Modern Playoff Tournament-Design Analysis Across Sports: An Operations Research Study using Statistical Learning and Monte-Carlo Simulation

1 Introduction

Sports have long been a popular form of entertainment, gaining more viewers with the digital age. The 2020 Summer Olympics attracted ~3-billion viewers, the 2022 FIFA World Cup Final ~1.5-billion, and the 2025 Super Bowl a record 127.7-million US-viewers.

Rules are fundamental to sport's functioning. Tournament-design refers to the set-of-rules governing tournament-structure—encompassing contestant-group, format, schedule, and ranking. Thus, "designing an optimal-contest is...of significant financial concern for organizers, participating-individuals, and teams, and...of consuming personal-interest for millions of fans" (Szymanski,2003). League tournament-design can raise sails or sink ships, where leagues globally have adopted new/hybrid playoff-formats recently.

We evaluate newly-adopted/hybrid versus classical tournament-designs across sports using three key-criteria: 1)**Efficacy**: revelation of true team-rankings; 2)**Fairness**: team win-likelihoods proportional to team-strengths; 3)**Attractiveness**: excitement/exposure/financial-viability.

2 Methods

Assume teams can be ranked best-to-worst and are numbered accordingly $1, 2, \cdots, n$. Let $P = (p_{ij})$ denote the probability-matrix, where p_{ij} represents probability team$_i$ beats team$_j$, $1 \leq i < j \leq n$.

2.1 Four-Team Knockout Playoffs

We first consider the 4-team single-elimination knockout playoff-design. Let $A = [(1, 4), (2, 3)]$, $B = [(1, 2), (3, 4)]$, $C = [(1, 3), (2, 4)]$ be the distinct possible-draws, where "A" is usually

considered best-draw *vis-a-vis* criteria (See Fig.1 for supporting result). A is used, e.g., in *Olympic Volleyball, NBA Play-In Tournament*, and other prominent sport-tournaments.

Page-Playoff-System (PPS) is an alternative 4-team hybrid knockout playoff-design, used in professional cricket (Indian Premier League, Pakistan Super League), curling, and soft-ball–providing possibly-asymmetric advantage and disadvantage to top-two seeds. Performance of PPS *vis-a-vis* criteria is heretofore-unknown. Fig.1 shows tournament-design draw-structures for A, PPS and 1-million tournament simulation-results for A, B, C, PPS.

Let $s_i \in \mathbb{R}$ be team$_i$ strength, $s_1 \geq s_2 \geq \cdots \geq s_n > 0$, $i = 1, 2, \cdots, n$. Using a Bradley-Terry nonparametric-statistical rank-learning model, we define $p_{ij} = \dfrac{s_i}{s_i + s_j}$. Monte-Carlo simulations verify theorem's results by simulating 1-million tournaments, $s_i \sim U(a_i, b_i)$.

2.2 College Football Playoffs

Furthermore, we analyze the new 12-team college football playoff-format and consider changes from 2024-25-to-2025-26. We simulate college football playoffs under 2025-26 rules, with-and-without home-advantage, for Round-1 games, and under 2024-25 playoff-rules, where four highest-ranked conference-champions received a first-round bye. See Fig.2 for details.

3 Results

Fig.1 shows PPS is an (optimizing-)elaboration of Draw-B with top-bracket double-elimination. PS increases chances of top-team(s) winning in four-team playoffs compared to efficacy/fairness-optimized single-elimination tournaments. It is especially important in tournaments with longer initial rounds, like round-robins, to recognize performance of top-seeded teams.

Fig.2 confirms that CFP made the correct decision to drop the first-round bye-rule for top-ranked conference-champs vis-a-vis criteria. The new Top-4 overall seed bye-rule improves efficacy/fairness.

4 Conclusion

This project prioritizes practitioners' interests by analyzing current tournament designs instead of introducing complex or model-based designs that organizers are unlikely to adopt. We believe the content is accessible and has the potential to influence practice.
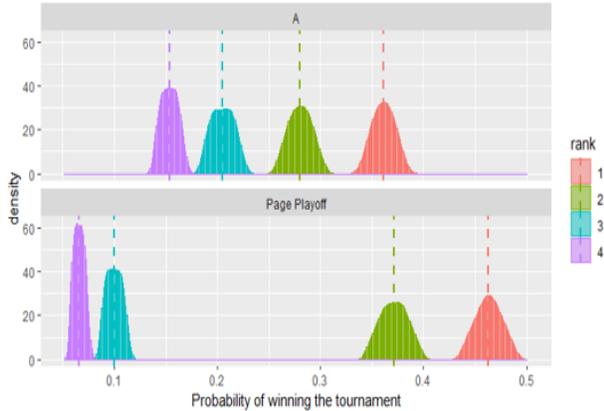
Page Playoff System

