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Chapter 5 Theoretical Pricing Models

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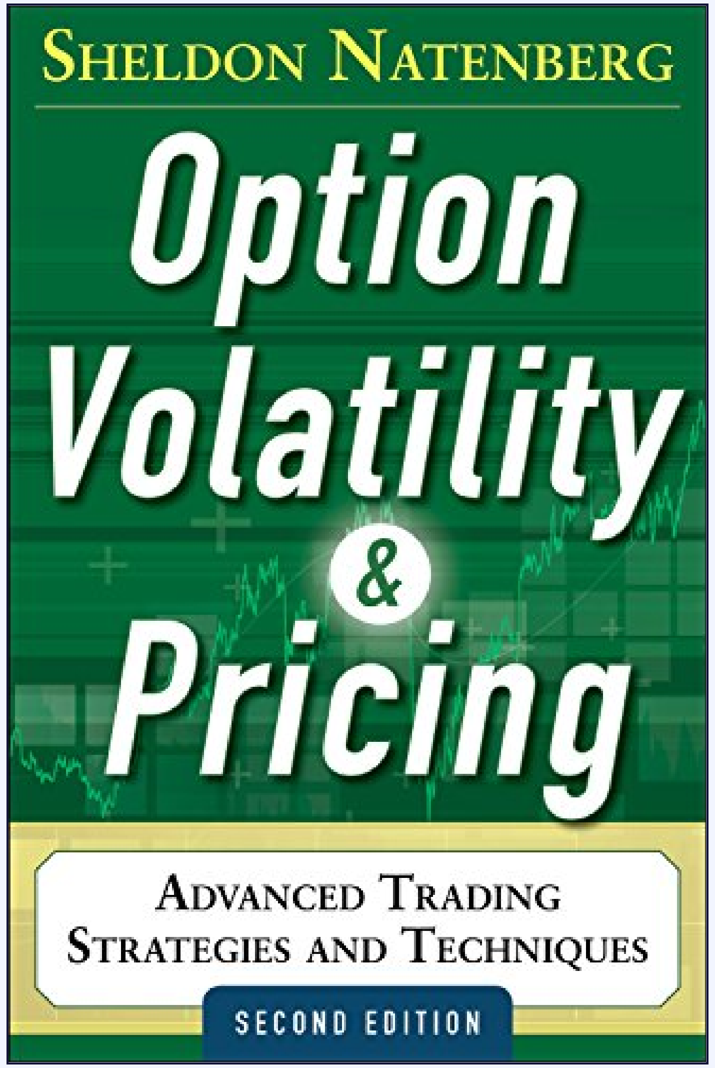
Chapter 5 — Theoretical Pricing Models



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Theoretical Pricing Models

In Chapter 4, we considered the value of an option and the profit or loss resulting from an option strategy at the moment of expiration. From the expiration profit and loss (P&L) graphs, we can see clearly that the direction in which an underlying contract moves can be an important consideration in choosing an option strategy. A trader who believes that the underlying market will rise will be more inclined either to buy calls or sell puts. A trader who believes that the underlying market will fall will be more inclined either to buy puts or sell calls. In each case, the directional movement in the underlying market will increase the likelihood that the strategy will be profitable.

However, an option trader has an additional problem that we might call the "speed" of the market. If we ignore interest and dividend considerations, a trader who believes that a stock will rise in price within a specified period can be reasonably certain of making a profit if he is right. He can simply buy the stock, wait for it to reach his target price, and then sell the stock at a profit.

The situation is not quite so simple for an option trader. Suppose that a trader believes that a stock will rise in price from $100, its present price, to $115 within the next five months. Suppose also that a $110 call expiring in three months is available at a price of $2. If the stock rises to $115 by expiration, the purchase of the $110 call will result in a profit of $3 ($5 intrinsic value minus the $2 cost of the option). But is this profit a certainty? What will happen if the price of the stock remains below $110 for the next three months and only reaches $115 after the option expires? Then the option will expire worthless, and the trader will lose his $2 investment.

Perhaps the trader would do better to purchase a $110 call that expires in six months rather than three months. Now he can be certain that when the stock reaches $115, the call will be worth at least $5 in intrinsic value. But what if the price of the six-month option is $6? In this case, the trader still might show a loss. Even if the underlying stock reaches the target price of $115, there is no guarantee that the $110 call will ever be worth more than its $5 intrinsic value.

