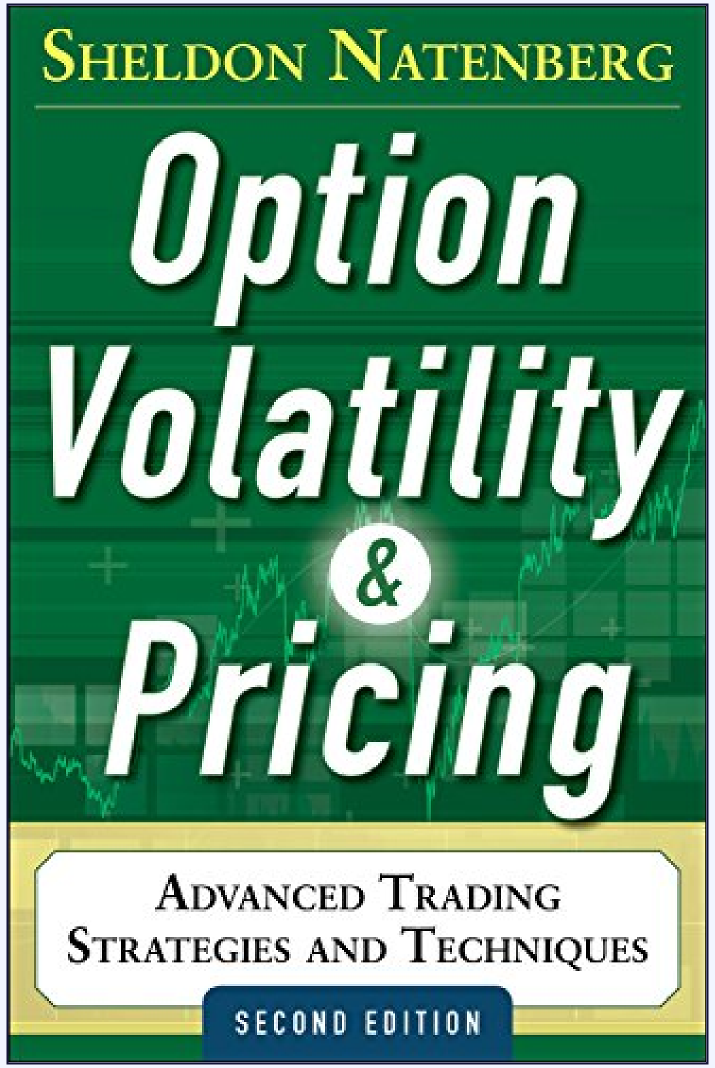
VolSignals

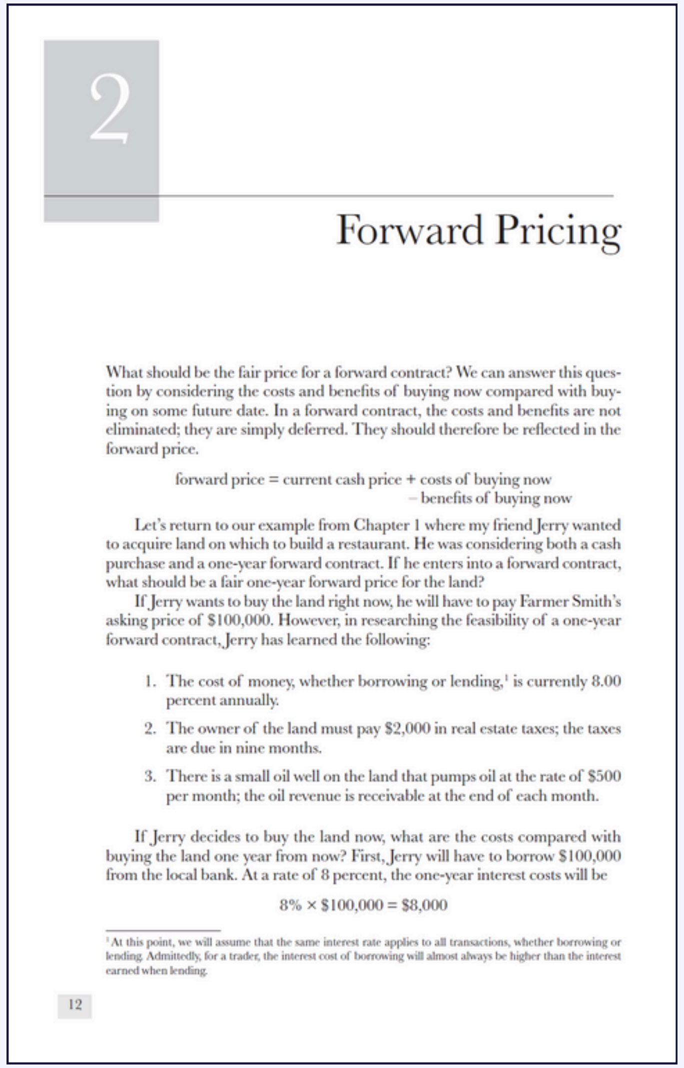
VolStudies

Chapter 2 Forward Pricing

VolStudies | Option Volatility & Pricing

Chapter 2 — Forward Pricing





2

Forward Pricing

What should be the fair price for a forward contract? We can answer this question by considering the costs and benefits of buying now compared with buying on some future date. In a forward contract, the costs and benefits are not eliminated; they are simply deferred. They should therefore be reflected in the forward price.

