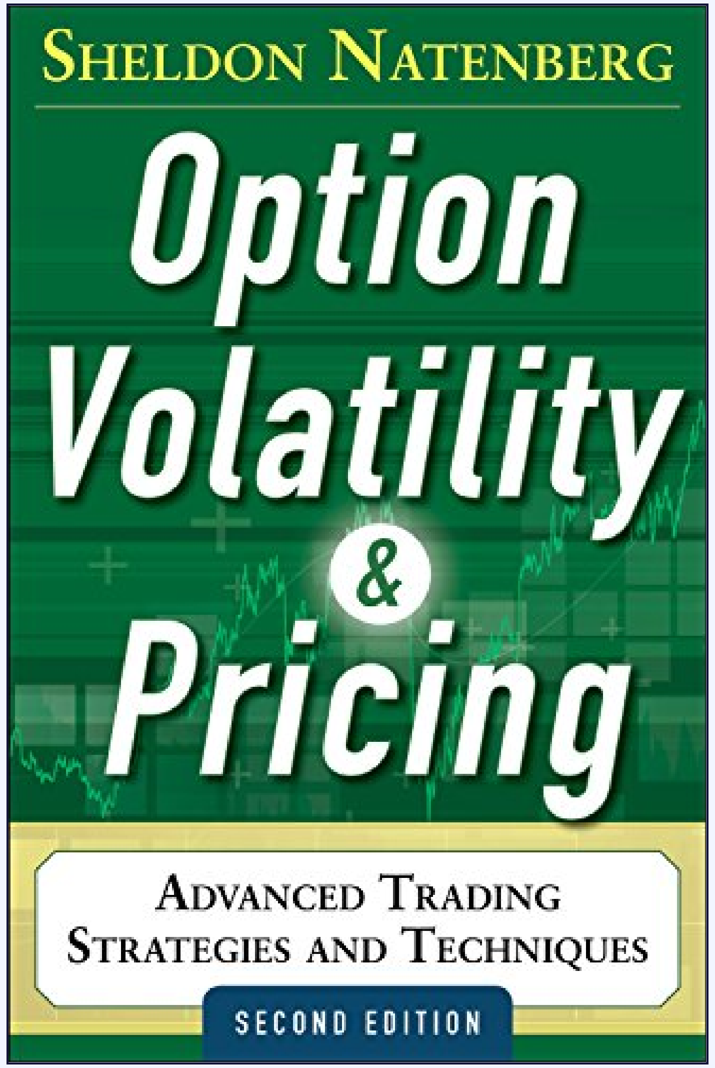
VolSignals

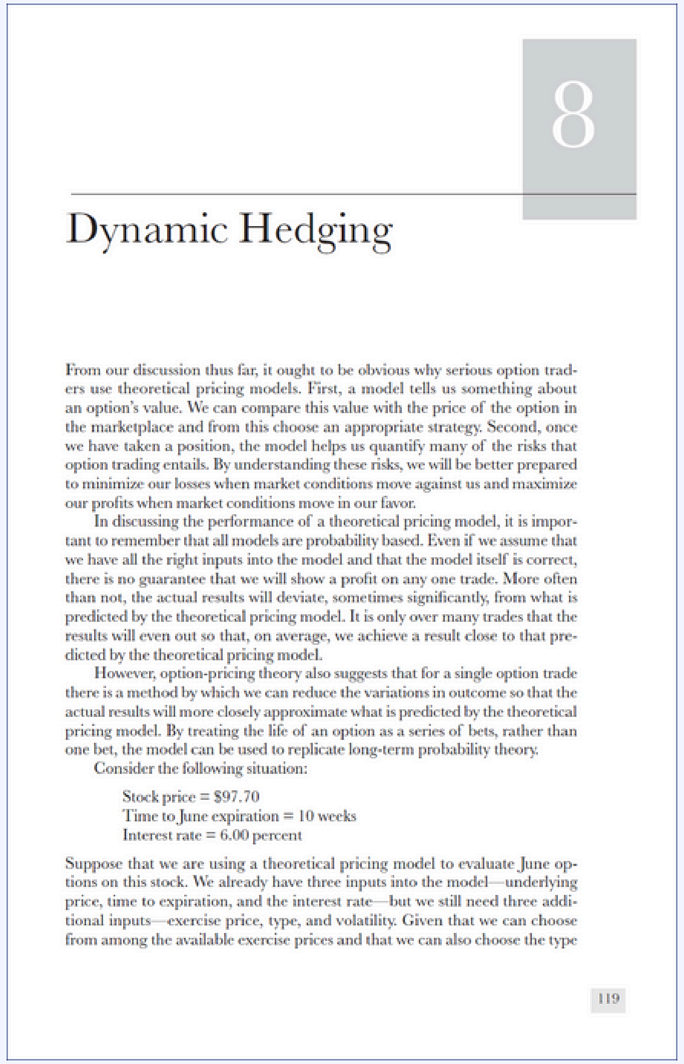
VolStudies

Chapter 8 Dynamic Hedging

VolStudies | Option Volatility & Pricing

Chapter 8 — Dynamic Hedging





8

Dynamic Hedging

From our discussion thus far, it ought to be obvious why serious option traders use theoretical pricing models. First, a model tells us something about an option's value. We can compare this value with the price of the option in the marketplace and from this choose an appropriate strategy. Second, once we have taken a position, the model helps us quantify many of the risks that option trading entails. By understanding these risks, we will be better prepared to minimize our losses when market conditions move against us and maximize our profits when market conditions move in our favor.