- The Importance of Probability
- A Simple Approach
- The Black-Scholes Model

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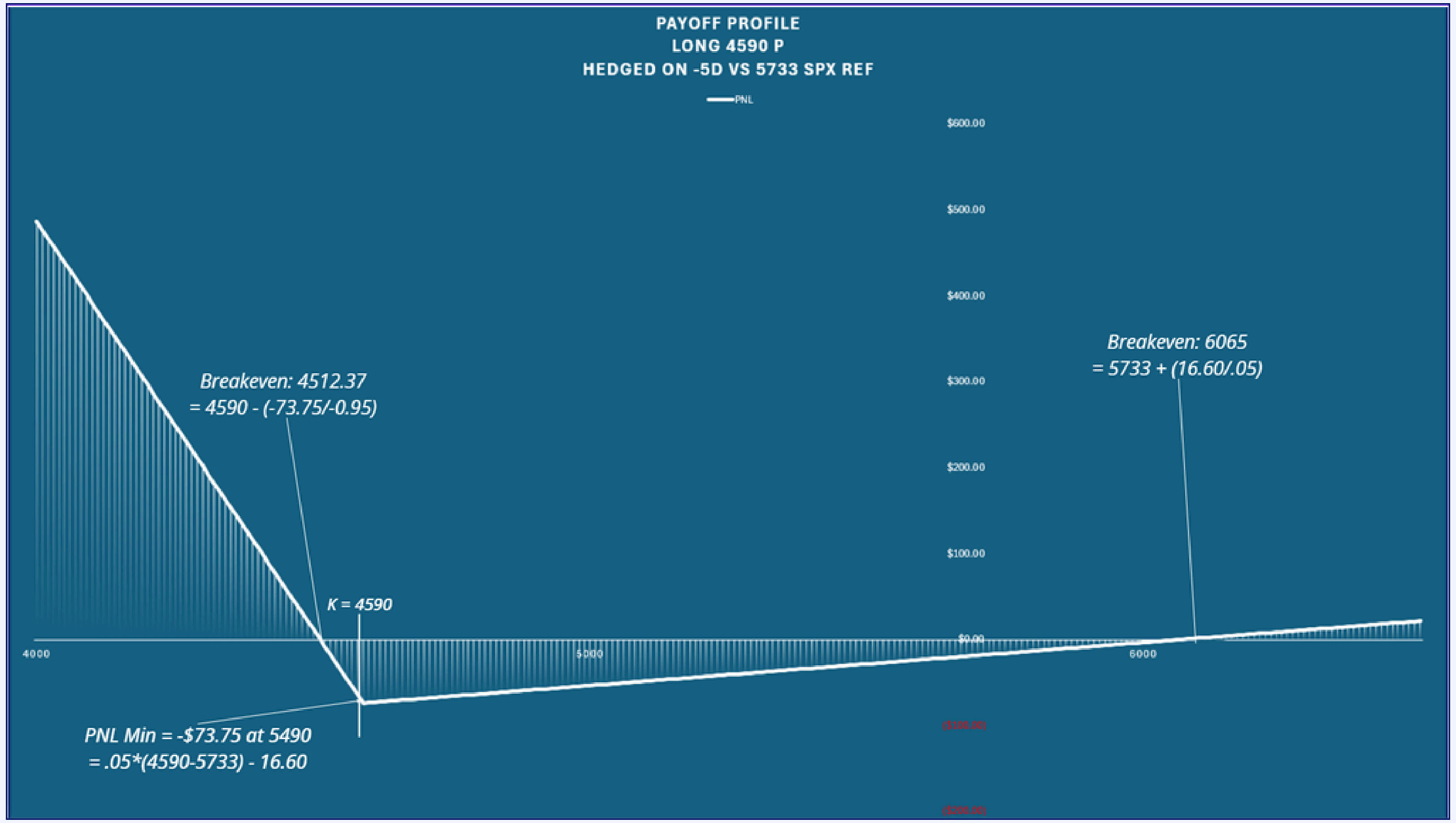
What are expiration PNL graphs lacking?



