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Chapter 7 Risk Measurement I

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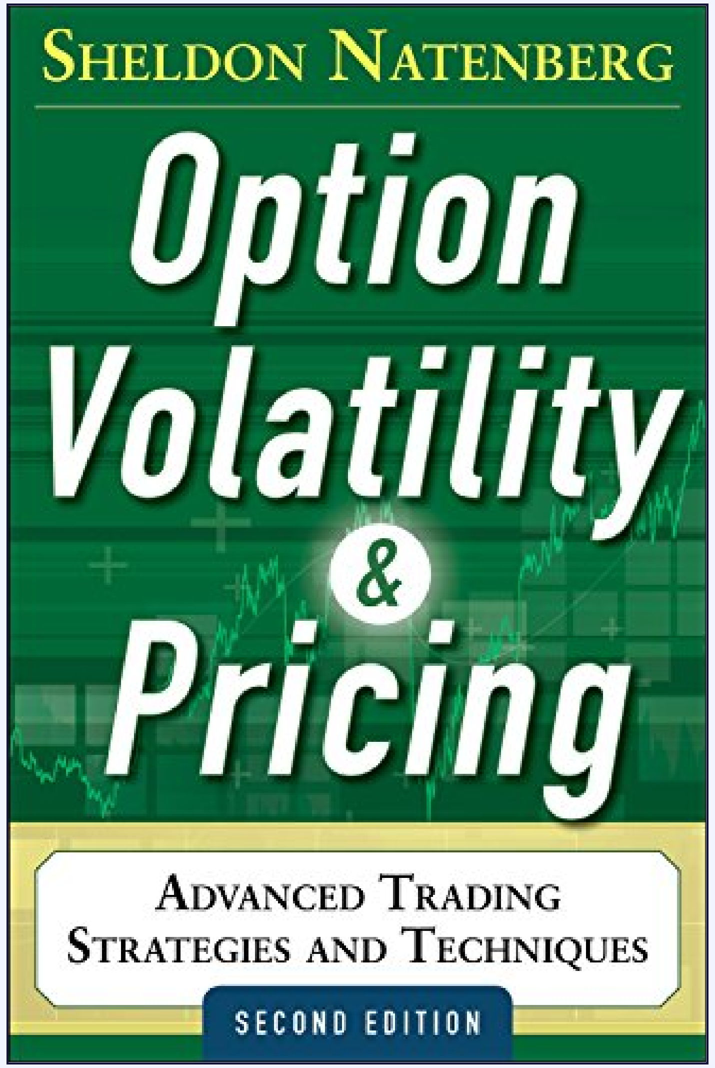
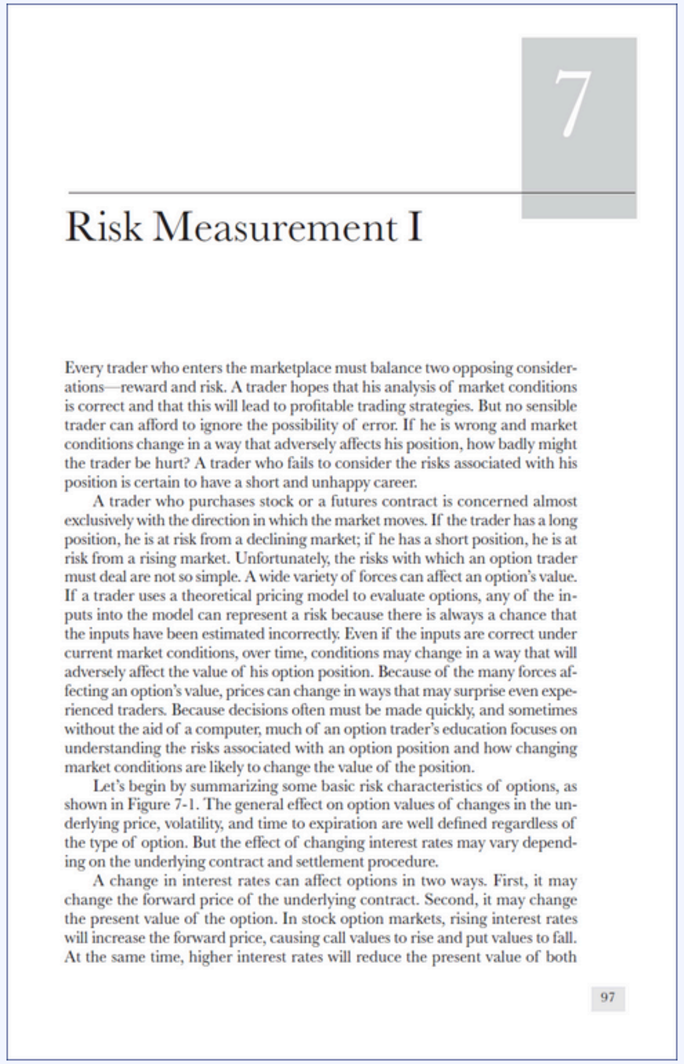
Chapter 7 — Risk Measurement I





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Risk Measurement I

Every trader who enters the marketplace must balance two opposing considerations—reward and risk. A trader hopes that his analysis of market conditions is correct and that this will lead to profitable trading strategies. But no sensible trader can afford to ignore the possibility of error. If he is wrong and market conditions change in a way that adversely affects his position, how badly might the trader be hurt? A trader who fails to consider the risks associated with his position is certain to have a short and unhappy career.

A trader who purchases stock or a futures contract is concerned almost exclusively with the direction in which the market moves. If the trader has a long position, he is at risk from a declining market; if he has a short position, he is at risk from a rising market. Unfortunately, the risks with which an option trader must deal are not so simple. A wide variety of forces can affect an option's value. If a trader uses a theoretical pricing model to evaluate options, any of the inputs into the model can represent a risk because there is always a chance that the inputs have been estimated incorrectly. Even if the inputs are correct under current market conditions, over time, conditions may change in a way that will adversely affect the value of his option position. Because of the many forces affecting an option's value, prices can change in ways that may surprise even experienced traders. Because decisions often must be made quickly, and sometimes without the aid of a computer, much of an option trader's education focuses on understanding the risks associated with an option position and how changing market conditions are likely to change the value of the position.