In discussing the performance of a theoretical pricing model, it is important to remember that all models are probability based. Even if we assume that we have all the right inputs into the model and that the model itself is correct, there is no guarantee that we will show a profit on any one trade. More often than not, the actual results will deviate, sometimes significantly, from what is predicted by the theoretical pricing model. It is only over many trades that the results will even out so that, on average, we achieve a result close to that predicted by the theoretical pricing model.

However, option-pricing theory also suggests that for a single option trade there is a method by which we can reduce the variations in outcome so that the actual results will more closely approximate what is predicted by the theoretical pricing model. By treating the life of an option as a series of bets, rather than one bet, the model can be used to replicate long-term probability theory.

Consider the following situation:

Stock price = \$97.70
Time to June expiration = 10 weeks
Interest rate = 6.00 percent

Suppose that we are using a theoretical pricing model to evaluate June options on this stock. We already have three inputs into the model—underlying price, time to expiration, and the interest rate—but we still need three additional inputs—exercise price, type, and volatility. Given that we can choose from among the available exercise prices and that we can also choose the type

“Serious options traders use theoretical pricing models”

. . .but why?

Natenberg offers two reasons:

- *“First, a model tells us something about an option’s value. We can compare this value with the price of the option in the marketplace and from this choose an appropriate strategy”*
- *“Second, once we have taken a position, the model helps us quantify many of the risks that option trading entails.”*

In practice?

"First, a model tells us something about an option's value. We can compare this value with the price of the option in the marketplace and from this choose an appropriate strategy"

- *The reality works in reverse… traded price IS the market's "value" and the model allows us to solve for implied volatility. This foundation allows us to compare option trades and eventually construct volatility surfaces*

"Second, once we have taken a position, the model helps us quantify many of the risks that option trading entails."

- *Quantifying the various risks allows us to HEDGE them* 🎯

Remember, a model has limitations…

Longevity in trading requires understanding the limitations of your model, and accounting for those limitations in your style of position management.

And all models in this context are probability based…

Even if your model is excellent and your inputs are correct-
…any particular trade can deviate tremendously from your expectation.

For a Market Maker, a solid model with limitations you can both understand AND manage is the foundation of your operation.

When your "edge" is sound and reproducible, what do you want?

…more bets.

Example

Given the following:

- *Stock price = $97.70*
- *Time Until June Expiration = 10 Weeks*
- *Interest Rate = 6.00%*

We still need:

- *Exercise Price*
- *Type*
- *Volatility**

Let's evaluate the 100 Strike Call with an IV of 37.62%

Black Scholes Option Pricer: (European Call, K=100, spot=97.7, t=(10/52), r=0.06, vol=.3762)

Input Data

Stock Asset Price:	97.70	US $
Option Strike Price:	100	US $
Maturity (Time Until Expiration):	0.191781	Years
Risk-Free Interest Rate:	.06	Annual %
Volatility:	.3762	Annual %