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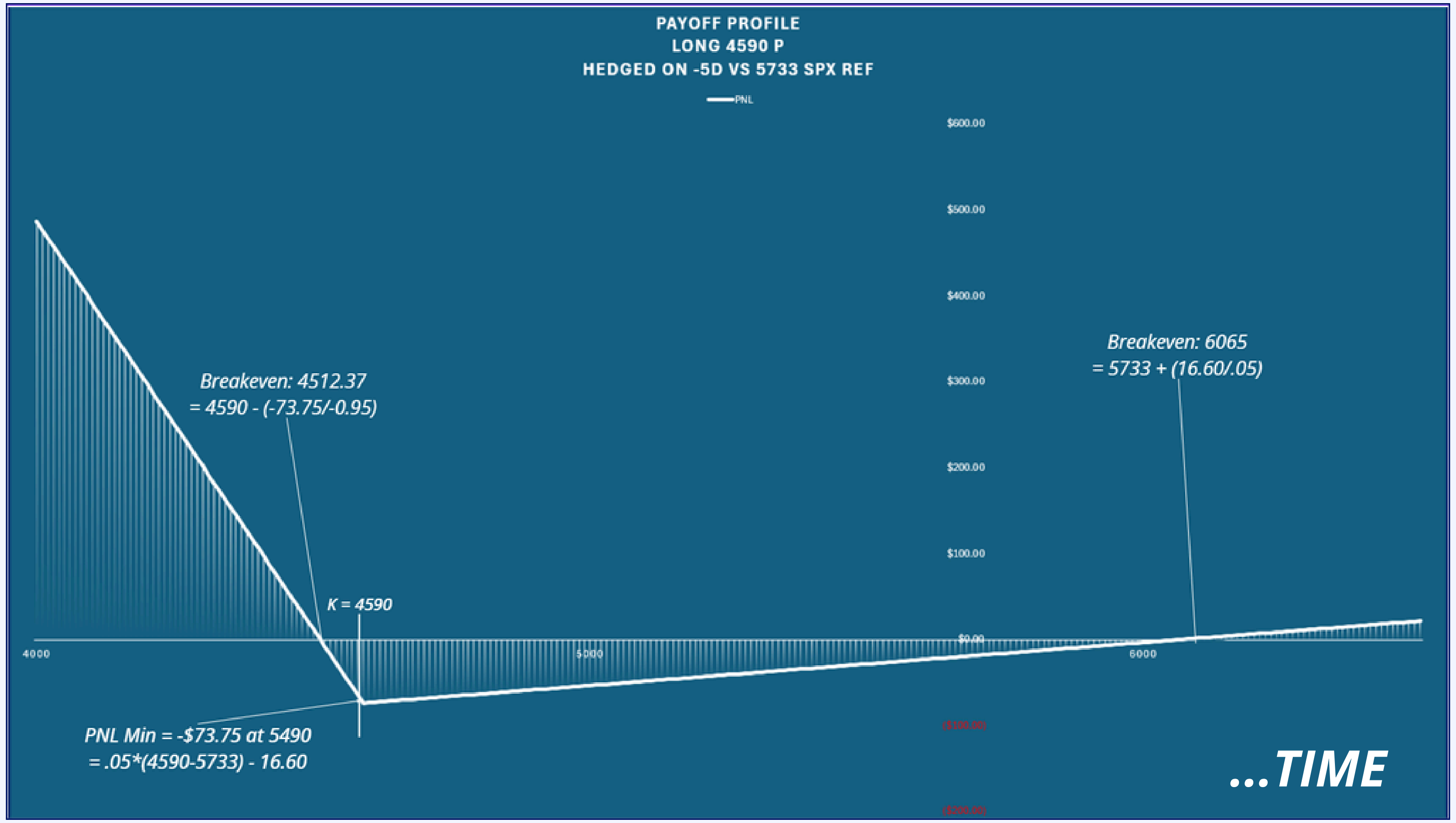

The Importance of Probability

No matter which angle you approach the subject from… paths and probabilities are at the heart of options

- *"If A, then D.. but if B, then C"- type thinking is fundamental (get comfortable with this)*
- *Low probabilities = cheaper bets/greater relative payouts, & vice versa*