Let's begin by summarizing some basic risk characteristics of options, as shown in Figure 7-1. The general effect on option values of changes in the underlying price, volatility, and time to expiration are well defined regardless of the type of option. But the effect of changing interest rates may vary depending on the underlying contract and settlement procedure.

A change in interest rates can affect options in two ways. First, it may change the forward price of the underlying contract. Second, it may change the present value of the option. In stock option markets, rising interest rates will increase the forward price, causing call values to rise and put values to fall. At the same time, higher interest rates will reduce the present value of both

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Chapter 7 — Risk Measurement I

- The Delta
- The Gamma
- The Theta
- The Vega
- The Rho
- Interpreting the Risk Measures

"A trader who fails to consider the risks associated with his position is certain to have a short and unhappy career."

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Chapter 7 — Risk Measurement I

Typically, a trader who purchases a stock or a futures contract is concerned with one risk- …"is my direction correct?"

Options traders are exposed to MORE risks- and more COMPLICATED risks.

Evaluating options requires a theoretical pricing model- and any of these inputs may be "incorrect" / may change / (maybe your model is wrong entirely!)

Because options traders (especially MMs) must decide and react QUICKLY-
it's critical to have a solid grasp of the fundamental ways in which changes to your model's "inputs" or changes in market conditions impact the value of your options position.

Example… the buyer in (1) got direction RIGHT, but after a $100 point move higher, their Call option is worth LESS 👀

13:26:29 Big vol crush/reversal today with VIX down 36%. QQQ ATM august vols are down 8 points on the day, SPY down 7 points & SMH down 10 vols -- some really crazy moves given a typical day, having a 1-2 move lower in vols is a LOT. To put the hit to vols in $$ perspective, here are some quick maths/examples:

1. We had a cust sell the Oct 5750 call @ $24 yesterday when spot was 5200. SPX is almost 100 points HIGHER and that call is currently trading $4 LOWER.

2. QQQ Sept 435 put (the at-the-money option on the close) closed yday worth $19. With QQQ up $10 today, you would expect to be down $4 on that 40D option on the delta move (i.e. trading $15). However, its trading $11.50, so the other $3.50 of lost value is from the vol crush (that option is down 6 vols and about ~.60 of vega). So 22% of option value loss is from Delta, and 18% from vol...

The Basics: Delta, Vega, Theta...

If....	Call values will....	Put values will....
The price of the underlying contract rises	rise	fall
The price of the underlying contract falls	fall	rise
Volatility rises	rise	rise
Volatility falls	fall	fall
Time passes	fall*	fall*

*In some unusual cases it may be possible for the value of an option to rise as time passes, even if all other market conditions are unchanged. The circumstances which can cause this will be discussed later.

*Interest rate changes are not *so* simple*⏩

A change in interest rates can impact an option in two distinct ways…
First, recall the forward equation:

$$F = [S \times (1 + r \times t)] - D$$

When interest rates RISE, forward values RISE

- *Call values go UP*
- *Put values go DOWN*

When interest rates DECLINE, forward values DECLINE

- *Call values go DOWN*
- *Put values go UP*

Remember- the FORWARD is the option's underlying, not the spot (current) price- theoretically.

Second- the option's PRESENT VALUE changes- higher rates = lower present values, via discounting

Chapter 7 — Risk Measurement I

In reality it’s not enough to just know simple directional influences.

We need to know something about the MAGNITUDE of these risks.

Enter the Greeks…

Δ δ
delta

Θ θ
theta

Ρ ρ
rho

Γ γ
gamma

~~Ν~~ ν
vega

The Delta

Delta measures how an option's price changes with respect to its underlying

- *A positive (+) delta means the option price moves UP as the underlying moves UP, and DOWN as the underlying moves DOWN*
- *A negative (-) delta means the option price moves DOWN as the underlying moves UP, and UP as the underlying moves DOWN*

As we'll see, there are a few different definitions- or "interpretations"- of Delta... all of which have their own analytical or practical utility.

Δ δ

delta

The Delta as a Rate of Change

At expiration, an option is worth only its intrinsic value (recall the payoff profiles?)
Delta in this state is easy to recognize as either 1/-1 or 0.
At expiration, there is no “in-between”

Prior to expiration, however, the option’s value changes along a curve representing the combination of intrinsic AND extrinsic value, or “time value”

Study the Call and Put value graphs on the following slide…

Note the similarity to the payoff profiles we constructed previously-

*The foundation is *still* the same payoff profile, with an added “time” value component*

Now, the delta is no longer binary (0 or +/- 1)… it’s the slope of the tangent line at the reference price

