlier, this definition of particles influences the definition of density as well. In order to determine the effect of the number of particles on the density and pressure of the particles in the star, ten trials in intervals of 100 particles were conducted. Qualitatively, as the number of particles increased, the density and pressure of the particles seemed to increase significantly, as can be seen in Figure 2. This is demonstrated by the graphs in Figure 3, which show the relationship between the dependent variables and the number of particles.

The graphs indicate a linear relationship between density and number of particles as well as a quadratic relationship between pressure and number of particles. Because of this, there is an overall quadratic relationship between pressure and density. This relationship makes sense since Equation (13) states that the pressure is proportional to the density to the power of $1 + \frac{1}{n}$. In this simulation, the value of the polytropic index, n, is one, which means that the pressure is proportional to the density to the power of two, or density squared. Therefore, the major result from these trials is that the density is directly proportional to the number of particles.

Molecular Cloud Mass

Similarly, there were ten trials that varied the mass of the initial molecular cloud by 0.5 solar masses. Figure 4 shows that as the mass increases, the density and pressure increase; however, this increase is not as significant as that of the number of particles. Additionally, the graphs, shown in Figure 5, demonstrate the quantitative relationship rather than a qualitative description.

In this case, there is a square root relationship for density versus mass and a linear relationship for pressure versus mass. However, the quadratic relationship between density and pres-

Variable	Description	Tested Values
N	Number of Particles	[100, 200, 300, 400, 500, 600, 700, 800, 900, 1000]
M	Molecular Cloud Mass (in solar masses)	[0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0]
R	Molecular Cloud Radius (in solar radii)	[0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5]
t_{Ends}	End Time of Simulation	[10, 20, 30, 40, 50, 60, 70, 80, 90, 100]
h	Smoothing Length	[0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20]

Table 1 Tested values of independent variables

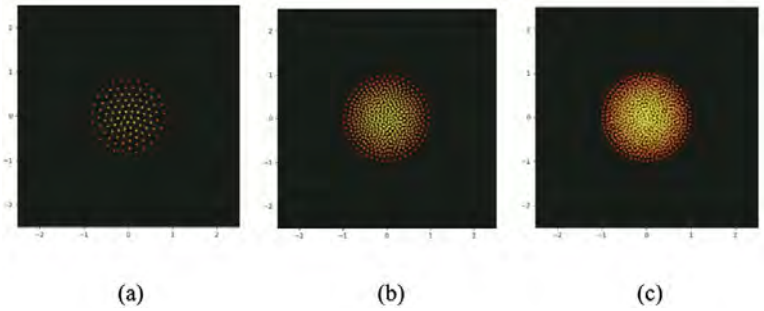

Fig. 2 Result of stellar formation simulation for (a) N=100, (b) N=500, and (c) N=1000.

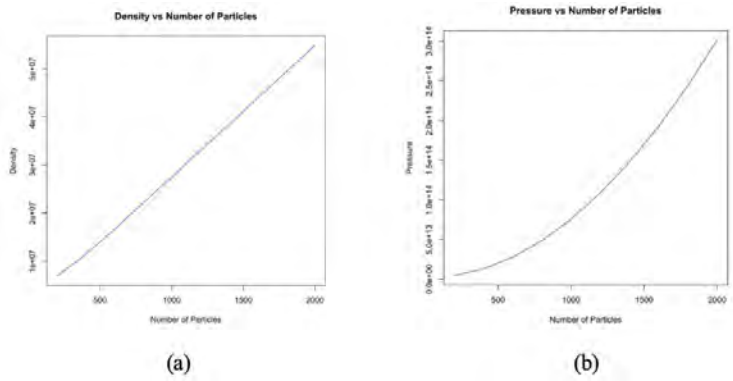

Fig. 3 Graphs of (a) density versus number of particles and (b) pressure versus number of particles.

