Primer / Refresher on Core Greeks Critical to Your Understanding

The following pages are excerpts from Sheldon Natenberg’s “Option Volatility and Pricing”

Many of you have likely read or heard of this book. It is widely used among market making firms as ‘required reading’ in its entirety for incoming classes. Word is one major Chicago firm contracted Sheldon himself to lead some of the training for new hires.

If you haven’t read the book, I do recommend it -> it’s a good transition into a more pragmatic understanding of the real world use of options from hedging to spreading, and it introduces the reader to thinking about the complexity of managing an options ‘book’ (helpful for our purposes).

Greeks, Hedging, & Dynamics (“Hedging Greeks as They Change”)

For this lesson, I’ve cut out the most important concepts for you to understand as we move forward.

If you are brand new -> Read this, twice. (As long as it takes to understand)

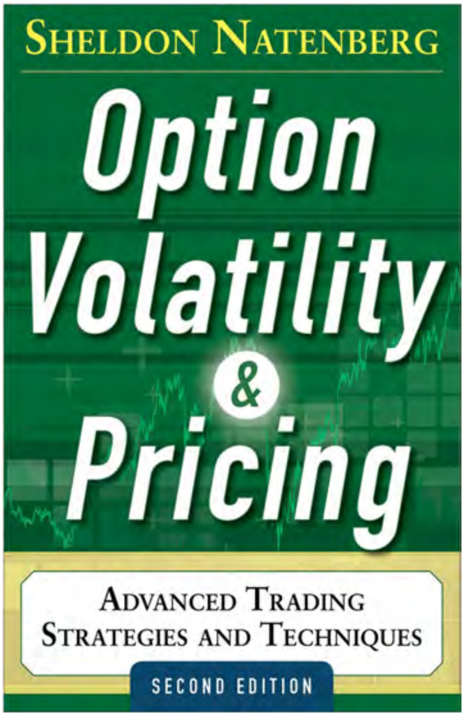
If you’ve read it before but haven’t thought about it in a while -> Read it once, closely.

If you intuitively grasp these concepts already -> Start with the Squeeze Metrics White Paper & refer back to these excerpts as-needed.

Upcoming Lesson + Notes

Consider the material herein a prerequisite for the upcoming talk.

Read and digest -> Next, we’ll talk you through (pragmatically) how to think about these Greeks, the hedging of them (in the aggregate), and we’ll discuss the GEX/VEX approach and its pros & cons.



While I'll make the book available in its entirety in our shared folder - for now, the excerpts below covering the Greeks we can hedge (and how they change) are good to read if you are new to the material or need a refresher:

The Delta

The *delta* (Δ) is a measure of an option's risk with respect to the direction of movement in the underlying contract. A positive delta indicates a desire for upward movement; a negative delta indicates a desire for downward movement. The delta has several different interpretations, any of which may be use-ful to a trader depending on the types of strategies being executed.

Rate of Change

At expiration, an option is worth exactly its intrinsic value. Prior to expiration, however, the theoretical value of an option is a curve that will approach intrin-sic value as the option goes very deeply into the money or very far out of the money. This is shown in Figure 7-4. As the underlying price rises, the slope of the graph approaches +1; as the underlying price falls, the slope of the graph approaches zero. The delta of the call at any given underlying price is the slope of the graph—the rate of change in the option's value with respect to movement in the underlying contract.

Assuming that all other market conditions remain unchanged, a call option can never gain or lose value more quickly than the underlying contract, nor can it move in the opposite direction of the underlying market. The delta of a call must therefore have an upper bound of 1.00 if the call is very deeply in the money and a lower bound of 0 if the call is very far out of the money. Most calls will have deltas somewhere between 0 and 1.00, changing value more slowly than changes in the price of the underlying contract. A call with a delta of 0.25 will change its value at 25 percent of the rate of change in the price of the underlying contract. If the underlying rises (falls) 1.00, the option can be expected to rise (fall) 0.25. A call with a delta of 0.75 will change its value at 75 percent of the rate of change in the price of the underlying contract. If the underlying rises (falls) 0.60, the option can be expected to gain (lose) 0.45 in value. A call with a delta close to 0.50 will rise or fall in value at just about half the rate of change in the price of the underlying contract.

Puts have characteristics similar to calls except that put values move in the opposite direction of the underlying market. In Figure 7-5, we can see that when the underlying price rises, puts lose value; when the underlying price falls, puts gain value. For this reason, puts always have negative deltas, ranging from 0 for far out-of-the-money puts to –1.00 for deeply in-the-money puts. As with call deltas, put deltas measure the rate of change in the put's value with respect to a change in the price of the underlying, but the negative sign indicates that the change will be in the opposite direction of the underlying contract.

Figure 7-4 Theoretical value of a call.