Δ δ
delta

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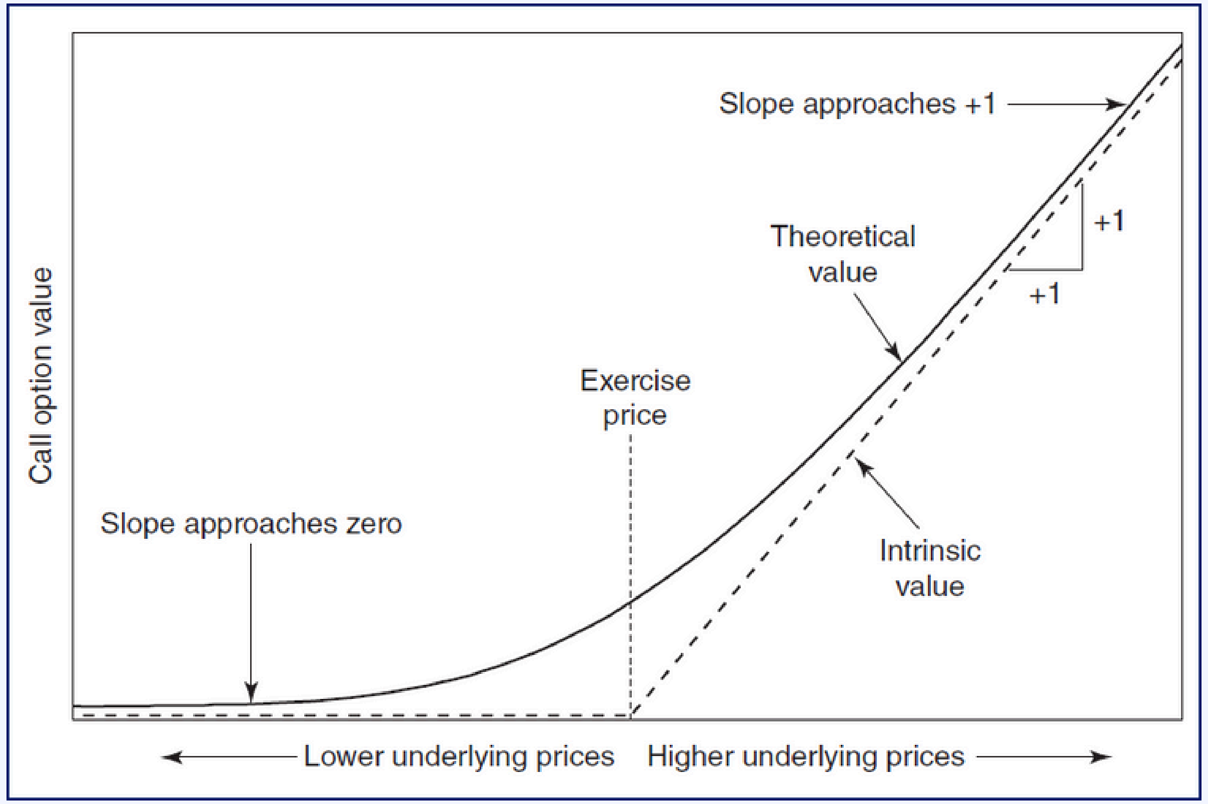

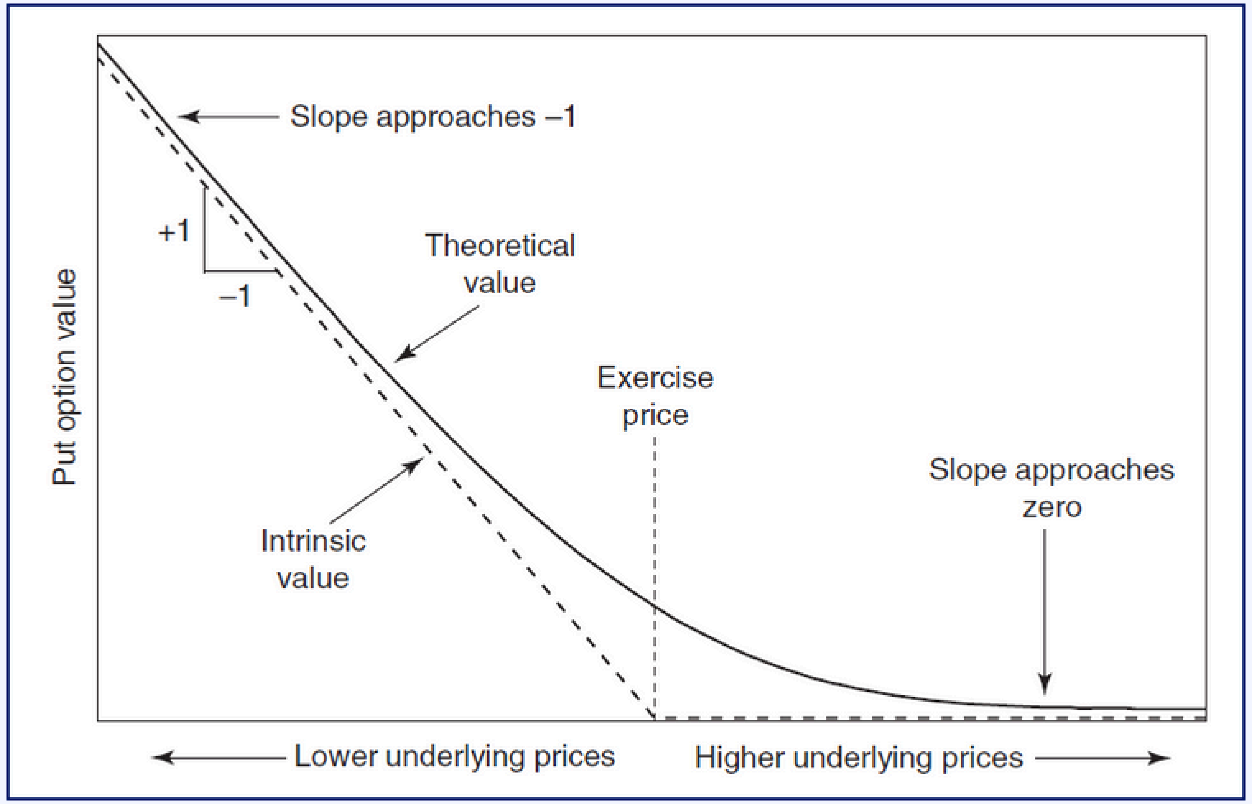

Δ δ

delta

The Delta as a Hedge Ratio

Pragmatically- the Delta of an option (or an option structure) can be used, in conjunction with its underlying (or hedge asset), to determine the "risk neutral" hedge.

For small price ranges, this "risk neutral" hedge offsets the directional exposure of the option and eliminates any local market risk.