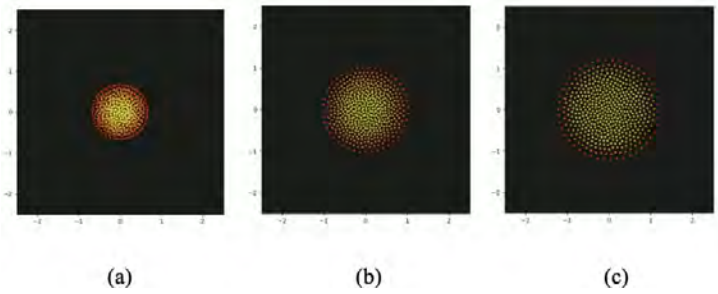

Fig. 4 Result of stellar formation simulation for (a) M=0.5 solar masses, (b) M=2.5 solar masses, and (c) M=5 solar masses.

sure still holds since the square of a square root is linear. Similar to the results of the number of particles trials, the important relationship is the square root curve in the density versus mass graph.

Molecular Cloud Radius

Unlike the previous results, varying the radius of the molecular cloud had no effect on the density or pressure of the particles. Despite changing the radius by 0.25 solar radii for ten trials,

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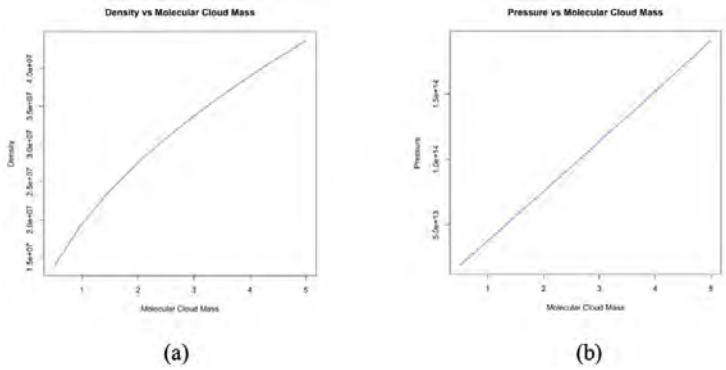

(a) (b)

Fig. 5 Graphs of (a) density versus molecular cloud mass and (b) pressure versus molecular cloud mass.

there was very little change in either value. The density was approximately $2.387.(10)^7$ kilograms per meters cubed while the pressure was approximately $5.700.(10)^{13}$ Pascals. Therefore, there is no relationship between the dependent variables and the radius.

End Time

The end time of the simulation is simply a measure of the length of the simulation. As the end time increases, the simulation is run for a longer period of time. The tested values of the end time of the simulation started at 10 and increased by 10 each trial. The results of this simulation were unique since there was no constant mathematical relationship, yet there was still some effect on the density and pressure. Similar to the molecular cloud radius trials, the density and pressure of the first trial were $2.671.(10)^7$ kilograms per meters cubed and $7.132.(10)^{13}$ Pascals., respectively. However, as the trials progressed, the density and pressure settled to approximately $2.738.(10)^7$ kilograms per meters cubed and $7.495.(10)^{13}$ Pascals, respectively, with small differences. This means that as the end time increased, there were very slight variations in both density and pressure.

Smoothing Length

The smoothing length is a property of the simulation that dictates the radius within which a particle interacts with its neighboring particles. If the smoothing length is low, then each particle does not interact much with its nearby particles. On the other hand, if the smoothing length is high, then there is a larger number of interactions that occur. When the smoothing length was changed, the results were very unpredictable. As displayed in Figure 6, it seems that for smoothing lengths of 0.02 and 0.04, there was no star that formed. At smoothing lengths between 0.06 and 0.12, the star formed seems to be similar to the stars in previous simulations. However, the outer edge of the star is disconnected from the inner layers when the smoothing length is between 0.14 and 0.2. In addition, the graphs in Figure 7 include a relationship that cannot be described mathematically.

This complicated relationship is likely caused by the presence of the smoothing length in the Gaussian kernel and, in turn, the density equation. Since it appears in the denominator of the coefficient as well as the denominator of the exponent, it does not have a straightforward correlation with the density or pressure.

Discussion

Analysis

First of all, the results indicate that there is a linear relationship between density and the number of particles. This makes sense because increasing the number of particles directly increases the interactions between nearby particles. This is due to the fact that there are simply more particles within the support radius of a specific particle. Because the formula for density involves adding up the product of the mass and Gaussian kernel for each nearby particle, it is directly proportional to the number of particles. Therefore, this relationship is valid based on the nature of smooth particle hydrodynamics and its governing equations.