Expected Value / EV as starting point

- *Straightforward concept:*
  - *Assign a value and a probability to each potential outcome*
  - *Expected value of the "bet" is the simple probability-weighted sum of outcome values*
  - *…this is effectively the same thing as a "theoretical value" (…or "theo" in options parlance)*

Expected value of a roll of a fair six-sided die

$$\mathrm{E}[X] = \sum_{i=1}^{k} x_i\, p_i = x_1 p_1 + x_2 p_2 + \cdots + x_k p_k$$

$$\mathrm{E}[X] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = \frac{7}{2} = 3.5$$

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Natenberg refers to how important it is in options to account for interest & dividends

...the fundamental point however is that all cash flows in or out must be accounted for (and properly discounted)

What's the goal of an options trader?

- *Positive EV (expected value) - this is what we call "edge"*
  - *dice game*
  - *vs "real world"*

We would use MODELS to describe the array of bet outcomes at any given time-

- *Connect things continuously = more important than "predict future accurately"*
- *Trading in this fashion required thinking in "levels relative to current model" (not prices)*
- *Models aren't perfect... MANY blow-ups arise out of a simple failure to grasp their very REAL limitations*

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From an entire set of outcomes…
-to an "option" on that distribution

*Simple ownership of the underlying implies that **today's price** is a probability-weighted sum of all possible future prices*

For an option, this just requires us to slice the possible final outcomes and evaluate the "bet" according to those outcomes which fall within its parameters (call/put, strike price)

$$(20\% \times \$80) + (20\% \times \$90) + (20\% \times 100) + (20\% \times \$110) + (20\% \times \$120) = \$100$$

From underlying…

…to the 100 strike Call

$$(20\% \times 0) + (20\% \times 0) + (20\% \times 0) + (20\% \times \$10) + (20\% \times \$20) = \$6$$

Call…

$$\sum_{i=1}^{n} p_i \max(S_i - X, 0)$$

Put…

$$\sum_{i=1}^{n} p_i \max(X - S_i, 0)$$

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What's wrong with this picture..?

Figure 5-1

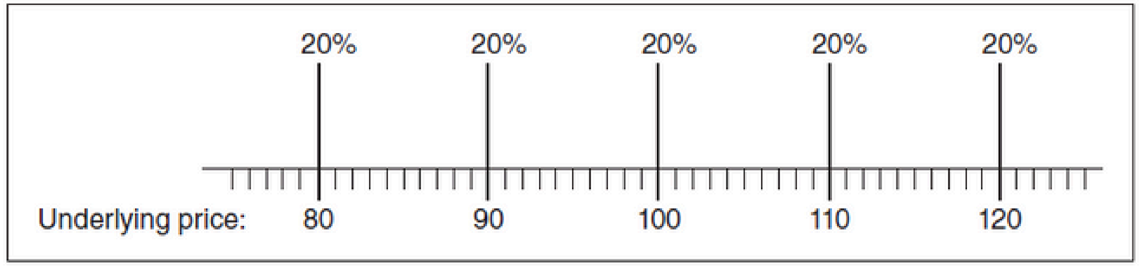
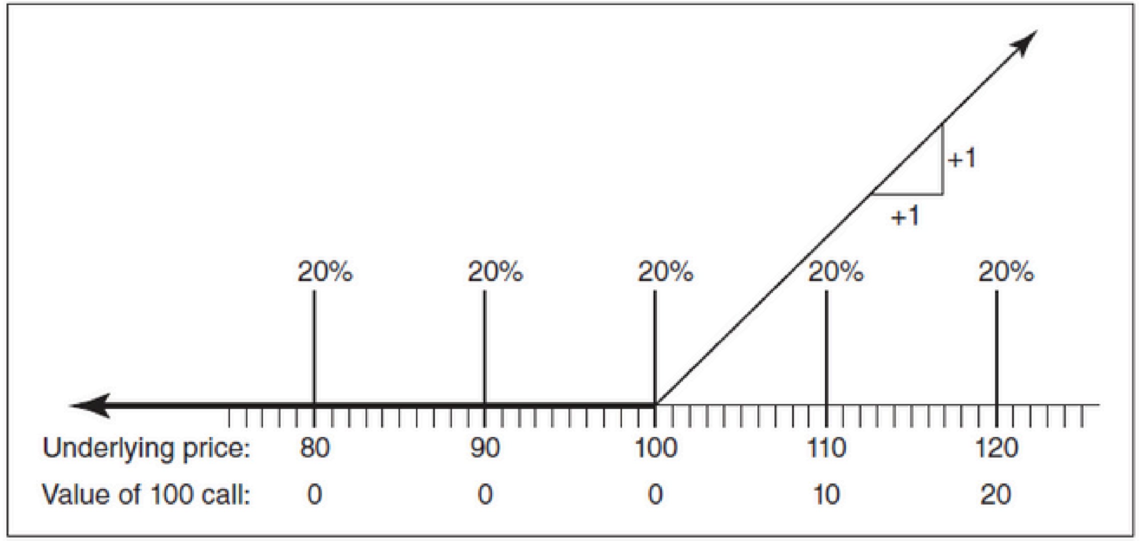