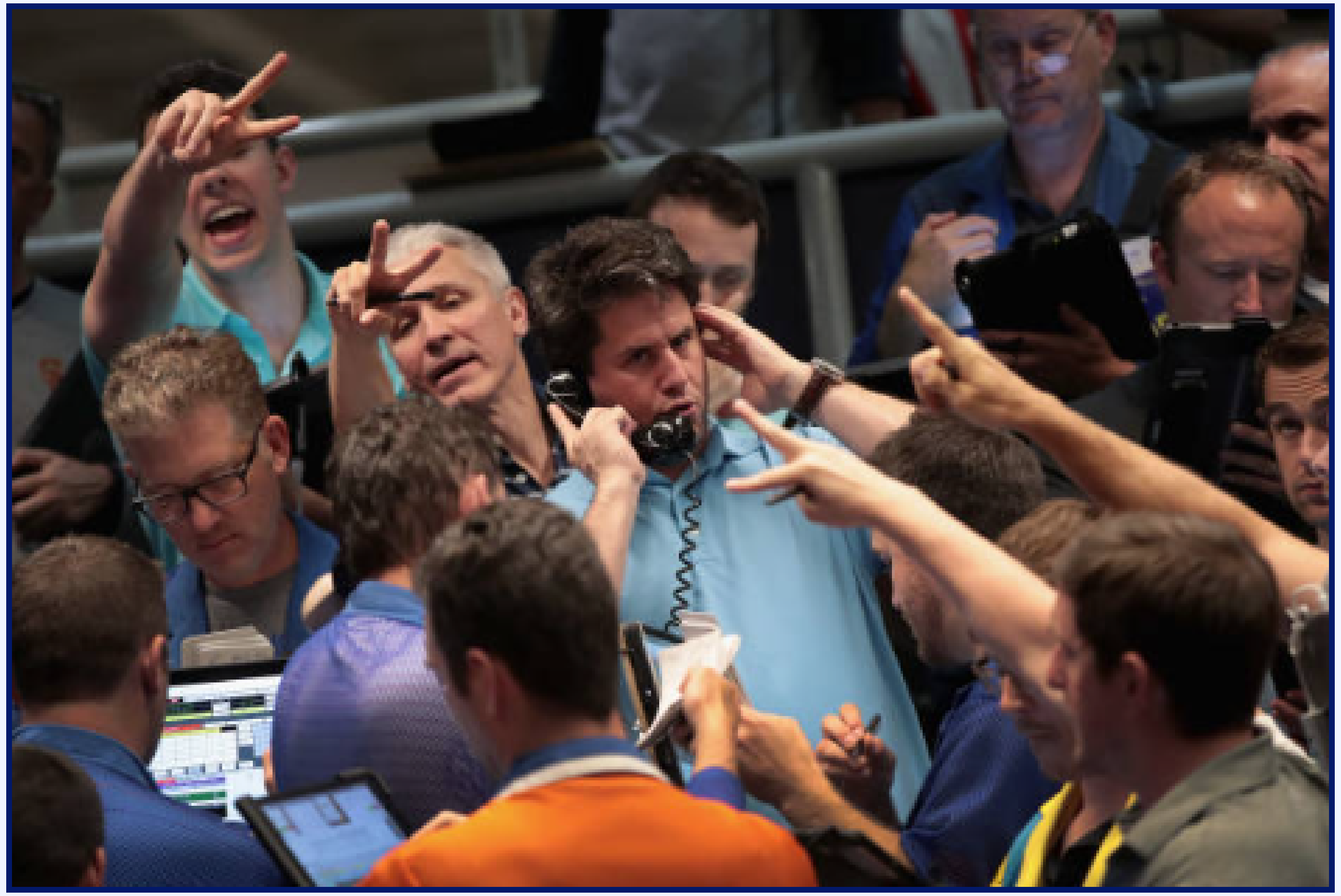
Calculate

Options (Fair Value)

European Call:	5.8905	US $

With our inputs, the theoretical fair value for our option is $5.89. Let's get a quote!

Chapter 8 — Dynamic Hedging



“Can I get a two-way on the June 100 Call tied to 97.70? Looking to pay ~$5.50”

“5.50? They’re at 5 to start”

“Pay it...”

Chapter 8 — Dynamic Hedging

We bought the June 100 C $0.89 under our theoretical ("theo") value—

Nice trade! *(Maybe…)*
Now what?

First order of business— the HEDGE

- *First step is to offset the model's option delta with a position in the underlying*
- *Converts directionality to volatility when done properly*
- *What about the language of our "quote" indicates this trade included a hedge?*

Carrying on with Natenberg's example from the text, our hedged position looks like this:

Position	Contract Delta	Delta Position
Long 1 June 100 Call	50	+50
Short 50 Shares	1	–50

Δ δ
delta

Now we have to “manage the position.”

This will involve tracking the underlying price movement and trading the underlying to maintain delta-neutrality. We’ll use weekly hedge intervals in this example.

**We are using the model strictly here, and assuming constant IV as well as Interest Rates- this means we are not “marking to market” our option in this example.*

After one week, the stock moves +1.80 to 99.50

Δ δ

delta

Chapter 8 — Dynamic Hedging

Using our model to reprice the option →

Black Scholes Option Pricer: (European Call, K=100, spot=99.50, t=(9/52), r=0.06, vol=.3762)

The updated Call price is $6.46 and the ***model returns a Delta of 54***

Our option Delta has changed… but our position in the underlying shares remains the same.

We have to rebalance the hedge in order to restore Delta-neutrality to the position.

Position	Contract Delta	Delta Position
Long 1 June 100 Call	54	+54
Short 50 Shares	1	−50

Δ δ

delta

Nothing tricky about this one-

Position	Contract Delta	Delta Position
Long 1 June 100 Call	54	+54
Short 54 Shares	1	–54

We have to sell 4 shares in order to adjust the overall position Delta back to zero.

"Filled… 4 @ $99.50"

Each re-hedge is like a new "bet."

This is "dynamic hedging", as opposed to "static hedging", which we'll explore later in the book.