$$\text{forward price} = \text{current cash price} + \text{costs of buying now} - \text{benefits of buying now}$$

Let's return to our example from Chapter 1 where my friend Jerry wanted to acquire land on which to build a restaurant. He was considering both a cash purchase and a one-year forward contract. If he enters into a forward contract, what should be a fair one-year forward price for the land?

If Jerry wants to buy the land right now, he will have to pay Farmer Smith's asking price of $100,000. However, in researching the feasibility of a one-year forward contract, Jerry has learned the following:

1. The cost of money, whether borrowing or lending,[1] is currently 8.00 percent annually.
2. The owner of the land must pay $2,000 in real estate taxes; the taxes are due in nine months.
3. There is a small oil well on the land that pumps oil at the rate of $500 per month; the oil revenue is receivable at the end of each month.

If Jerry decides to buy the land now, what are the costs compared with buying the land one year from now? First, Jerry will have to borrow $100,000 from the local bank. At a rate of 8 percent, the one-year interest costs will be

$$8\% \times \$100{,}000 = \$8{,}000$$

[1] At this point, we will assume that the same interest rate applies to all transactions, whether borrowing or lending. Admittedly, for a trader, the interest cost of borrowing will almost always be higher than the interest earned when lending.

12

VolSignals

- Physical Commodities
- Stock
- Bonds and Notes
- Foreign Currencies
- Stock and Futures Options
- Arbitrage
- Dividends
- Short Sales

Pricing forwards or futures contracts
comes down to eliminating arbitrage-
by weighing up the costs & benefits of buying now vs. agreeing to buy on a certain date in the future.

General Case for a (fair) forward price =
Forward price *=* ***current cash (spot) price*** *+**COSTS** of buying now —**BENEFITS** of buying now*

*...what are these **costs** or **benefits**?* ⏩

Instrument	Costs of Buying Now	Benefits of Buying Now
Physical commodity	Interest on cash price Storage costs Insurance costs	Convenience yield (to be discussed)
Stock	Interest on stock price	Dividends (if any) Interest on dividends
Bonds and notes	Interest on bond or note price	Coupon payments Interest on coupon payments
Foreign currency	Interest cost of borrowing the domestic currency	Interest earned on the foreign currency

Physical Commodities

Grains, Energy Products, Precious Metals, etc.

BUYING NOW…

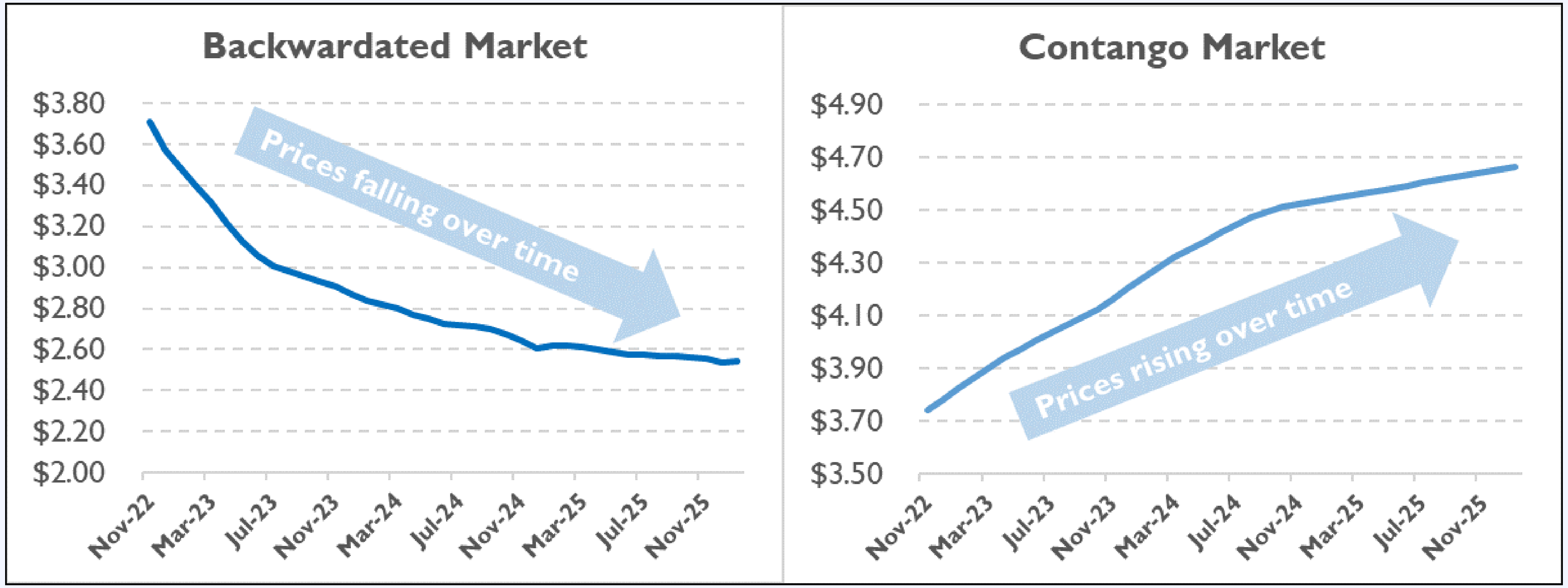
✅*Pay current price up front (cash outflow)*
✅*Forego the interest on this amount*
✅*Storing the asset until maturity/ end-of-contract*
✅*Insuring the asset until maturity*