The next result is that density and the molecular cloud mass have a square root relationship, which means that pressure and the molecular cloud mass have a linear relationship. This relationship can be seen in various stellar interior equations in which pressure is linearly dependent on the initial mass[6]. This confirms the result of this paper that the initial mass of the molecular cloud is one of the factors of density and pressure; additionally, it confirms that the relationships are square root and linear, respectively.

While it seems that radius should influence the stellar formation process, the key point is that this study used the radius of the initial molecular cloud rather than the radius of the star. Therefore, the only change that occurred when the radius changed was that the particles were spread farther apart initially. However, the density and pressure that are measured are the final values,

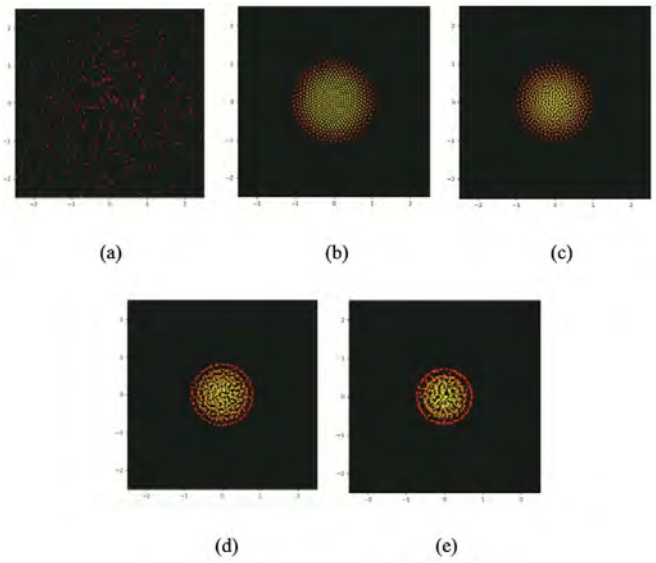

Fig. 6 Result of stellar formation simulation for (a) h=0.02, (b) h=0.06, (c) h=0.10, (d) h=0.16, and (e) h=0.2.

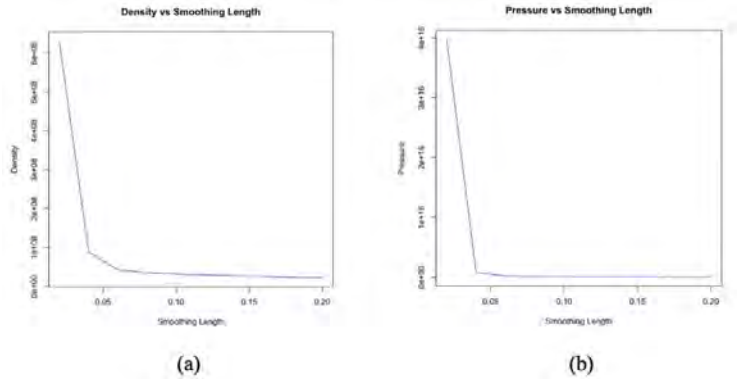

Fig. 7 Graphs of (a) density versus smoothing length and (b) pressure versus smoothing length.

which only changes very slightly due to the randomness in the simulation. Thus, it makes sense that the radius does not have an impact on the formation of the star since it is the molecular cloud radius.

Because the end time represents the time of the simulation, the negligible impact of this factor can be attributed to the stability of the star after sufficient time has passed. At first, there are relatively large changes, but as time passes, there is no measurable influence that the end time has on the density and pressure of the star since the protostar has settled. On the other hand, the smoothing length had a tangible impact throughout the trials; however, the mathematical relationship could not be determined because of its chaotic nature. This is likely because the smoothing length influences the number of particles that a given particle interacts with, but due to the randomness of the simulation, it is impossible to determine the specific effect that it would have. Therefore, while the smoothing length does change the values of density and pressure, it is unclear the nature of the impact.

Limitations and Future Research

This study used smooth particle hydrodynamics and a spherical accretion model to approximate the stellar formation process; however, there are multiple factors that are considered negligible in such a simulation. Some of these include the rotation of the gas and the presence of magnetic fields during the collapse.

