WMML
Meet #1
Oct. 1, 2019

Name _____

School _____

Arithmetic and Number Theory

1) Find the smallest composite number that has no prime factors less than 10.

1._____121_____

The answer is the product of the two smallest prime numbers greater than 10, and they don't need to be unique.

$$11 \times 11 = 121$$

2) Each week, between 30 and 50 mathletes show up for math practice. Usually the mathletes break up into equal sized groups, but this week the two coaches noticed that the mathletes could not break up into groups of equal size. If the two coaches joined the mathletes, then they still could not break up into groups of equal size. How many mathletes showed up for practice?

2._____41_____

The answer must be the lower number in a pair of twin primes between 30 and 50. The only possibility is 41.

3) What is the smallest possible 4-digit number that gives a quotient of 432 with a remainder of 2 when divided by a positive 1-digit number?

3._____1298_____

$$a = b \cdot 432 + 2$$
$$434 = 1 \cdot 432 + 2$$
$$866 = 2 \cdot 432 + 2$$
$$1298 = 3 \cdot 432 + 2$$

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Algebra 1

1) A gardener is planning a rectangular garden area in a community garden. His garden will be next to an existing 12-ft fence. The gardener has a total of 44 ft of fencing to build the other three sides of the garden. How long will the garden be if the width is 12 ft?

$$x + 12 + x = 44$$
$$x = 16$$

1. 16 feet

2) Write the equation of the line (in slope-intercept form) that has the same slope as the line $3x - 5y = 7$ and the same y-intercept as the line $2y - 9x = 8$.

Slope from: $y = \frac{3}{5}x - \frac{7}{5}$

Y-intercept from: $y = \frac{9}{2}x + 4$

Answer: $y = \frac{3}{5}x + 4$

2. $y = \frac{3}{5}x + 4$

3) Suppose that $(0,4)$ and $(2,36)$ are on the graph of an exponential function in the form of $y = a \cdot b^x$. What is the value of $y$ when $x = 4$?

$$4 = a \cdot b^0 = a$$
$$36 = 4 \cdot b^2$$
$$b = 3$$
$$y = 4 \cdot 3^4 = 324$$

3. 324

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Geometry

1) You can block out the moon by holding a coin up at a distance from your eye that is 110 times the diameter of the coin. The moon is roughly 3640 kilometers in diameter. How far away is it?

1.____400,400 km_____

$$110 = \frac{x}{3640}$$
$$x = 400{,}400\ km$$

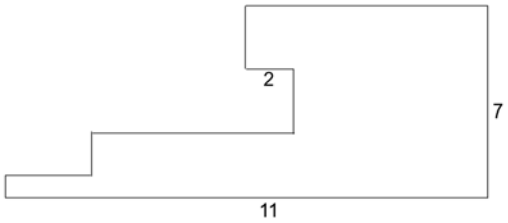
2) Two angles in an equilateral triangle have measures $3x + 27°$ and $2y - 4°$. Find $x + y$.

2._____43_____

$$3x + 27 = 60$$
$$x = 11$$
$$2y - 4 = 60$$
$$y = 32$$
$$11 + 32 = 43$$

3) All sides of the building shown below meet at right angles. If three of the sides measure 2 meters, 7 meters, and 11 meters as shown, then what is the perimeter of the building in meters?

3._____40_____




$$7 + 7 + 11 + 11 + 2 + 2 = 40$$

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Algebra 2

1) Let $f(x) = \sqrt{4-x}$ and $g(x) = \sqrt{2x-6}$. What is the domain of $f \cdot g$?

$3 \leq x \leq 4$

1._____

Domain of $(x) = \sqrt{4-x} : x \leq 4$

Domain of $(x) = \sqrt{2x-6} : x \geq 3$

Domain of product is the overlap: $3 \leq x \leq 4$

2) What is the product of the quotient and the remainder when $3x^2 + 3x - 7$ is divided by $x + 6$?

$249x - 1245$

2._____

| -6 | 3 | 3 | -7 |
|---|---|---|---|
| | | -18 | 90 |
| | 3 | -15 | 83 |

$83(3x - 15) = 249x - 1245$

3) Find the value of $x$ if $x$ is positive and $x - 1$ is the reciprocal of $x + \frac{1}{2}$.

3._____3/2_____

$$\frac{1}{x+\frac{1}{2}} = x - 1$$
$$1 = x^2 - \frac{1}{2}x - \frac{1}{2}$$
$$2x^2 - x - 3 = 0$$
$$(x+1)(2x-3)$$

$x = -1$ and $x = \frac{3}{2}$, but $x$ must be positive.

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Trigonometry and Complex Numbers

1) What is the value of $\sec(2745°)$?

$-\sqrt{2}$

1._____

$$\sec(2745°) = \sec(225°) = -\frac{1}{\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

2) $ABC$ is a triangle such that $AC = 5$, $BC = 11$, and $\angle ACB = 30°$. What is the area of $\Delta ABC$?

2._____55/4_____

$$Area = \frac{1}{2}(11)(5\sin(30°)) = \frac{55}{4}$$

$\sqrt{2}$

3) Define a sequence of complex numbers by the following:

3._____

$z_1 = 0,\ z_{n+1} = z_n^2 + i$ for $n \geq 1$.

How far away from the origin is $z_{111}$?

$$z_2 = 0^2 + i = i$$
$$z_3 = i^2 + i = -1 + i$$
$$z_4 = (-1 + i)^2 + i = -i$$

After the first two terms the pattern alternates between $-1 + i$ and $-i$, landing on $z_{111} = -1 + i$.

$$|z_{111}| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

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Precalculus

1) Find the degree of $f(g(x))$ given that $f$ and $g$ are polynomials such that the degree of $f$ is $m$ and the degree of $g$ is $n$, where $m$ and $n$ are nonnegative integers.

1._____*mn*_____

By the power rule of exponents, the degree would be the product of the degrees: $mn$.

2) Let $\boldsymbol{u}$ and $\boldsymbol{v}$ be vectors such that $\boldsymbol{u} = 5\boldsymbol{i} - 12\boldsymbol{j}$ and $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j}$.

Find the dot product $(\boldsymbol{u} - \boldsymbol{v}) \cdot (\boldsymbol{u} + \boldsymbol{v})$.

2._____144_____

$$\left(\begin{pmatrix}5\\-12\end{pmatrix} - \begin{pmatrix}3\\4\end{pmatrix}\right) \cdot \left(\begin{pmatrix}5\\-12\end{pmatrix} + \begin{pmatrix}3\\4\end{pmatrix}\right)$$
$$= \begin{pmatrix}2\\-16\end{pmatrix} \cdot \begin{pmatrix}8\\-8\end{pmatrix} = 16 + 128 = 144$$

3) Let $f(x^2 + 1) = x^4 + 5x^2 + 3$. What is $f(x^2 - 1)$?

3._____$x^4 + x^2 - 3$_____

Set $t = x^2 + 1$, then $x^2 = t - 1$ and $x^4 = (t - 1)^2$.

$$f(t) = f(x^2 + 1) = x^4 + 5x^2 + 3 = (t - 1)^2 + 5(t - 1) + 3$$
$$= t^2 - 2t + 1 + 5t - 5 + 3 = t^2 + 3t - 1$$

So $f(x^2 - 1) = (x^2 - 1)^2 + 3(x^2 - 1) - 1 = x^4 + x^2 - 3$