WMML
Meet #5
Mar. 29, 2022

Name _____

School _____

Arithmetic and Number Theory

240

1) What is the smallest number of people that can be divided equally into groups of 15 people and also divided equally into groups of 48 people?

1._____

$$15 = 3 \cdot 5$$

$$48 = 2^4 \cdot 3$$

$$lcm(15,48) = 2^4 \cdot 3 \cdot 5 = 240$$

880

2) Find the sum of the five smallest positive multiples of 8 that are perfect squares.

2._____

$8 = 2^3$, but perfect squares must have all even powers in the prime factorization, so all multiples must contain 2^4 as a factor.

$$(2^4 \cdot 1^2) + (2^4 \cdot 2^2) + (2^4 \cdot 3^2) + (2^4 \cdot 4^2) + (2^4 \cdot 5^2)$$

$$= 16 + 64 + 144 + 256 + 400 = 880$$

64

3) How many positive integers are divisors of 999,999?

3._____

$$999{,}999 = 3^3 \cdot 7^1 \cdot 11^1 \cdot 13^1 \cdot 37^1$$

$$(3+1)(1+1)(1+1)(1+1)(1+1) = 64$$

WMML
Meet #5
Mar. 29, 2022

Name _____

School _____

Algebra 1

1) What is the domain of the function $f(x) = \sqrt{-2x+7}$?

$$-2x + 7 \geq 0$$
$$2x \leq 7$$
$$x \leq \frac{7}{2}$$

$x \leq \frac{7}{2}$

1._____

2) What is the maximum value that the function $f(x) = 2022 - 30x - x^2$ can be?

$$x = \frac{-b}{2a} = \frac{30}{2(-1)} = -15$$
$$f(-15) = 2022 - 30(-15) - (-15)^2$$
$$= 2022 + 450 - 225 = 2247$$

2247

2._____

3) Let $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 - bx + c$. If $f(1) = g(1) + 2$ and $f(2) = 2$, find $g(2)$.

$f(1) = g(1) + 2 \longrightarrow a + b + c = a - b + c + 2 \longrightarrow b = 1$

$f(2) = 2 \longrightarrow 4a + 2b + c = 2 \longrightarrow 4a + c = 2 - 2b = 0$

$g(2) = 4a - 2b + c = (4a + c) - 2b = 0 - 2 = -2.$

-2

3._____

WMML
Meet #5
Mar. 29, 2022

Name _____

School _____

Geometry

5

1) If the following lines are graphed in the same coordinate plane, how many points of intersection are created?

1._____

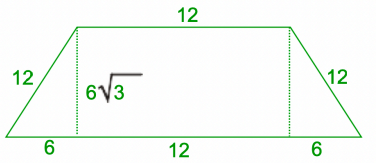
$$y = 5x - 1$$
$$y = 2x + 1$$
$$y = -3x + 2$$
$$y = 5x + 3$$

Two lines are parallel, which creates two points of intersection with each of the other lines as transversals. The two transversals also intersect once, making 5 points of intersection.

$108\sqrt{3}$

2) Determine the exact area of an isosceles trapezoid whose legs each measure 12, larger base measures 24, and whose base angles measure $60°$.

2._____

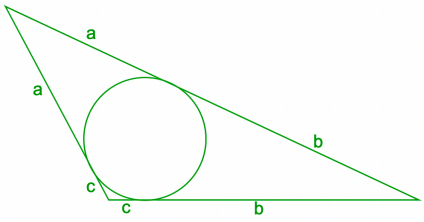


$$\text{Area} = \frac{1}{2}(12 + 24)(6\sqrt{3}) = 108\sqrt{3}$$

1

3) The sides of a triangle have lengths 10, 12, and 18. Find the distance from the midpoint of the longest side to the point at which that side is tangent to a circle inscribed in the triangle.

3._____



$$a + b = 18$$
$$b + c = 12$$
$$a + c = 10$$
$$a - c = 18 - 12 = 6$$
$$2a = 16$$
$$a = 8$$

$$9 - 8 = 1$$

WMML
Meet #5
Mar. 29, 2022

Name _____

School _____

Algebra 2

1) If the equation of the line that passes through the points $(-3,-5)$ and $(4,30)$ is written in the form $y = mx + b$, then what is the value of $\sqrt{m^2 + b^2}$?

$$m = \frac{30-(-5)}{4-(-3)} = \frac{35}{7} = 5$$

$$y - 30 = 5(x-4) \quad \longrightarrow \quad y = 5x + 10$$

$$\sqrt{5^2 + 10^2} = \sqrt{125} = 5\sqrt{5}$$

$5\sqrt{5}$

1._____

2) Find all ordered pairs (x, y) that satisfy both $x^2 + xy = 28$ and $y^2 + xy = -12$.