Δ δ
delta

Chapter 8 — Dynamic Hedging

Here's what happens if we keep this up until the contract's expiration:

Stock price = 97.70 — Time to June expiration = 10 weeks; Interest rate = 6.00%; Volatility = 37.62%

June 100 call: Price = 5.00 (Implied volatility = 32.40%); Theoretical value = 5.89; Delta = 50

Week	Share Price	Delta of 100 Call	Total Delta Position	Adjustment (Contracts)	Total Adjustments	Adjustment Cash Flow	Interest on Adjustments
0	97.70	50	0				
1	99.50	54	+4	Sell 4	Short 4	+398.00	+4.12
2	92.75	35	−19	Buy 19	Long 15	−1762.25	−16.22
3	95.85	43	+8	Sell 8	Long 7	+766.80	+6.18
4	96.20	43	0	None	Long 7	0	0
5	102.45	62	+19	Sell 19	Short 12	+1946.55	+11.20
6	93.30	28	−34	Buy 34	Long 22	−3172.20	−14.60
7	91.15	17	−11	Buy 11	Long 33	−1002.65	−3.46
8	95.20	27	+10	Sell 10	Long 23	+952.00	+2.19
9	102.80	72	+45	Sell 45	Short 22	+4626.00	+5.32
10	103.85			Buy 22		−2284.70	

Δ δ

delta

Incorporating the Option Settlement & Original Hedge:

We paid $5.00 for the June 100 Call & sold 50 shares at 97.70 to open the position.

At June expiration, the stock went out 103.85 leaving us with leg PNLs of:

- *-$115.00 on the option purchase (-5.00 + (103.85-100.00)) x 100*
- *-$307.50 on the original hedge (-50 x (97.70-103.85))*

Total loss on original position = -$422.50

Adjustments & Cumulative Interest Paid / Received:

Note, our hedging activity (the adjustments) produced positive cash flow throughout the 10 weeks...

Adding all the adjustment cash flows indicates a profit of $467.55

Adding up the interest (paid or received) associated with each adjustment nets a loss of -$5.28

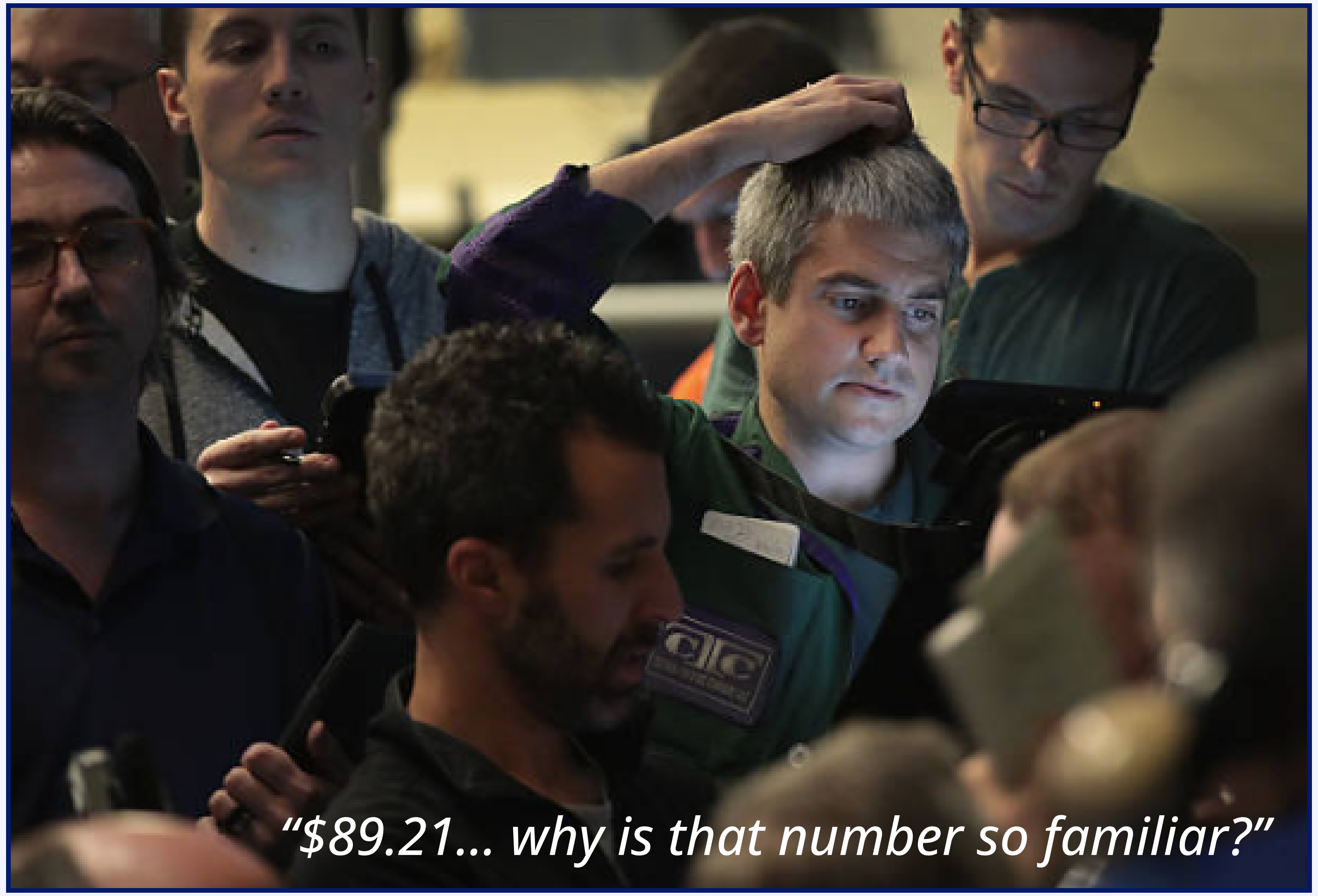
The option position cost $5.75 to finance (-$5.75 cash flow),
...but we earned $56.21 in interest on the $4,885 balance associated with our initial stock sale.