We will use SPX as the option and ES as the hedge asset, but this logic is extensible to any option/underlying pair

Note- pay attention to MULTIPLIERS ✅

Δ δ
delta

The Math (with the Multipliers)

The SPX option multiplier is 100

For any option contract, we will shift the decimal accordingly when discussing delta…

- *A -0.05d Put becomes a "-5 Delta Put"*
- *A +1.0d Call becomes a "100 Delta Call"*

The E-mini S&P 500 Future (ES) is the most common hedge asset- its multiplier is 50

To hedge a long (+1) 100 Delta SPX Call requires an offsetting short of TWO ES (-2)
Why? -> $100 + 50X = 0$. . . $50X = -100$. . . $X = -100 / 50$. . . $X = -2$

The General Form

*Hedge Quantity = -1 * (Option Qty * Option Delta) / (Hedge Asset Multiplier)*

Δ δ
delta

Theoretical or Equivalent Underlying Position

Remember- the "underlying" contract always has a Delta of 100. Its Delta does not vary.

- *This is where the term "D1", or "delta-one" arises from… the linearity of stock & futures assets*
- ***NOTE - Our hedge ratio approach accounted for MULTIPLIERS. This is logically consistent* 👍

Options exposure can be thought of as a theoretical replacement / equivalent of the underlying-

For example, if you purchase 100 SPX 20 Delta Calls, you may equate this with:

- *Long 2,000 Units of the SPX (100 * 20 = 2,000)*
    - *Remember- we already include the option multiplier (100) when we drop the Delta's decimal (.20 = 20)*

Δ δ

delta

Quick note on the SPX product complex-

SPX Options have a multiplier of 100 relative to the benchmark SPX Index...

Adjusting for product multipliers and/or Index sizes (as in SPY),
we consider 1 SPX Option equal to:

- *10 XSP Options*
- *10 SPY Options*
- *2 ES Options*
- *20 MES Options*

The Delta as Probability

Ignore the sign (+ or -)... and the Delta is approximately equal to the probability of the option finishing "in-the-money"

Δ δ
delta

The Gamma

For any option, the Delta will change as the underlying changes...

- *approaching 0 as an option moves farther OTM (out-of-the-money)*
- *approaching 100 (-100 for Puts) as an option moves farther ITM (in-the-money)*

Gamma describes how fast this change is taking place locally- it is also sometimes called the "curvature" of the option

Gamma = rate of change in the Delta as the underlying price changes

- *= d(delta)/d(spot)*

For example, an ATM (at-the-money) SPX Call option with a Delta of 50 and a Gamma of 1.5:

- *If SPX rises by 2.00, that same Call option now has a Delta of 53 (50 + 2.00 * 1.5)*
- *If SPX declines by 1.00, the option now has a Delta of 48.5 (50 + -1.00 * 1.5)*

Γ γ

gamma

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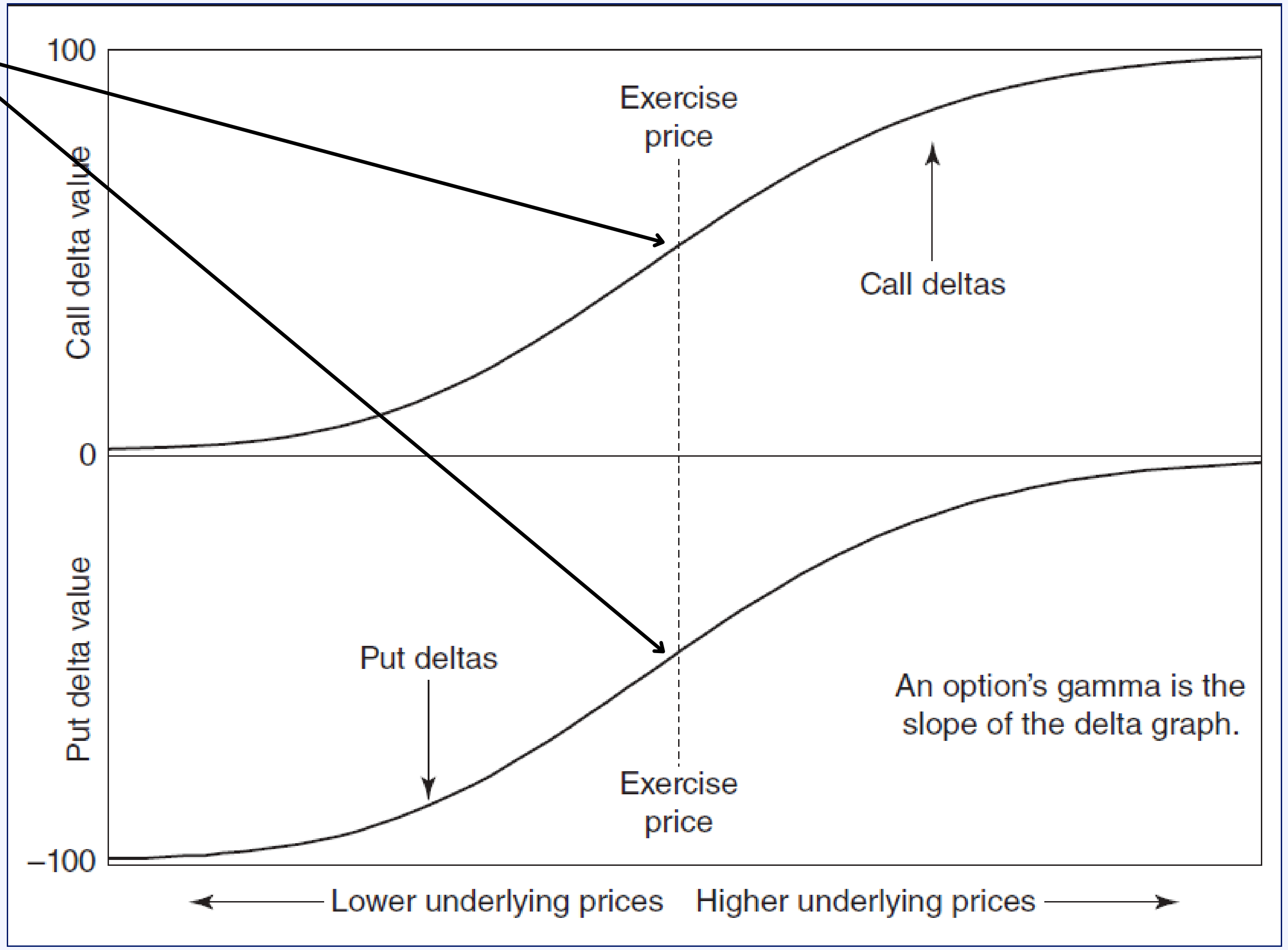
An option's Gamma is highest when ATM (at-the-money)



Calls & Puts have positive Gamma-

*As the underlying rises, a Call's Delta increases (becomes more positive) while a Put's Delta increases (becomes less negative)**

Gamma is additive.
The Gamma of a portfolio of options is equivalent to the SUM of the individual option Gammas, scaled by their contract quantity and +/- according to whether held long or short.