QUALIFIER 1
1st vs 2nd

Loser

Winner

QUALIFIER 2
Q1 Loser vs E Winner

Winner

FINAL
Q1 Winner vs Q2 Winner

ELIMINATOR
3rd vs 4th

Winner

4 Team Single Elimination (Draw A)

SEMI FINAL 1
1st vs 4th

Winner

FINAL

SEMI FINAL 2
2nd vs 3rd

Winner

Theorem: Assume (AI) $0.5 \leq p_{ij} \leq 1$, $0 \leq p_{ji} \leq 0.5$ for $i \leq j$; (A2) $p_{ij} + p_{ji} = 1$ (No Draws); (A3) p_{ij} is non-decreasing in j. Then among draws A, B, C, and PPS:

(X1) PPS maximizes probability of best-team winning tournament, where A maximizes among classical-designs A,B,C.

(X2) A and PPS are fair (chance of winning tournament proportional to team strength).

### Page Playoff System maximizes the chance of top team winning and is fair

Draw A is also fair but has smaller win probability than Page playoff. Dotted lines represent mean.
A million tournament simulations with spaced out strengths and theorem conditions satisfied.

1a

Page Playoff System and draw A show robustness

A million tournament simulations with overlapping strengths for top 2 seeds and
theorem conditions are not satisfied. Note the overlap in win probability distributions.
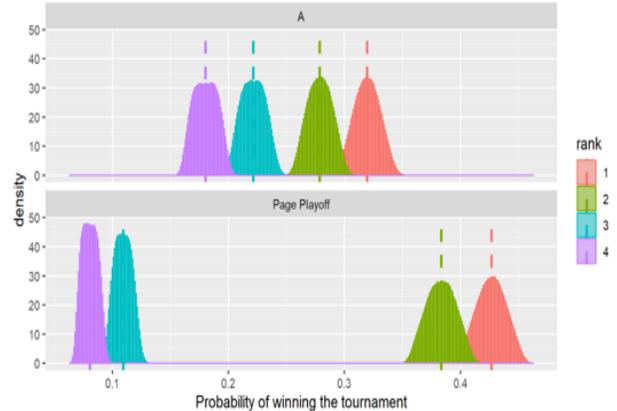
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Figure 1: PPS v. Draw-A Simulations

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	1	2	3	4	5	6	7	8	9	10	11	12
2025-26	22.7	18.5	15.4	12.4	8.4	6.7	4.9	3.9	2.7	2.0	1.5	1.0
2025-26 (w/o home advantage)	22.4	18.6	15.3	12.5	8.5	6.6	4.9	3.5	2.9	2.0	1.7	1.2
2024-25 (conf. champs got 1st round bye)	**17.2**	10.5	6.24	3.1	**19.9**	14.7	10.3	6.8	4.2	3.2	2.2	1.4

Seed

*Probability estimates (in percentage %) of winning the tournament, for given seed and tournament design, from 50000 simulated tournaments. The tournament game outcomes are generated conditional on the simulated θ_i with equally spaced strengths & equal and moderate uncertainty. An interesting bi-modal pattern emerges for the 24-25 season where conference champs got seeded higher. Since conference champs could have been ranked lower but got moved to the higher seed, the shift disrupts the probability distribution of the winner wrt seed and lowers the efficacy and fairness.

**Considering nature-of-sport and existing modeling-literature, we assume Thurstone-Mosteller stochastic-transitivity model, $p_{ij} = \Phi(\theta_i - \theta_j + \eta)$, where θ_i is team i's strength parameter, η accounts for home-advantage, and Φ is standard normal CDF. We assume $\theta = (\theta_1, \theta_2, \cdots, \theta_n)$ follows a multivariate normal distribution. Strength-parameters θ_i are estimated using historical data and a Bayesian state-space model.

***We simulate college football playoffs under 2025-26 playoff-rules, with and without home-advantage, for Round-1 games, and under 2024-25 playoff-rules, where four highest-ranked conference-champions received a first-round bye.

Figure 2: NCAA CFP Simulations Old v. New Format