C = commodity price[2]
t = time to maturity of the forward contract
r = interest rate
s = annual storage costs per commodity unit
i = annual insurance costs per commodity unit[3]

Forward Price (F)

$$F = C \times (1 + r \times t) + (s \times t) + (i \times t)$$

Contango & Backwardation



*Given positive interest rates & positive costs for storage & insurance… backwardation *shouldn't* be the norm…*
UNLESS end-users need immediate access to the physical.
*—the divergence this produces from **fair value** is known as the **convenience yield***

Stock / Equity ETFs

Tradeoff for equities?

Pay cash up front for stock...

forego the interest you could otherwise receive ✕

entitled to dividends ✓

S = stock price
t = time to maturity of the forward contract
r = interest rate over the life of the forward contract
d_i = each dividend payment expected prior to maturity of the forward contract

Forward Price (F) using discrete dividends

$$F = S + (S \times r \times t) - [d_1 \times (1 + r_1 \times t_1)] - \cdots - [d_n \times (1 + r_n \times t_n)]$$
$$= [S \times (1 + r \times t)] - \Sigma[d_n \times (1 + r_n \times t_n)]$$

$$F = [S \times (1 + r \times t)] - D$$

Bonds

...think of coupon payments like dividends 👍

B = bond price
t = time to maturity of the forward contract
r = interest rate over the life of the forward contract
c_i = each coupon expected prior to maturity of the forward contract
t_i = time remaining to maturity after each coupon payment
r_i = applicable interest rate from each coupon payment to maturity of the forward contract

Forward Price (F) of a Bond Future

$$F = B + (B \times r \times t) - [c_1 \times (1 + r_1 \times t_1)] - \cdots - [c_n \times (1 + r_n \times t_n)]$$
$$= [B \times (1 + r \times t)] - \Sigma[c_n \times (1 + r_n \times t_n)]$$

VolStudies | Option Volatility & Pricing

Chapter 2 — Forward Pricing

Currency Forwards / Futures

Tradeoffs are respective rates available for each currency

See example from text->

$$S = \frac{C_d}{C_f}$$

Suppose that we have a domestic rate r_d and a foreign rate r_f. What should be the forward exchange rate at the end of time t? If we invest C_f at r_f and we invest C_d at r_f, the exchange rate at time t ought to be

$$F = \frac{C_d \times (1 + r_d \times t)}{C_f \times (1 + r_f \times t)}$$

$$= \frac{C_d}{C_f} \times \frac{1 + r_d \times t}{1 + r_f \times t}$$

$$= S \times \frac{1 + r_d \times t}{1 + r_f \times t}$$

For example, suppose that €1.00 = \$1.50. Then

$$S = \frac{1.50}{1.00} = 1.50$$

If

$$\text{Dollar interest rate } r_\$ = 6.00\%$$

$$\text{Euro interest rate } r_€ = 4.00\%$$

then the six-month forward price is

$$F = \frac{1.50 \times (1 + 0.06 \times 6/12)}{1.00 \times (1 + 0.04 \times 6/12)}$$

$$= \frac{1.50}{1.00} \times \frac{1 + 0.06 \times 6/12}{1.00 \times (1 + 0.04 \times 6/12)}$$

$$= \frac{1.50 \times 1.30}{1.02} = \mathbf{1.5147}$$

Stock and Futures Options

Option underlyings are NOT equivalent throughout time…

Some misleading tendencies arise out of the ignorance of this fact:

- *Misinterpretation of what is “at- the- money” when comparing tenors*
- *Leads to errors in comparing Implied Vols across calendar spreads*
    - *for example- if you are comparing ATM vol in 1M vs 1Y and using spot as the reference strike in both, you are biased towards observing a steeper term structure due to skew in the longer tenor*
    - *Correct for this by using the proper forward price when comparing strikes (spot check w/delta)*
- *Attribution errors when comparing richness/cheapness of structures expressed as %-spot price*
    - *Correct for this by defining structures according to their **delta** instead*

Examples to follow (product portal & Discord discussion)

VolSignals

VolStudies