Long options = positive Gamma
Short options = negative Gamma

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The Gamma

When estimating an option's price change over a given move in the underlying, we incorporate the Gamma and use the AVERAGE Delta as follows:

$$\text{Average delta} = [\Delta + \Delta + (S_1 - S_2) \times \Gamma]/2 = \Delta + (S_1 - S_2) \times \Gamma/2$$

For example…

Given an ATM Put Option with a Delta of -50 and a Gamma of 1.5…
We would expect that for a $4.00 RISE in the underlying, the Put value will decline by $1.88

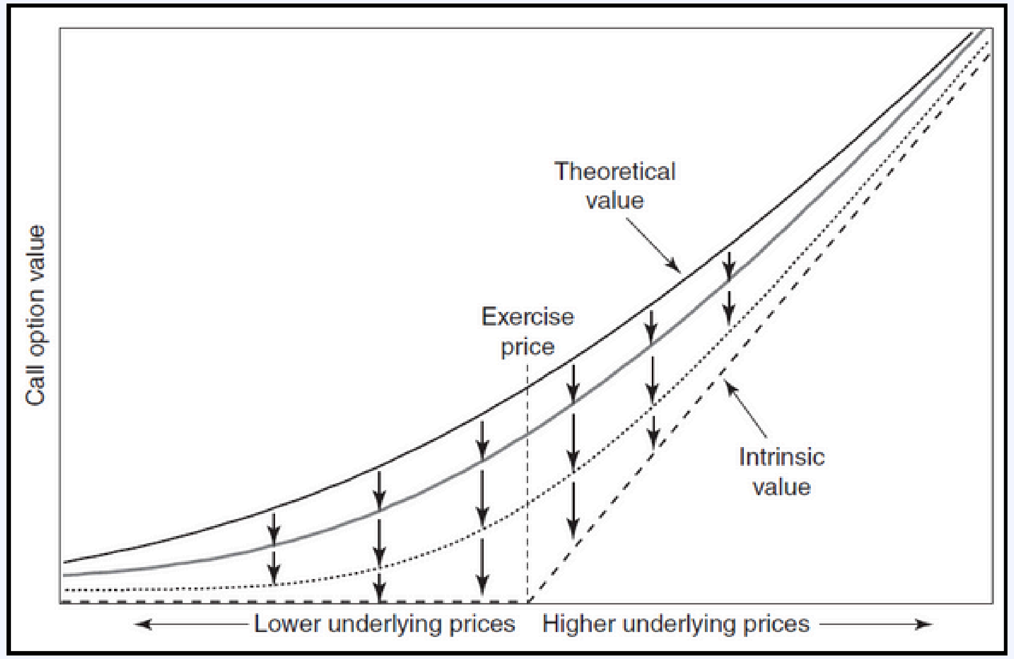
*Price Change = 4.00 * (-.50 +((4.00*1.50)/2))) = 4.00 * (-.50 + 3.00) = 4.00 * -.47 = -$1.88*