Figure 5-2



What's <u>wrong</u> with this picture..?

"All price outcomes are equally likely"

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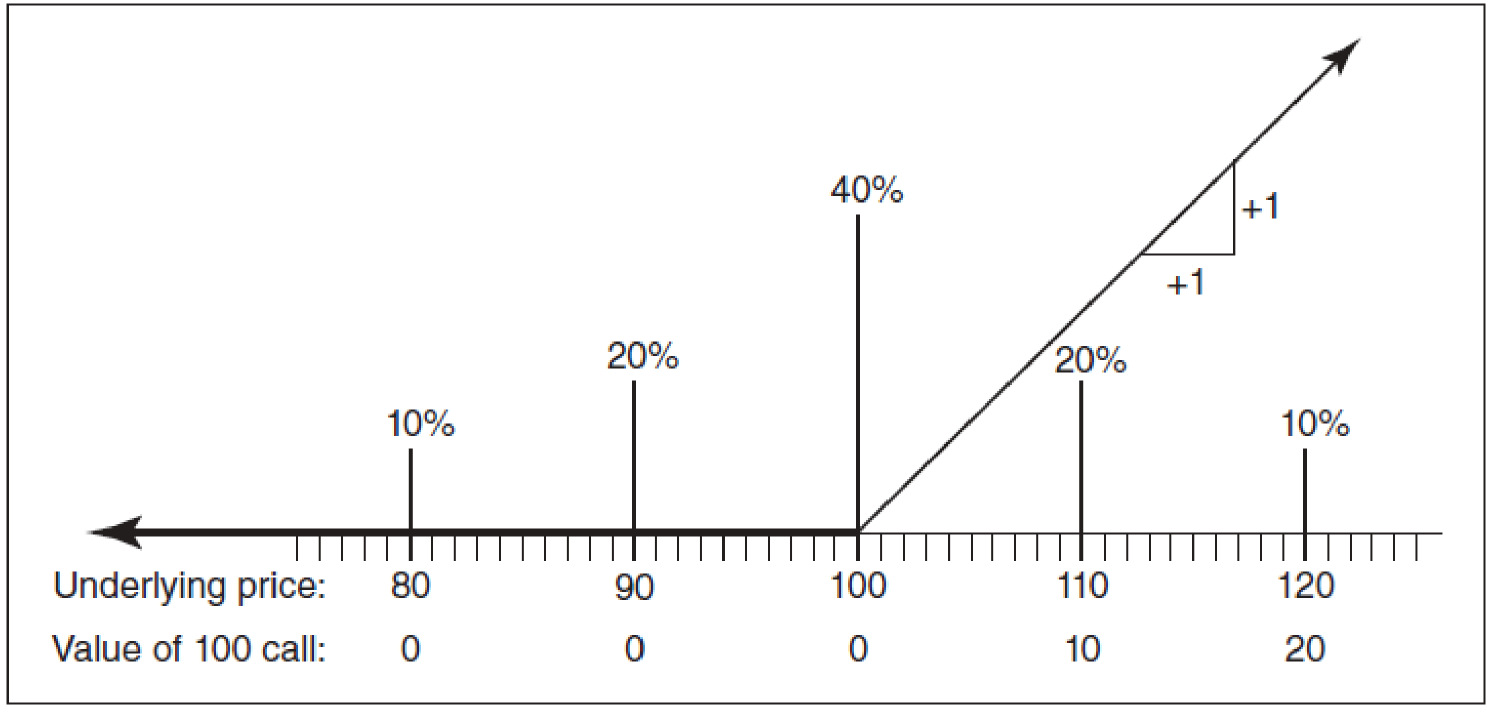
Alright- that's (getting) better…