Adding all cash flows yields a total P&L of $90.24. Discounting to account for the 10 week period-

Initial Value of Total P&L = $89.21

VolStudies | Option Volatility & Pricing

Chapter 8 — Dynamic Hedging

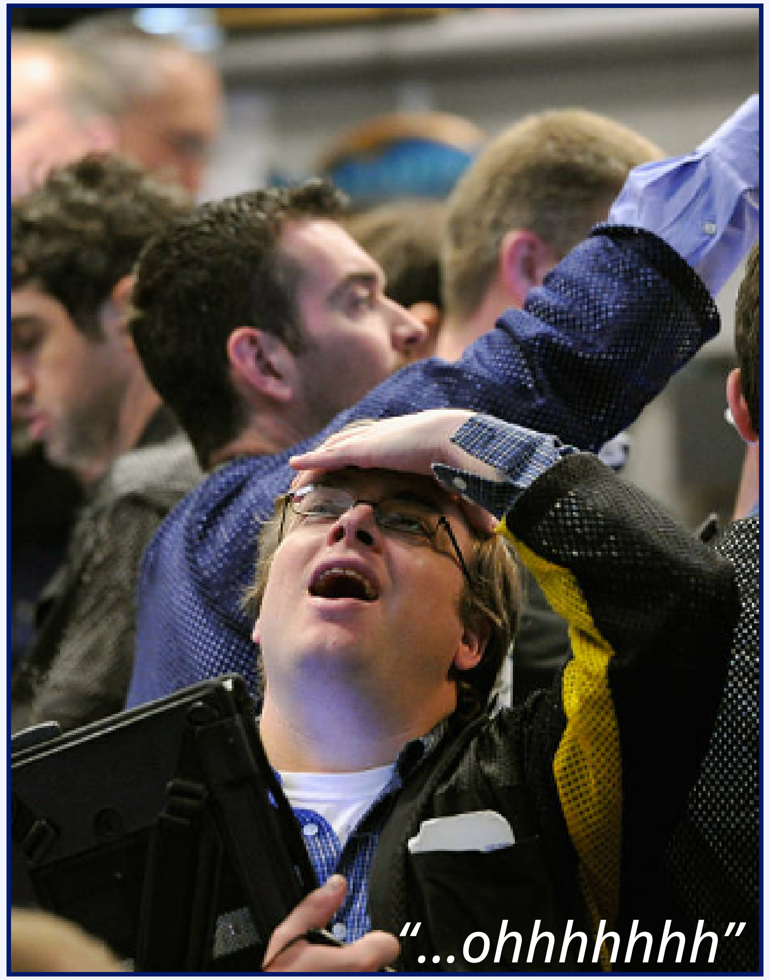


VolSignals

VolStudies | Option Volatility & Pricing

Chapter 8 — Dynamic Hedging

Chapter 8 — Dynamic Hedging



A complete look— individual contributions to total P&L:

Dynamic Hedging Results		
Original hedge P&L:		–422.50
Option P&L	100 × (3.85 – 5.00) = –115.00	
Stock P&L	50 × (97.70 – 103.85) = –307.50	
Adjustment P&L:		+467.55
Carry (interest) on the options:		
	100 × –5.00 × 6.00% × 70/365 = –5.75	–5.75
Carry (interest) on the stock:		
	50 × +97.70 × 6.00% × 70/365 = +56.21	+56.21
Interest on the adjustments:		–5.27
Total cash flow:		+90.24
Discounted cash flow:	90.24/(1 + 0.06 × 70/365) = 89.21	+89.21
Predicted P&L:	100 × (5.89 – 5.00) = 100 × 0.89 = 89.00	+89.00

Was this example realistic?

Yes... but we relied on some basic assumptions:

- *Interest rates are constant*
- *Volatility is constant*
- *Volatility compounds continuously*
- *Traders buy/sell the underlying with no friction*
- *Traders can borrow/lend freely at the same rate*
- *It's free to trade!*
- *(What are taxes?)*