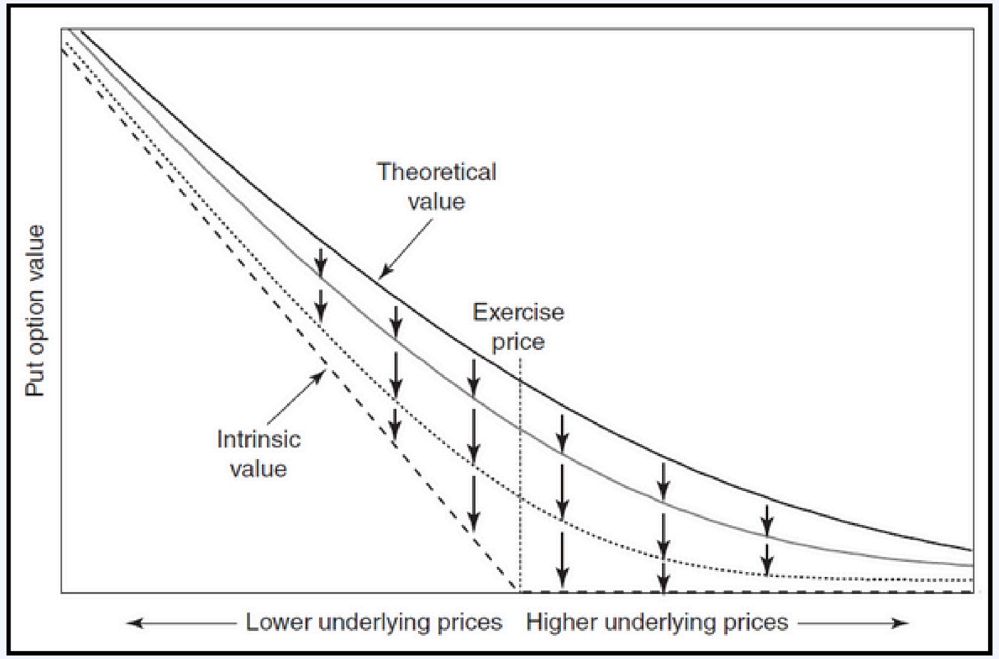
$\Gamma \ \gamma$

gamma

The Theta

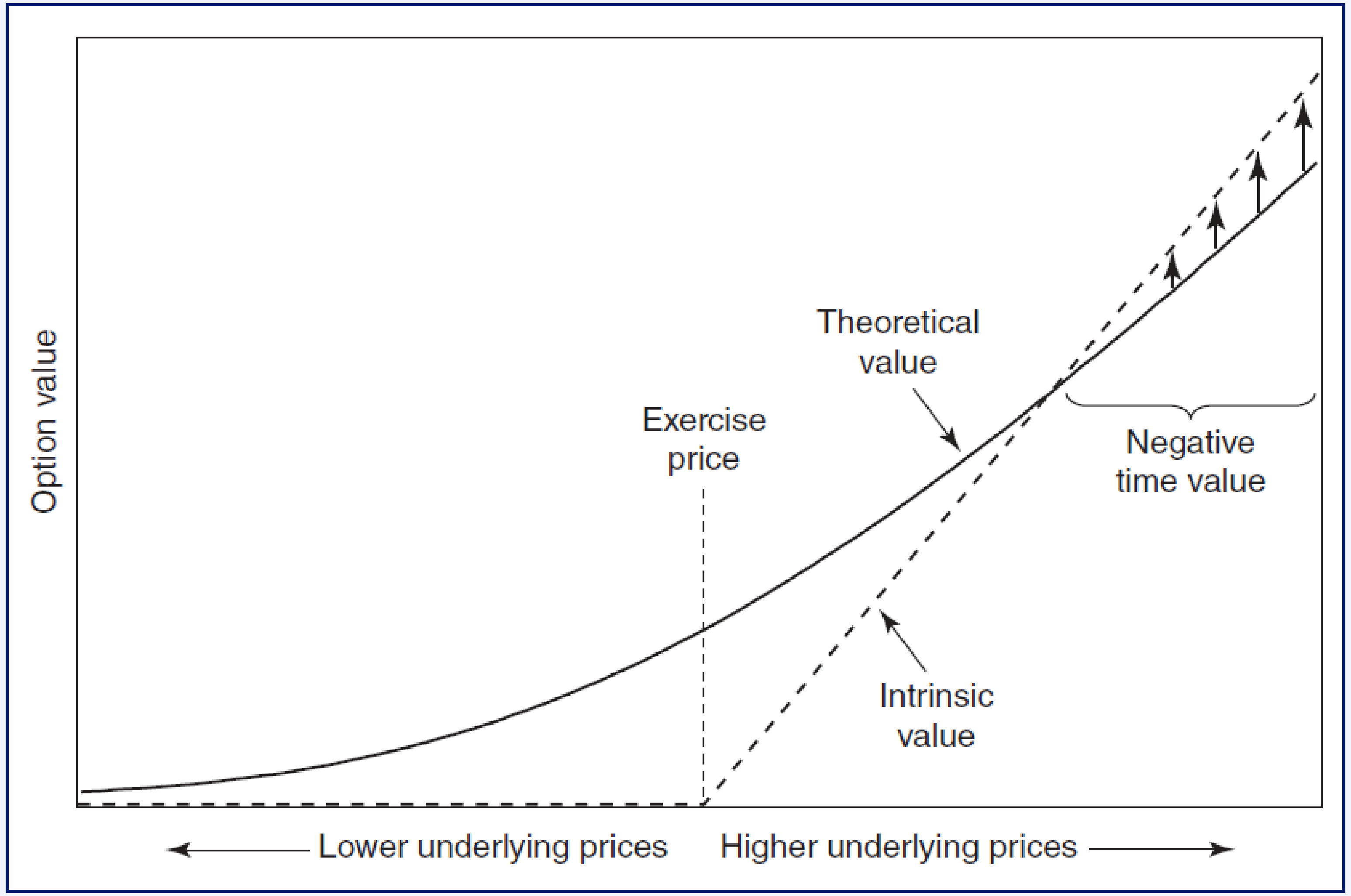
Theta, or "time decay", is the rate at which an option LOSES value as time passes, holding other variables constant.
Typically expressed as price change per day, every option loses value as time passes*
Highest for ATM options… and for ATM options the Theta will increase as the option nears expiration





Θ θ

theta



Negative time decay??
Technically, all options are losing "extrinsic premium" associated with volatility as time passes.

However- for options which are

- *deep ITM and thus*
- *have a large premium, and*
- *are subject to stock-type settlement, and*
- *cannot be exercised early (EURO options)*

the theta will become negative at a certain point, as the PV of the option premium is subject to a smaller discount factor with every passing day.

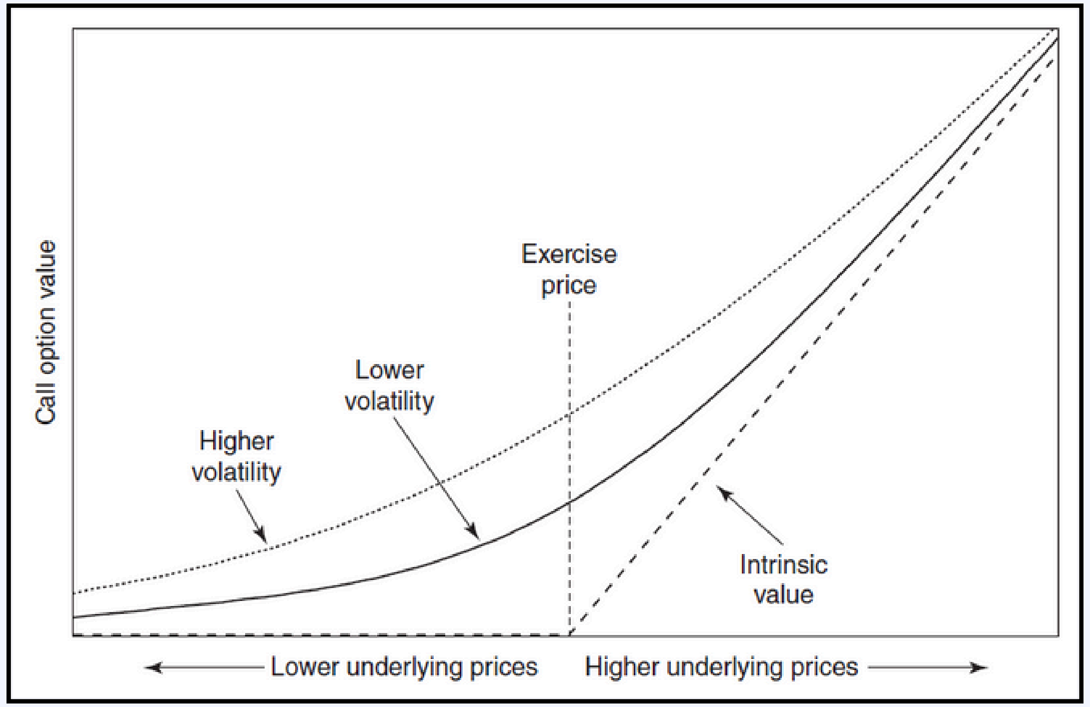
This intrinsic value is often referred to as an option's Box Premium