Figure 5-3



$$(0\% \times 0) + (20\% \times 0) + (0\% \times 0) + (20\% \times \$10) + (10\% \times \$20) = \$4$$

In the real world…

- *Probabilities do not need to be symmetrical around the price-*
- *…but the implication is that the current price IS the fair value (however distorted the distribution)*
- *Always account for cash flow timing, and discount appropriately when necessary (EV accounts for PV)*
- *Infinite outcomes (conceptually)*

Real world requires a continuous distribution…

The Black-Scholes Model

A "BS Model" for the real world…

Formative approach to options modeling and risk management

Originally:

- *European ("euro") options only*
- *Non-dividend paying stocks*

Modifications emerged quickly, iterations continued as applications w/other assets grew- but generally they are all the same thing
**in reality, current models are heavily modified versions of "BS"*

The Black-Scholes Model calculation requires:

- *Option's exercise price*
- *Time to expiry*
- *Current underlying (ignore forwards here)*
- *Interest rate (assume continuous over life of option)*
- *Volatility of the underlying contract**

*This is your **known unknown***
*In practice you *back into* this part, "implying" it from tradable prices.*

"Riskless hedge"

- *...DELTA!*
- *(aka 'hedge ratio', aka 'probability of settling in-the-money')*
- *This is the concept of "option replication"*

*Delta hedging must *offset* your directional exposure...*

Option Position	Corresponding Market Position	Appropriate Hedge
Buy call(s)	Long	Sell underlying
Sell call(s)	Short	Buy underlying
Buy put(s)	Short	Buy underlying
Sell put(s)	Long	Sell underlying

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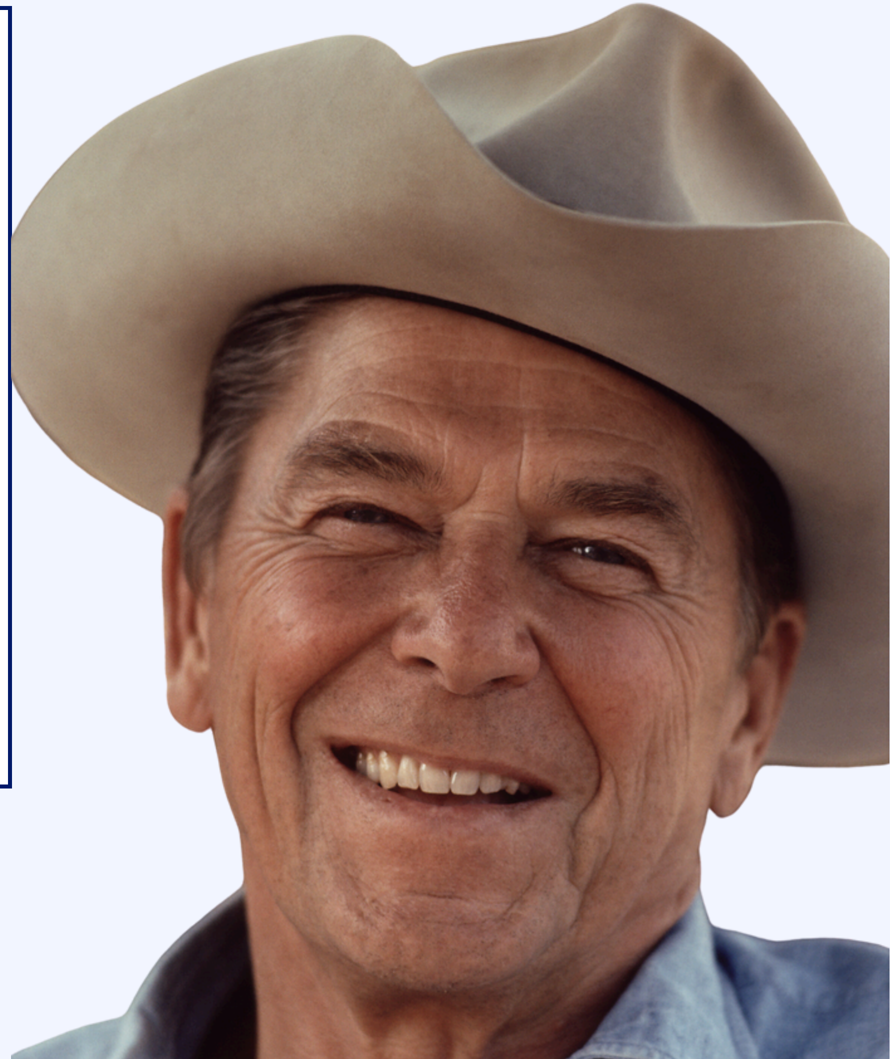
Colloquially,
*when you mistakenly hedge in the same direction and *double* your risk instead of eliminate it…*

