Name _____

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Arithmetic and Number Theory

1) Find the units digit of $2019^{11} + 2019^{12}$.

1._____

2) Find the units digit of $2019(7^{2019} - 6^{2019})^{2019}$.

2._____

3) Let x be a six-digit number whose first three digits are 523 such that the integer is divisible by each of 7, 8, and 9. Find the sum of all possible values of x.

3._____

Algebra 1

1) The sum of the squares of the roots of the equation

$x^2 + 2hx = 3$ is 10. Find $|h|$.

1._____

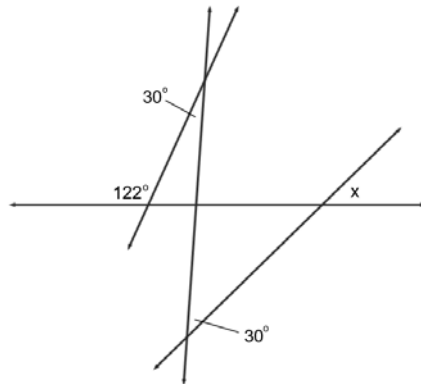2) Simplify $\sqrt{4 + \sqrt{7}} - \sqrt{4 - \sqrt{7}}$.

2._____

3) AfterMath Airlines currently charges $200 per ticket, and sells 40,000 tickets in a year. For every $10 they increase the ticket price, they sell 1000 fewer tickets. How much should they charge to maximize their revenue?
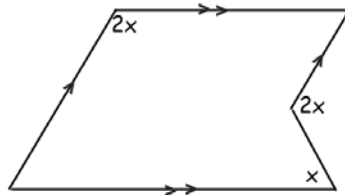
3._____

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Geometry

1) What is the value of x in the diagram below?
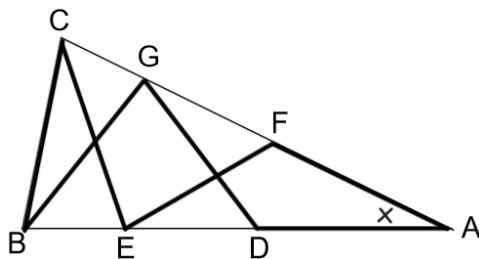
1._____

30°

122°

x

30°

2) What is the value of x in the diagram below?

2._____

2x

2x

x

3) In $\triangle ABC$ in the diagram below, D and E are points on side $\overline{AB}$, and F and G are points on side $\overline{AC}$, such that $AD = DG = GB = BC = CE = EF = FA$. Find the value of x.

3._____

C

G

F

x

B   E   D   A

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Algebra 2

1) Find the value of k if 3 is a root of $f(x) = x^3 + 3x^2 + kx - 9$.

1._____

2) What is the sum of all possible values of c for which the equation $\frac{x^2-x+c}{x^2-8x+15}$ has exactly one vertical asymptote?

2._____

3) Find the sum of the real roots of the equation

$$(2 + (2 + (2 + (2 + x)^2)^2)^2)^2 = 15129.$$

3._____

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Trigonometry and Complex Numbers

1) Let $\triangle ABC$ be a right triangle with $\angle ABC = 90°$, $AB = 12$, and $AC = 20$. Find the value of

$$\frac{\sin(A)\cos(A)}{\tan(A)}.$$

1._____

2) Four complex numbers lie at the vertices of a square in the complex plane. Three of the numbers are $1 + 2i$, $-2 + i$, and $-1 - 2i$. What is the fourth number?

2._____

3) Right triangle ABC has $\angle ABC = 90°$. Let the midpoint of $\overline{AB}$ be M. The perpendicular bisector of $\overline{AB}$ intersects $\overline{AC}$ at D, and $\sin(\angle MDB) = \frac{1}{5}$. If $MD = 1$, then what is AC?

3._____

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Precalculus

1) Find all values of a for which the line $y = x + a$ passes through the vertex of the parabola $y = x^2 + a^2$.

1._____

2) Find the area of the region enclosed by the graph of the equation $y = \frac{(x-4)^2}{12} + \frac{(y+2)^2}{12} = 1$.

2._____

3) A set of n consecutive positive integers beginning with 1 is written on a blackboard. One number is erased, and the average of the remaining $n - 1$ integers is $35\frac{7}{17}$. What number is erased?

3._____

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Team Round

1) Find the value of n if 2^n is the highest power of 2 that evenly divides 30!.

1._____

2) Find the real values of K for which the equation
$$x = K^2(x-1)(x-2) \text{ has real roots.}$$

2._____

3) A triangle has sides measuring 13 cm, 13 cm, and 10 cm. A second triangle is drawn with sides measuring 13 cm, 13 cm, and x cm, where x is a whole number other than 10. If the two triangles have equal areas, what is the value of x?

3._____

4) Find all values of x such that $2\log_3(x+4) - \log_3(4x-11) = 2$.

4._____

5) Find all acute angles θ such that
$$\sin(\theta) + \sin(2\theta) = \cos(\theta) + \cos(2\theta).$$

5._____

6) Find r if r is positive and the line whose equation is $x + y = r$ is tangent to the circle whose equation is $x^2 + y^2 = r$.

6._____