When a molecular cloud collapses, it tends to spin with an angular momentum per mass of approximately 1021 to 1022 centimeters squared per second[7]. This value decreases throughout the stellar formation and evolution process until the main sequence star is formed. Despite this, the rate of rotation has a significant impact on the formation of a protostar. This is due to the fact that rotation causes a centripetal acceleration, which opposes the gravitational collapse. The increased acceleration

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in a plane that is perpendicular to the axis of rotation would lead to the formation of a circumstellar disk[7]. Since the rotation opposes the gravitational collapse, it would decrease the effect that the number of particles and mass have on the density and pressure. While these quantities would still increase, it would likely be at a rate that is less steep than those of the current relationships. Finally, the density redistribution caused by the conservation of angular momentum in a rotating protostar implies that the radius would have an effect on the process. It would influence the stability and shape of the resulting protostar, which means that it would likely lead to a complex relationship between radius and the other variables.

Similarly, the existence of magnetic fields in the initial interstellar matter would lead to more complicated results. First of all, it has been found that magnetic fields can influence the direction of large-scale gas flows in molecular clouds. This would change the accretion process since it would no longer be perfectly spherical accretion[8]. In addition to this, the pressure created by magnetic fields can counteract the gravitational collapse[8]. Similar to rotation, the existence of magnetic fields would cause a slower formation process and less steep relationships between the various factors.

Due to these two limitations, future research could clarify the true effect that the mass, the radius, and the number of particles have on the stellar formation process. It would likely involve creating a Python simulation that uses magnetohydrodynamics. While this would make the simulation more complicated, it would also produce more accurate results. The inclusion of rotation in the simulation would be necessary as well.

Despite these limitations, this research provides insight into the early stages of star formation. This is only a first step in the quest to understand the universe. The unanswered questions, from star formation and evolution to black holes and dark matter, are the vast unknowns that scientists strive to comprehend. This research is a stepping stone toward unraveling the secrets of creation itself.

Conclusion

Overall, the results of this simulation reveal various factors that influence the formation of a star. First of all, the density is linearly-related to the number of particles, which is a discrete approximation of the amount of gas present. The density and the entire mass of the molecular cloud have a square root relationship. Since the pressure is proportional to the square of the density, the pressure has a quadratic and linear relationship to the number of particles and mass, respectively. However, the radius of the initial cloud does not have any impact on the density or pressure.

Because the density and pressure seemed to oscillate slightly around a fixed value as the end time increased, it can be concluded that the star has finished forming when the end time is twenty, and the remainder of the variations is slow expansion and contraction. Finally, the complex relationship with the smoothing length is a curious phenomenon that is likely caused by the multiple occurrences of the smoothing length in the formula for the Gaussian kernel. Thus, the major physical factors that have a large impact on stellar formation are the amount and mass of the initial material. This is a significant result because it clarifies which of the five prominent factors actually have a noticeable effect on the process of star formation. The experimental hypothesis was accepted for the number of particles and the molecular cloud mass. However, the experimental hypothesis was rejected for the molecular cloud radius, the end time, and the smoothing length.