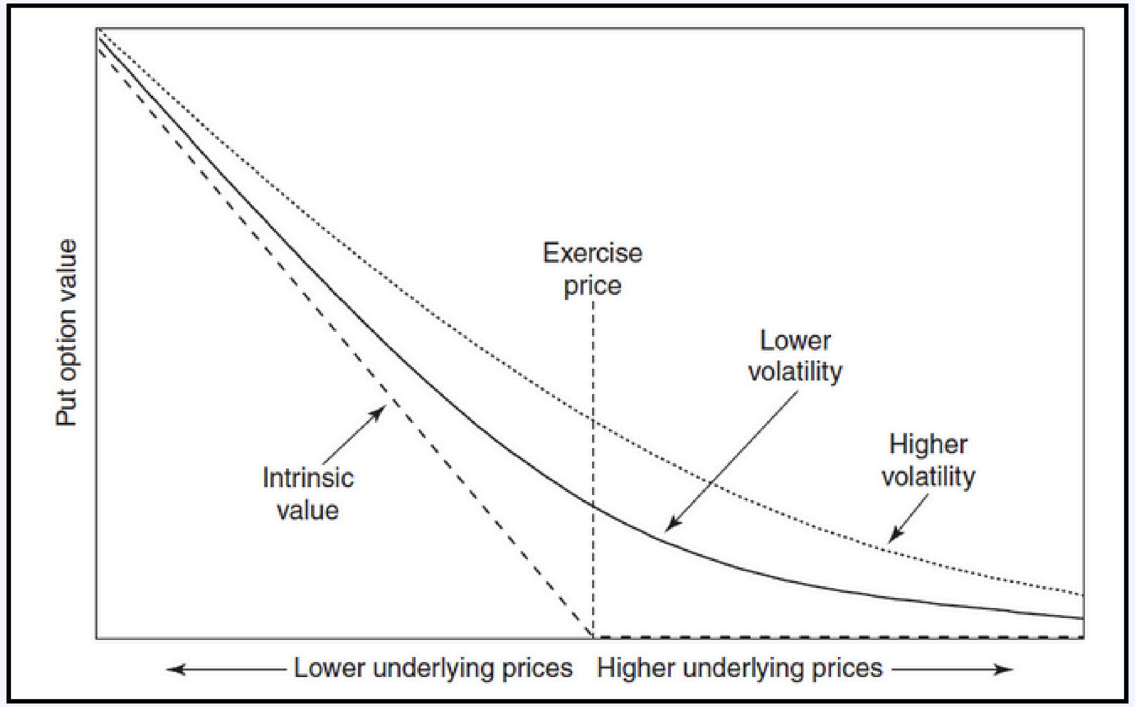
Θ θ

theta

The Vega

Vega refers to an option's change in theoretical value per 1.00% change in Implied Volatility
All options have positive Vega. Vega is highest for ATM options, and generally increases WITH time to expiration.





ν
vega

The Rho

Rho describes the sensitivity of an option's theoretical value to a 1% change in interest rates

Rho depends on option & underlying-

SPX options are treated as "stock options," per the adjacent table.

In practice, Rho is "downstream" of the aforementioned Greeks.

Can be hedged with SOFR future strips (formerly Eurodollar futures)

	If domestic rates rise	If domestic rates fall	If foreign rates rise	If foreign rates fall
stock option calls will	rise	fall	not applicable	not applicable
stock option puts will	fall	rise	not applicable	not applicable
futures option calls				
(stock-type settlement)	fall	fall	not applicable	not applicable
futures option puts				
(stock-type settlement)	fall	fall	not applicable	not applicable
futures option calls	no	no		
(futures-type settlement)	effect	effect	not applicable	not applicable
futures option puts	no	no		
(futures-type settlement)	effect	effect	not applicable	not applicable
foreign currency option calls	rise	fall	fall	rise
foreign currency option puts	fall	rise	rise	fall

Ρ ρ

rho

Interpreting the Risk Measures

If you are . . .	Your delta position is	Your gamma position is	Your theta position is	Your vega position is	Your rho position is
Long the underlying contract	+	0	0	0	0
Short the underlying contract	−	0	0	0	0
Long calls	+	+	−	+	+ (on stock) − (on futures)*
Short calls	−	−	+	−	− (on stock) + (on futures)*
Long puts	−	+	−	+	− (on stock) − (on futures)*
Short puts	+	−	+	−	− (on stock) + (on futures)*

*This applies when options on futures are subject to stock-type settlement.
If options on futures are subject to futures-type settlement, the effective rho is zero.