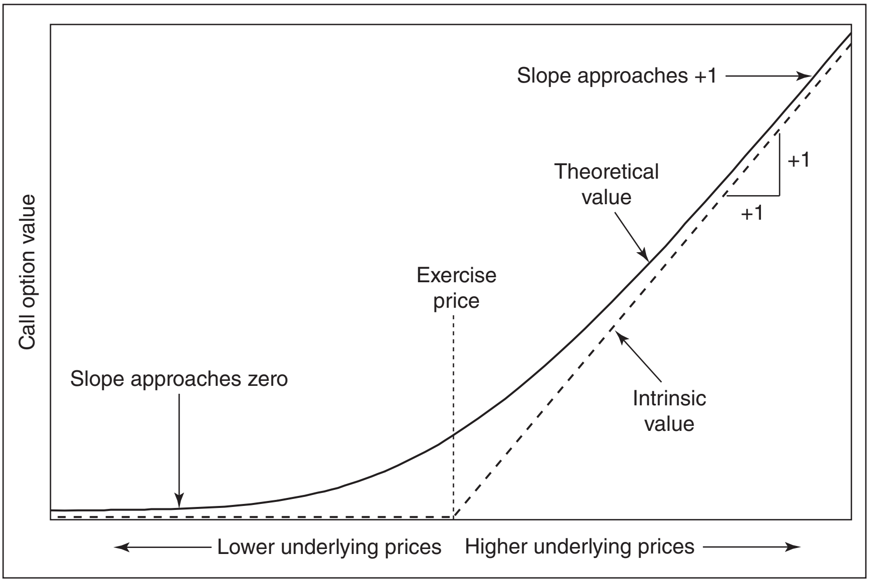
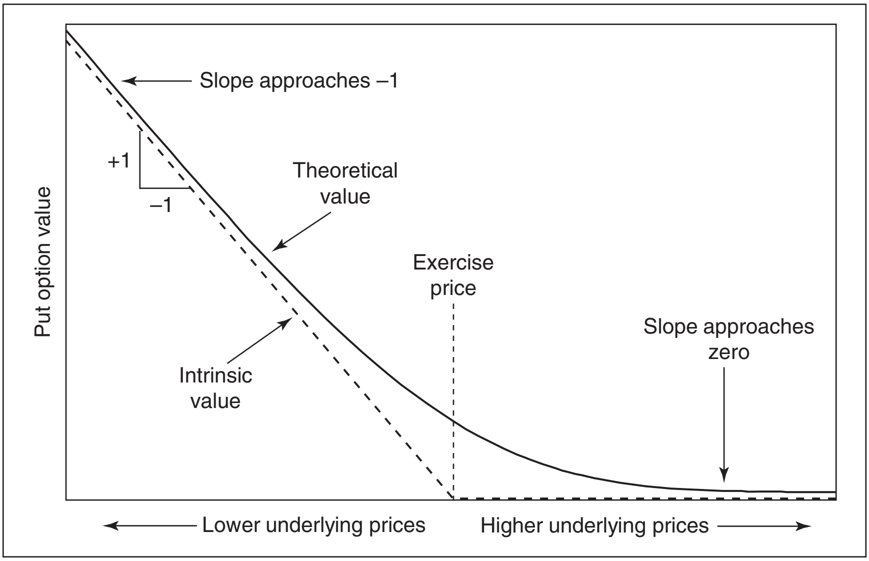


Figure 7-5 Theoretical value of a put.



A put with a delta of –0.10 will change its value at 10 percent of the rate of change in the price of the underlying contract, but in the opposite direction. If the underlying moves up (down) 0.50, the put can be expected to lose (gain) 0.05 in value. A put with a delta of –0.50 will change its value at approximately half the rate of the underlying, but in the opposite direction.

An option position is often combined with a position in the underlying contract. To determine the total risk of a combined position, we will need to assign a delta value to the underlying contract. Logically, a position in the underlying contract will gain or lose value at exactly the rate of change in the underlying price. Therefore, regardless of whether the underlying is stock, a futures contract, or some other instrument, the underlying contract always has a delta of 1.00.

Although delta values range from 0 to 1.00 for calls and from 0 to –1.00 for puts, it has become common practice among many option traders to express delta values as a whole number by dropping the decimal point, a convention that we will follow in this text.[2] Using this format, the delta of a call will fall within the range of 0 to 100, and the delta of a put within the range of –100 to 0. An underlying contract will always have a delta of 100.

Hedge Ratio

In Chapter 5, we introduced the concept of a *riskless*, or *neutral*, hedge, a position that, within a small price range, will neither gain nor lose value as the

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[2] This convention originated in the U.S. stock option market, where it became common for stock option traders to equate one delta with one share of stock. Because the underlying contract consisted of 100 shares, traders assigned a delta of 100 to the underlying contract. Many futures option traders also express the delta using this whole-number format.

price of the underlying contract moves up or down. We can determine the proper number of underlying contracts to option contracts required for such a hedge by dividing 100 (the delta of the underlying contract) by the option's delta. For a call option with a delta of 50, the proper hedge ratio is 100/50, or 2/1. For every two options purchased (sold), we need to sell (buy) one underlying contract to establish a neutral hedge. A call option with a delta of 40 requires the sale (purchase) of two underlying contracts for every five options purchased (sold) because 100/40 = 5/2.