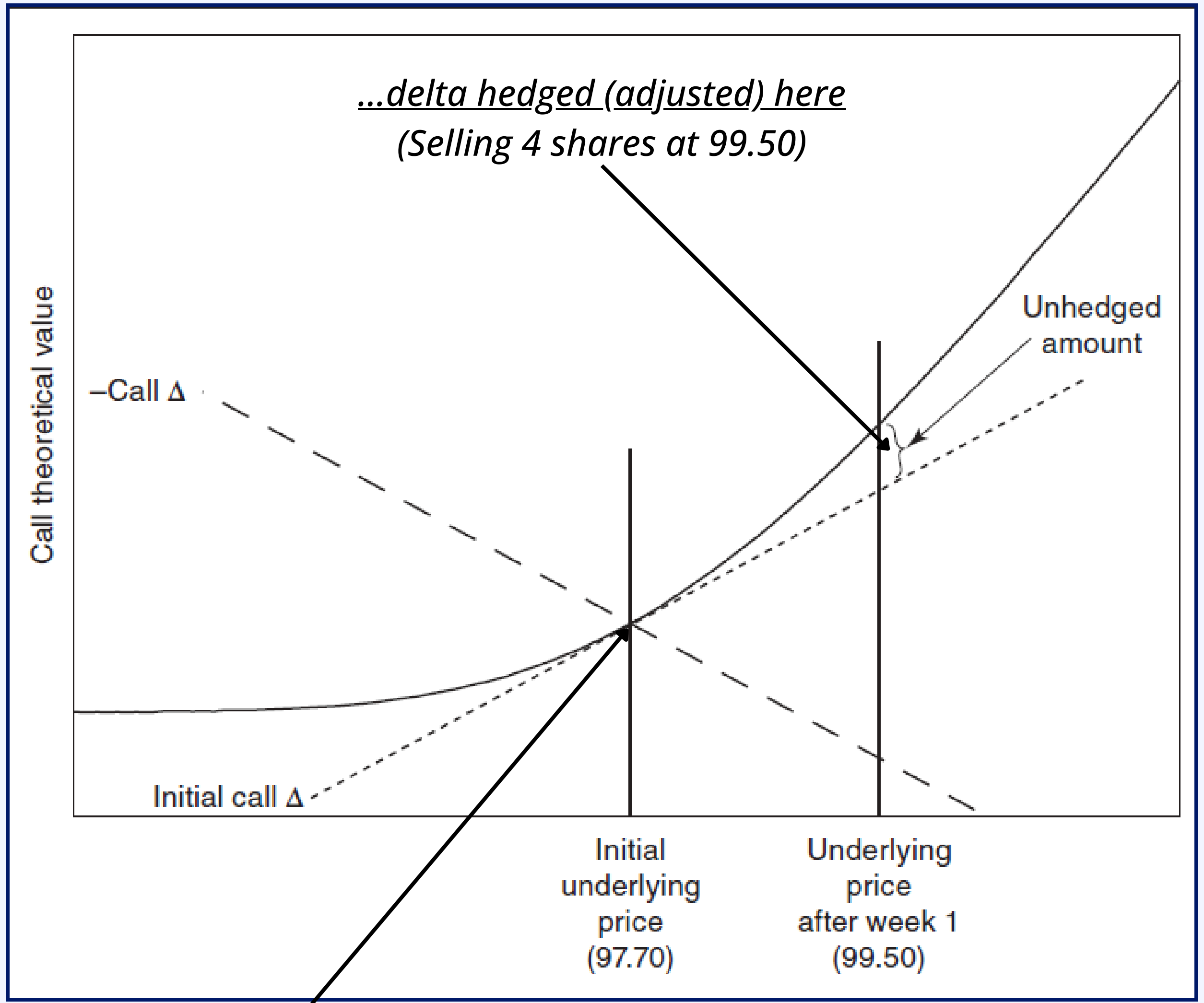
These all reflect reality... right?

The reality:

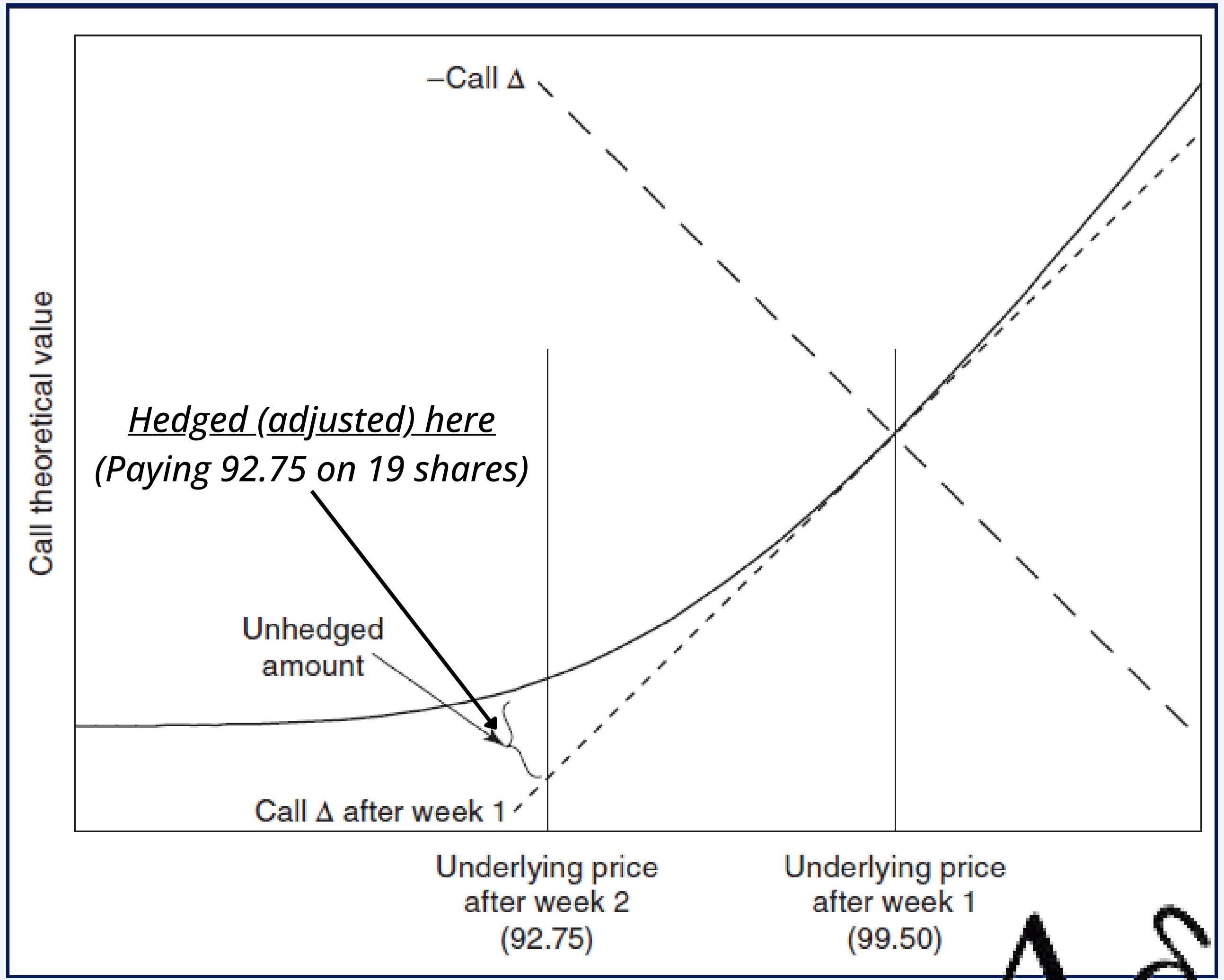
- *Interest rates are constant*
  - *Nope*
- *Volatility is constant*
  - *Nope. Not even close.*
- *Volatility compounds continuously*
  - *LOL*
- *Traders buy/sell the underlying with no friction*
  - *Limits? Gaps?*
- *Traders can borrow/lend freely at the same rate*
  - *Not so easily*
- *It's free to trade*
  - *Not for us it isn't!*
- *(What are taxes?)*
  - *-No comment-*

VolStudies | Option Volatility & Pricing

Chapter 8 — Dynamic Hedging



Initially hedged to delta neutral here
(Selling 50 shares at 97.70)



Recall, each re-hedge (adjustment) is like a new bet

Δ δ
delta

Chapter 8 — Dynamic Hedging

Delta Hedging Considerations-

Delta hedging should be continuous… but in reality:

- *Discrete time intervals*
- *Delta tolerance*
- *Underlying moves**

Greater hedge frequency should reduce deviation from theoretical edge (in theory)

Realized Volatility Considerations…

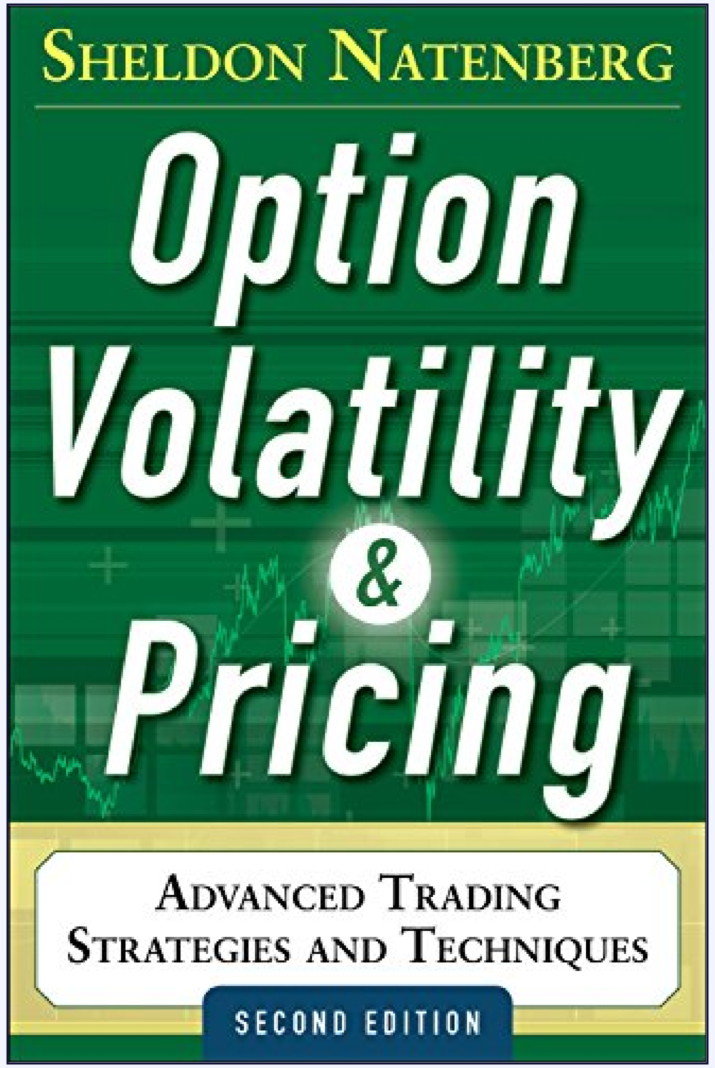
- *If actual Volatility was less than 32.40%, the overall position would have shown a LOSS*
- *If you were the MM that sold the option in the example, you'd have the opposite P&L*