Origin [edit]

"Texas Hedging" dates back to the early days of the CBOE and CME, when options were gaining in popularity.

Notoriously aggressive traders were known to push markets their way when buying cheap near-term options. The earliest known reference comes from James Gilbert, a former TransMarket Group floor clerk, who relayed the following to Trader Magazine in 2006: "*I was a runner at the time. And I see this guy signal to buy 500 futures. Big futures. And I know he just bought calls. So I yell - hey, you are backwards on your hedge! And he looks me square in the face with these eyes of cobalt, not an ounce of joviality in his veins. He says 'Boy - I'm from Texas. We don't hedge when we're right in Texas. We double-down, son.' The whole pit must've heard him, because from that moment on, any time any trader mistakenly hedged backwards, they would say 'I TEXASED' and the whole trading crowd would point and laugh. Everybody but Bill, from Texas. He would just stare, with those piercing cobalt eyes.*"[citation needed]

In livestock trading, the Texas hedge typically refers derisively to Texan cattle ranchers who might buy cattle futures contracts while already owning cattle, thereby doubling their risk exposure.[1]



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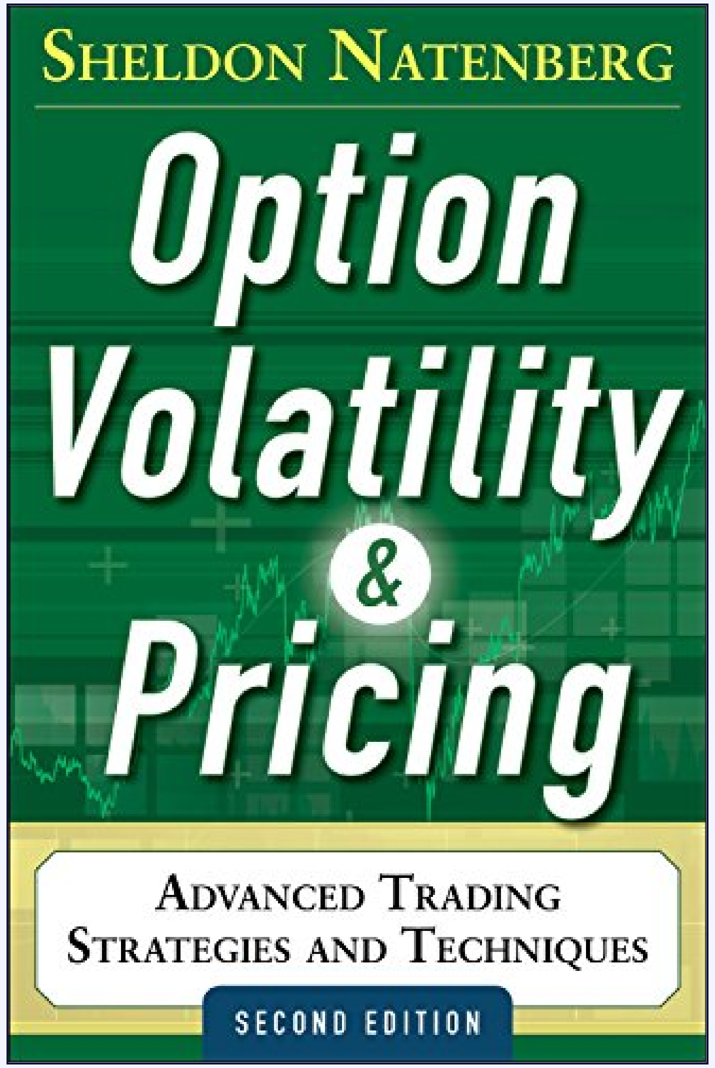
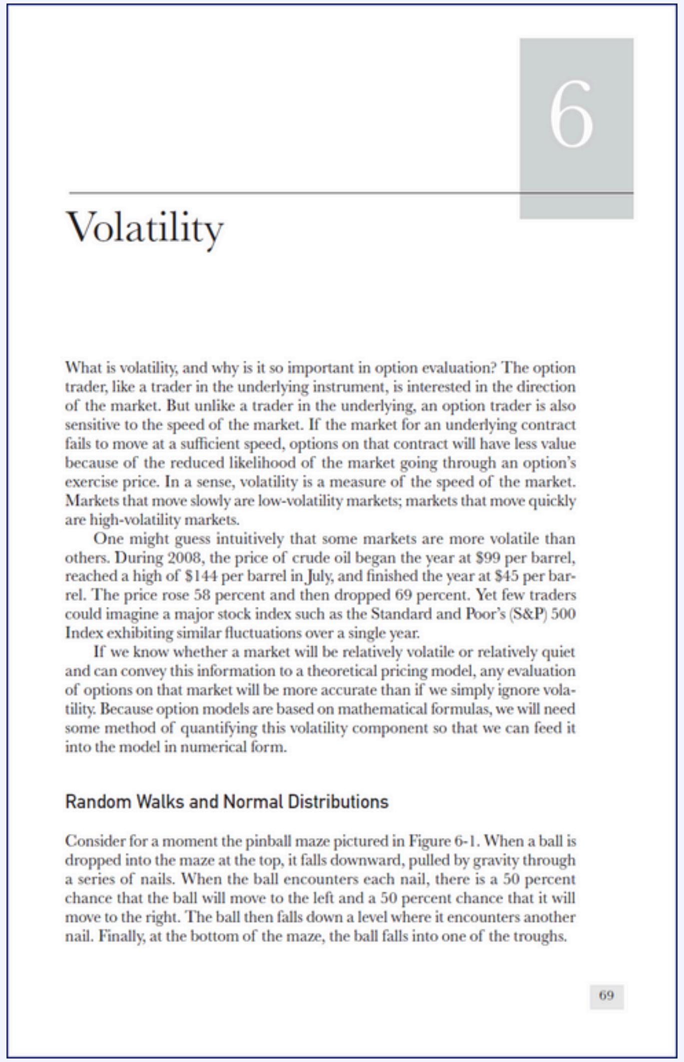
Next up…





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Volatility

What is volatility, and why is it so important in option evaluation? The option trader, like a trader in the underlying instrument, is interested in the direction of the market. But unlike a trader in the underlying, an option trader is also sensitive to the speed of the market. If the market for an underlying contract fails to move at a sufficient