$$x^2 + xy + y^2 + xy = x^2 + 2xy + y^2 = (x+y)^2 = 28 + (-12) = 16$$

$$x + y = 4: \quad y = 4 - x \quad \longrightarrow \quad x^2 + x(4-x) = 28 \quad \longrightarrow \quad x = 7$$

$$y = 4 - 7 = -3$$

$$x + y = -4: \quad y = -4 - x \quad \longrightarrow \quad x^2 + x(-4-x) = 28 \quad \longrightarrow \quad x = -7$$

$$y = -4 - (-7) = 3$$

$(7, -3)$

and

$(-7,3)$

2._____

3) Determine the value of k that makes both solutions of the equation $kx^2 - 12x + 8 = 0$ equal.

$$(-12)^2 - 4(k)(8) = 0$$

$$144 - 32k = 0$$

$$32k = 144$$

$$k = \frac{144}{32} = \frac{9}{2}$$

$\frac{9}{2}$

3._____

WMML
Meet #5
Mar. 29, 2022

Name _____

School _____

Trigonometry and Complex Numbers

1) What is the value of $|5-3i|$?

$\sqrt{34}$

1._____

$$|5-3i| = \sqrt{5^2+3^2} = \sqrt{34}$$

$\frac{3+\sqrt{2}}{6(2+\sqrt{2})}$

2) If $3+\sqrt{2}$ is a solution to the equation $x^2 + (\sin(\theta)\tan(\theta))x + 1 = 0$, then what is the value of $\csc(\theta)\cot(\theta)$?

2._____

$3+\sqrt{2}$	1	b	1
		$3+\sqrt{2}$	$(3+\sqrt{2})(3+\sqrt{2}+b)$
1		$3+\sqrt{2}+b$	0

$$(3+\sqrt{2})(3+\sqrt{2}+b)+1=0$$

$$9+3\sqrt{2}+3b+3\sqrt{2}+2+b\sqrt{2}+1=0$$

$$b = \frac{-12-6\sqrt{2}}{3+\sqrt{2}} \quad \longrightarrow \quad \frac{1}{b} = -\frac{3+\sqrt{2}}{6(2+\sqrt{2})}$$

$\frac{-23}{9}$

3) If a and b are solutions to the equation $3x^2 - 5x + 8$, then what is the value of $a^2 + b^2$?

3._____

$$\frac{5 \pm \sqrt{(-5)^2 - 4(3)(8)}}{2(3)} = \frac{5 \pm \sqrt{-71}}{6}$$

$$a^2 = \left(\frac{5}{6} + \frac{\sqrt{-71}}{6}\right)\left(\frac{5}{6} + \frac{\sqrt{-71}}{6}\right) = \frac{25}{36} + \frac{10\sqrt{-71}}{36} + \frac{-71}{36}$$

$$b^2 = \left(\frac{5}{6} - \frac{\sqrt{-71}}{6}\right)\left(\frac{5}{6} - \frac{\sqrt{-71}}{6}\right) = \frac{25}{36} - \frac{10\sqrt{-71}}{36} + \frac{-71}{36}$$

$$a^2 + b^2 = \frac{25}{36} + \frac{25}{36} + \frac{-71}{36} + \frac{-71}{36} = \frac{-92}{36} = \frac{-23}{9}$$

WMML
Meet #5
Mar. 29, 2022

Name _____

School _____

Precalculus

2022

1) If the first two terms of an arithmetic sequence are -9 and -6, find the 678^{th} term of the sequence.

1._____

$$d = -6-(-9) = 3$$

$$-9+3(677) = 2022$$

9

2) The continued radical below converges to a positive number. What is that number?

2._____

$$\sqrt{54+3\sqrt{54+3\sqrt{54+3\sqrt{54+\cdots}}}}$$

$$x = \sqrt{54+3\sqrt{54+3\sqrt{54+3\sqrt{54+\cdots}}}} = \sqrt{54+3x}$$

$$x^2 = 54+3x$$

$$0 = x^2-3x-54 = (x-9)(x+6)$$

$$x = 9$$

$\log_5(3)$

3) Let $f(x) = \sqrt{5^x}$. Find the value of k if $f(x+2k) = 3 \cdot f(x)$. Write your answer in the form $\log_a(b)$.

3._____

$$5^{\frac{1}{2}(x+2k)} = 3 \cdot 5^{\frac{1}{2}x}$$

$$5^{\frac{1}{2}x+k-\frac{1}{2}x} = 3$$

$$5^k = 3$$

$$k = \log_5(3)$$