Acknowledgments

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Appendix A: Source Code of Simulation

```
import numpy as np
import matplotlib.pyplot as plt
import os
import glob
import tqdm
import cv2
import random
import math

# Constants

Ns = [200, 400, 600, 800, 1000, 1200,
1400, 1600, 1800, 2000]  # Number of particles
t = 0  # start time of simulation
tEnd = 10  # end time for simulation
dt = 0.05  # timestep

M_sun = 1.989e30
M = 2 # star mass
R_sun = 6.957e8
R = 0.75  # star radius

h = 0.1 * R  # smoothing length
k = 0.1  # equation of state constant
n = 1  # polytropic index
nu = 1  # damping
lmbda = 2.01  # lambda for gravity

# Initial Conditions

def initial(N, R):
    # Set random number generator seed

    np.random.seed(42)

    # Set randomly selected positions and velocities

    pos = np.random.randn(N, 2) * R # scaled
    with radius
    vel = np.zeros(pos.shape)
    return pos, vel

# Gaussian Smoothing Kernel

"""
  Inputs:
    x : matrix of x positions
    y : matrix of y positions
    h : smoothing length

  Output:
    w : evaluated smoothing function
  """

def kernel(x, y, h):
    # Calculate |r|

    r = np.sqrt((x ** 2) + (y ** 2))

    # Calculate value of smoothing function

    w = (1.0 / ((h ** 3) * (np.pi ** 1.5))) *
    np.exp(-(r ** 2) / (h ** 2))
    return w

# Gaussian Smoothing Kernel Gradient

"""
  Inputs:
    x : matrix of x positions
    y : matrix of y positions
    h : smoothing length

  Output:
    wx, wy : evaluated gradient
  """

def grad_kernel(x, y, h):
    # Calculate |r|

    r = np.sqrt((x ** 2) + (y ** 2))

    # Calculate scalar part of gradient

    n = (-2.0 / ((h ** 5) * (np.pi ** 1.5)))
    * np.exp(-(r ** 2) / (h ** 2))

    # Calculate vector parts of gradient

    wx = n * x
    wy = n * y

    return wx, wy

# Magnitude of Distance in Density Equation

"""
  Inputs:
    ri : M x 2 matrix of positions
    rj : N x 2 matrix of positions

  Output:
    dx, dy : M x N matrix of separations
  """

  def magnitude(ri, rj):
    M = ri.shape[0]
    N = rj.shape[0]

    # Calculate x, y of ri

    rix = ri[:, 0].reshape((M, 1))
    riy = ri[:, 1].reshape((M, 1))

    # Calculate x, y of rj

    rjx = rj[:, 0].reshape((N, 1))
    rjy = rj[:, 1].reshape((N, 1))

    # Calculate separations

    dx = rix - rjx.T
    dy = riy - rjy.T

    return dx, dy
```


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```
# Density Equation


"""
  Inputs:
    r : M x 3 matrix of sampling locations positions
    pos : N x 3 matrix of particle positions
    m : particle mass
    h :smoothing length

  Output:
    rho : M x 1 vector of densities
  """


def density(r, pos, m, h):
    M = r.shape[0]

    # Calculate density

    dx, dy = magnitude(r, pos)
    rho = np.sum(m * kernel(dx, dy, h),
    1).reshape((M, 1))

    return rho


# Pressure Equation


"""
  Inputs:
    rho : vector of densities
    k : equation of state constant
    n : polytropic index

  Output:
    P : pressure
  """


def pressure(rho, k, n):
    # Calculate pressure

    P = k * (rho ** (1 + (1 / n)))

    return P


# Acceleration Equation


"""
  Inputs:
    pos : N x 2 matrix of positions
    vel : N x 2 matrix of velocities
    m : particle mass
    h : smoothing length
    k : equation of state constant
    n : polytropic index
    lmbda : external force constant
    nu : viscosity

  Output:
    a : N x 2 matrix of accelerations
  """


def acceleration(pos, vel, m, h, k, n, lmbda, nu):
    N = pos.shape[0]

    # Calculate densities

    rho = density(pos, pos, m, h)

    # Calculate pressures

    P = pressure(rho, k, n)

    # Calculate pairwise distances and gradients

    dx, dy = magnitude(pos, pos)
    dWx, dWy = grad_kernel(dx, dy, h)

    # Add pressure contribution to acceleration

    ax = -np.sum((m * ((P / (rho ** 2)) +
    (P.T / (rho.T ** 2))) * dWx),
    1).reshape((N, 1))
    ay = -np.sum((m * ((P / (rho ** 2)) +
    (P.T / (rho.T ** 2))) * dWy),
    1).reshape((N, 1))

    # Pack acceleration components

    a = np.hstack((ax, ay))

    # Add external forces

    a += -(lmbda * pos) - (nu * vel)

    return a


# Folder Creation

if not os.path.exists('output'):
    os.mkdir('output')
else:
    files = glob.glob('output/*.png')
    for f in files:
        os.remove(f)

results_file = open("results/N/data.txt", "w")
results_file.write("N\tAverage
Density\tAverage Pressure\n")  # Add a header row

# Plot

for N in Ns:
  m = M / N
  Nt = int(np.ceil(tEnd / dt))
  pos, vel = initial(N, R)

  for i in tqdm.tqdm(range(Nt)):
      acc = acceleration(pos, vel, m, h, k, n, lmbda, nu)
      vel += acc * dt
      pos += vel * dt
      scaled_pos = pos*R_sun
      scaled_m = m*M_sun
```