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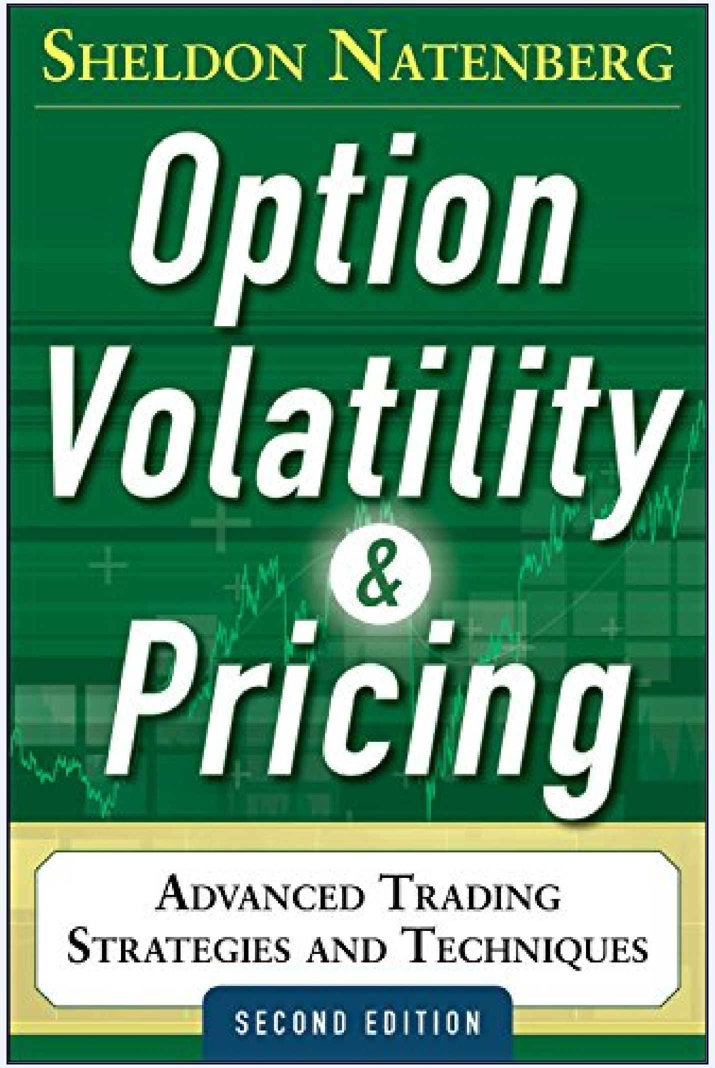
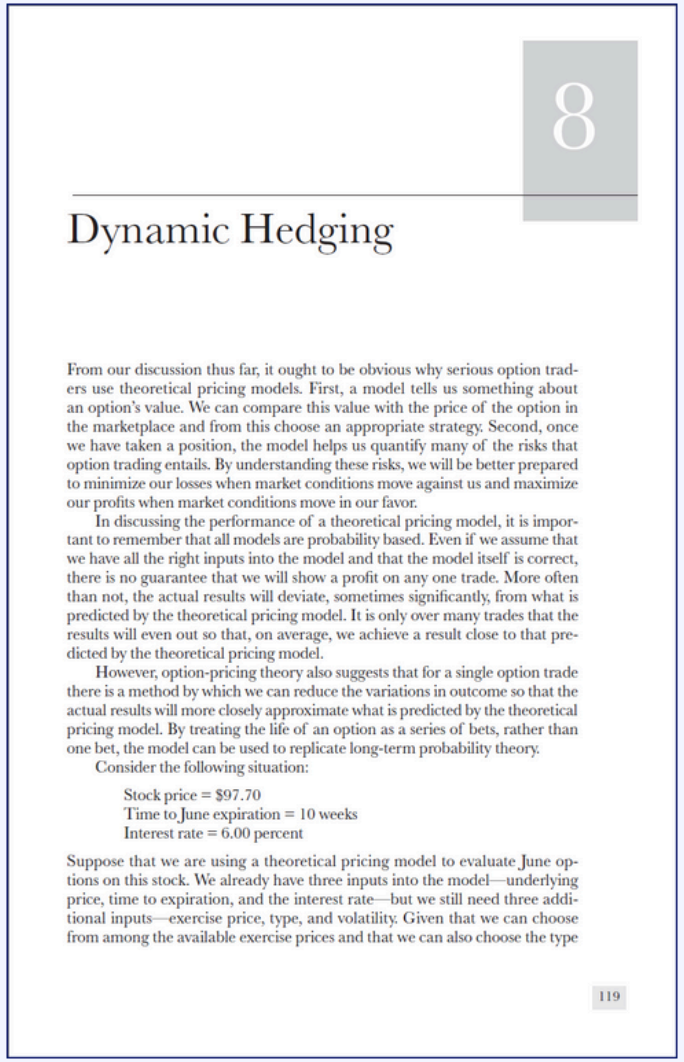
Next up...





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Dynamic Hedging

From our discussion thus far, it ought to be obvious why serious option traders use theoretical pricing models. First, a model tells us something about an option's value. We can compare this value with the price of the option in the marketplace and from this choose an appropriate strategy. Second, once we have taken a position, the model helps us quantify many of the risks that option trading entails. By understanding these risks, we will be better prepared to minimize our losses when market conditions move against us and maximize our profits when market conditions move in our favor.

In discussing the performance of a theoretical pricing model, it is important to remember that all models are probability based. Even if we assume that we have all the right inputs into the model and that the model itself is correct, there is no guarantee that we will show a profit on any one trade. More often than not, the actual results will deviate, sometimes significantly, from what is predicted by the theoretical pricing model. It is only over many trades that the results will even out so that, on average, we achieve a result close to that predicted by the theoretical pricing model.

However, option-pricing theory also suggests that for a single option trade there is a method by which we can reduce the variations in outcome so that the actual results will more closely approximate what is predicted by the theoretical pricing model. By treating the life of an option as a series of bets, rather than one bet, the model can be used to replicate long-term probability theory.

Consider the following situation:

Stock price = $97.70
Time to June expiration = 10 weeks
Interest rate = 6.00 percent

Suppose that we are using a theoretical pricing model to evaluate June options on this stock. We already have three inputs into the model—underlying price, time to expiration, and the interest rate—but we still need three additional inputs—exercise price, type, and volatility. Given that we can choose from among the available exercise prices and that we can also choose the type

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