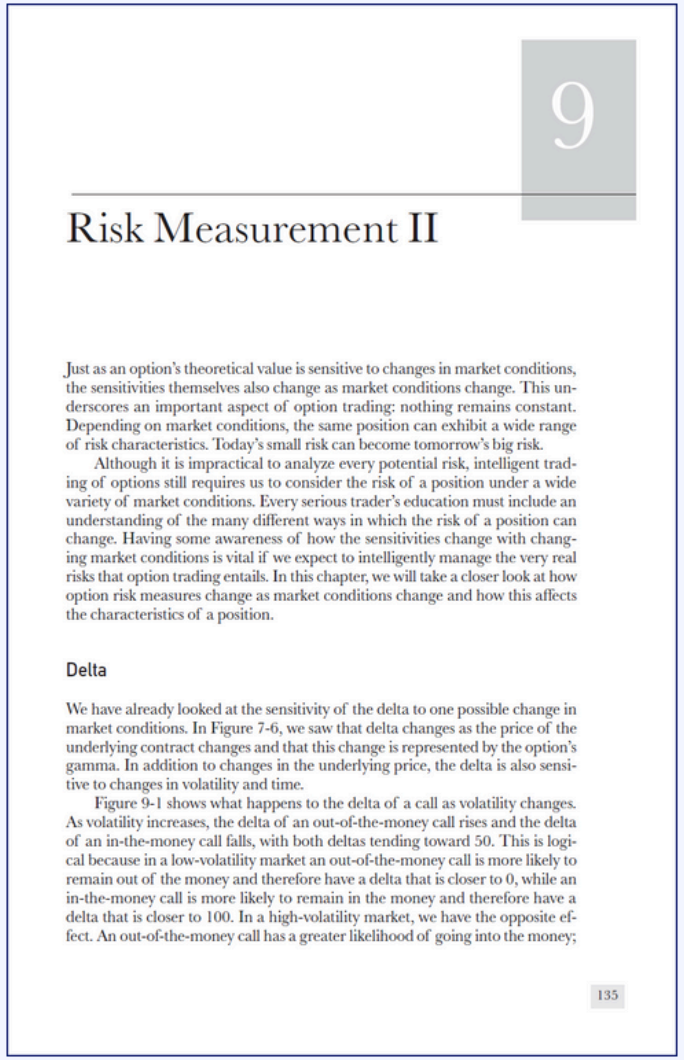
Δ δ

delta

VolStudies | Option Volatility & Pricing

Next up...





9

Risk Measurement II

Just as an option's theoretical value is sensitive to changes in market conditions, the sensitivities themselves also change as market conditions change. This underscores an important aspect of option trading: nothing remains constant. Depending on market conditions, the same position can exhibit a wide range of risk characteristics. Today's small risk can become tomorrow's big risk.

Although it is impractical to analyze every potential risk, intelligent trading of options still requires us to consider the risk of a position under a wide variety of market conditions. Every serious trader's education must include an understanding of the many different ways in which the risk of a position can change. Having some awareness of how the sensitivities change with changing market conditions is vital if we expect to intelligently manage the very real risks that option trading entails. In this chapter, we will take a closer look at how option risk measures change as market conditions change and how this affects the characteristics of a position.

Delta

We have already looked at the sensitivity of the delta to one possible change in market conditions. In Figure 7-6, we saw that delta changes as the price of the underlying contract changes and that this change is represented by the option's gamma. In addition to changes in the underlying price, the delta is also sensitive to changes in volatility and time.

Figure 9-1 shows what happens to the delta of a call as volatility changes. As volatility increases, the delta of an out-of-the-money call rises and the delta of an in-the-money call falls, with both deltas tending toward 50. This is logical because in a low-volatility market an out-of-the-money call is more likely to remain out of the money and therefore have a delta that is closer to 0, while an in-the-money call is more likely to remain in the money and therefore have a delta that is closer to 100. In a high-volatility market, we have the opposite effect. An out-of-the-money call has a greater likelihood of going into the money;

135

Chapter 9 — Risk Measurement II

VolSignals

VolStudies