```
        scaled_h = h*R_sun
        rho = density(scaled_pos, scaled_pos,
        scaled_m, scaled_h)
        # rho = density(pos, pos, m, h)

        fig, ax = plt.subplots(figsize=(6, 6))
        plt.sca(ax)
        plt.cla()

        cval = np.clip((rho - np.min(rho))
        /(np.max(rho) - np.min(rho)), 0, 1)
        plt.scatter(pos[:, 0], pos[:, 1],
        c=cval, cmap=plt.cm.autumn, s=5,
        alpha=0.75)

        ax.set(xlim=(-2.5, 2.5), ylim=(-2.5, 2.5))
        ax.set_aspect('equal', 'box')
        ax.set_facecolor('black')
        ax.set_facecolor((.1, .1, .1))

        plt.savefig(f'output/{i}.png')
        plt.close()

    # Animation

    img_array = []

    print("Reading Frames")

    imgs_list = glob.glob('output/*.png')
    lsorted = sorted(imgs_list, key=lambda x:
    int(os.path.splitext(x[7:])[0]))

    for filename in tqdm.tqdm(lsorted):
        img = cv2.imread(filename)
        height, width, layers = img.shape
        size = (width, height)
        img_array.append(img)

    out = cv2.VideoWriter('results/N/' + str(N)
    + '.mp4', cv2.VideoWriter_fourcc(*'MP4V'),
    30, size)

    print("Writing Frames")
    for i in tqdm.tqdm(range(len(img_array))):
        out.write(img_array[i])

    rho_avg = sum(rho)
    p_avg = pressure(rho_avg, k, n)

    print("Average density for N = ", N, ": ", rho_avg)
    print("Average pressure for N = ", N, ": ", p_avg)
    results_file.write(str(N) + "\t" +
    str(rho_avg[0]) + "\t" + str(p_avg[0]) +
    "\n")

    out.release()
```

* This code is specifically for changing the number of particles. When testing other independent variables, the code was slightly changed to cycle through the values of the other variables.

Appendix B: Raw Data

N	Average Density	Average Pressure
200	6847006.918	4688150373815
400	11380893.41	12952473469613
600	16644591.07	27704241197426
800	22049701.9	48618935394482
1000	27496283.04	75604558100479
1200	32963013.22	108656024031713
1400	38435580.75	147729386731678
1600	43918086.03	192879828014077
1800	49406944.51	244104616629334
2000	54892425.29	301317835438624

Table 2 Raw data for trials of number of particles

M	Average Density	Average Pressure
0.5	13765077.33	18947735402990
1	19446611.96	37817071660484
1.5	23810708.96	56694986101193
2	27496283.04	75604558100479
2.5	30756403.81	94595637526846
3	33713503.76	113660033595429
3.5	36437744.37	132770921463278
4	38992063.85	152038104334599
4.5	41395809.05	171361300689612
5	43685012.93	190838035446736

Table 3 Raw data for trials of molecular cloud mass

R	Average Density	Average Pressure
0.25	23477256.66	55118158032629
0.5	23675515.34	56053002665344
0.75	23776223.76	56530881634056
1	23843273.43	56850168791871
1.25	23896719.65	57105321014525
1.5	23940611.61	57315288419333
1.75	23980281.89	57505391939979
2	24014964.17	57671850397199
2.25	24048858.83	57834761103449
2.5	24078591.49	57977856796963

Table 4 Raw data for trials of molecular cloud radius

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tEnd	Average Density	Average Pressure
5	26705749.77	71319707058526
10	27496283.04	75604558100479
15	27381751.55	74976031803695
20	27376740.8	74948593675357
25	27377242.72	74951341884471
30	27377166.59	74950925059898
35	27377088.91	74950499716353
40	27377028.78	74950170461913
45	27376980.37	74949905437550
50	27376940.31	74949686065910

Table 5 Raw data for trials of end time

h	Average Density	Average Pressure
0.02	630302434.1	3.97E+16
0.04	88571073.28	784483502170217
0.06	42920970.68	184220972380198
0.08	35747911.55	127791318035279
0.1	31910344.7	101827009867894
0.12	29230415.96	85441721746313
0.14	27173683.07	73840905156118
0.16	25542776.52	65243343239536
0.18	24216049.2	58641703870849
0.2	23132536.62	53511425041986

Table 6 Raw data for trials of smoothing length