speed, options on that contract will have less value because of the reduced likelihood of the market going through an option's exercise price. In a sense, volatility is a measure of the speed of the market. Markets that move slowly are low-volatility markets; markets that move quickly are high-volatility markets.

One might guess intuitively that some markets are more volatile than others. During 2008, the price of crude oil began the year at $99 per barrel, reached a high of $144 per barrel in July, and finished the year at $45 per barrel. The price rose 58 percent and then dropped 69 percent. Yet few traders could imagine a major stock index such as the Standard and Poor's (S&P) 500 Index exhibiting similar fluctuations over a single year.

If we know whether a market will be relatively volatile or relatively quiet and can convey this information to a theoretical pricing model, any evaluation of options on that market will be more accurate than if we simply ignore volatility. Because option models are based on mathematical formulas, we will need some method of quantifying this volatility component so that we can feed it into the model in numerical form.

Random Walks and Normal Distributions

Consider for a moment the pinball maze pictured in Figure 6-1. When a ball is dropped into the maze at the top, it falls downward, pulled by gravity through a series of nails. When the ball encounters each nail, there is a 50 percent chance that the ball will move to the left and a 50 percent chance that it will move to the right. The ball then falls down a level where it encounters another nail. Finally, at the bottom of the maze, the ball falls into one of the troughs.

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Chapter 6 — Volatility

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