The hedge ratio interpretation also applies to puts, except that when we buy puts, we need to buy the underlying contract, and when we sell puts, we need to sell the underlying contract. A put with a delta of –75 will require the purchase (sale) of three underlying contracts for each four puts purchased (sold) because 100/–75 = 4/–3.

A position is neutrally hedged, or *delta neutral*, if the total of all the deltas that make up the position add up to 0. If we buy two calls with a delta of 50 each and sell one underlying contract, the total delta position is

$$\begin{array}{r} +2 \times \ \ 50 \\ \underline{-1 \times 100} \\ 0 \end{array}$$

If we sell four puts with a delta of –75 each and sell three underlying contracts, the total delta position is

$$\begin{array}{r} +4 \times \ -75 \\ \underline{-3 \times 100} \\ 0 \end{array}$$

Both positions are delta neutral.[3]

A position that is delta neutral has no particular preference for either upward or downward movement in the price of the underlying contract. Although a trader may take whatever delta position he feels is appropriate, either bullish (delta positive) or bearish (delta negative), we will see in Chapter 8 that a trader who is trying to capture the theoretical value of an option must start with and maintain a delta-neutral position over the entire life of an option.

Theoretical or Equivalent Underlying Position

Many option traders come to the option market after trading in the underlying contract. Futures option traders often start their careers by trading futures; stock option traders often start by trading stock. If a trader has become accustomed to evaluating his risk in terms of the number of underlying contracts bought or sold (either futures contracts or shares of stock), he can use the delta to equate the directional risk of an option position with a position of similar size in the underlying market.

Because an underlying contract always has a delta of 100, in terms of directional risk, each 100 deltas in an option position is theoretically equivalent

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[3] It is customary to indicate the purchase of a contract or contracts with a plus sign (a long contract position) and the sale of a contract or contracts with a negative sign (a short contract position).

to one underlying contract. A trader who owns an option with a delta of 50 is long, or controls, approximately half of an underlying contract. If he owns 10 such contracts, he is long 500 deltas or, in equivalent terms, five underlying contracts. If the underlying is a futures contract, the trader is theoretically long five such contracts. If the underlying is a stock contract consisting of 100 shares of stock, he is theoretically long 500 shares of stock. The trader has a similar theoretical position if he sells 20 puts with a delta of –25 each because –20 × –25 = +500.

It is important to emphasize the theoretical aspect of the delta interpretation as an equivalent to an underlying position. An option is not simply a surrogate for an underlying position. An actual underlying position is almost exclusively sensitive to directional moves in the underlying market. An option position, while sensitive to directional moves, is also sensitive to other changes in market conditions. An option trader who looks only at his delta position may be ignoring other factors that could have a far greater impact on his position. The delta represents an equivalent underlying position only under very narrowly defined market conditions.

Which interpretation—rate of change in the theoretical value, the hedge ratio, or the equivalent underlying position—should a trader use? That depends on how the trader intends to use the delta. A trader who has a delta position of +500 knows that he has a position that is similar to being long five underlying contracts (the equivalent-underlying-position interpretation). If he is a disciplined theoretical trader striving to maintain a delta-neutral position, he must sell five underlying contracts (the hedge-ratio interpretation). And finally, if he is bullish and maintains his current delta position of +500, the value of his position will change at approximately five times, or 500 percent, of the rate of change in the price of the underlying contract (the rate-of-change interpretation). If the price of the underlying contract rises by 2.00, the trader's position should gain approximately 10.00. If the price of the underlying contract falls by 1.25, the trader's position should lose approximately 6.25. Mathematically, all these interpretations are the same. A trader will choose a delta interpretation that is consistent with his approach to trading.

Probability

There is one other interpretation of the delta that is perhaps of less practical use, but is still worth mentioning. If we ignore the sign of the delta (positive for calls, negative for puts), the delta is approximately equal to the probability that the option will finish in the money. A call with a delta of 25 or a put with a delta of –25 has approximately a 25 percent chance of finishing in the money. A call with a delta of 75 or a put with a delta of –75 has approximately a 75 percent chance of finishing in the money. As an option's delta moves closer to 100, or –100 for puts, the option becomes more and more likely to finish in the money. As the delta moves closer to 0, the option becomes less and less likely to finish in the money. This also explains why at-the-money options tend to have deltas close to 50. If we assume that price changes are random, there is half a chance that the market will rise (the option goes into the money) and half a chance that the market will fall (the option goes out of the money).[4]

Of course, the delta is only an approximation of the probability because interest considerations and, in the case of stock options, dividends may distort this interpretation. Moreover, most option strategies depend not only on whether an option finishes in the money but also by how much. If a trader sells an option with a delta of 10 in the belief that the option will expire worthless nine times out of 10, he may indeed be correct. But, if on the tenth time he loses an amount greater than the total premium he took in the nine times the option expired worthless, the trade will result in a negative expected return. To trade options intelligently, we need to consider not only how often a strategy wins or loses but also how much it wins or loses. Every experienced trader is willing to accept several small losses if he can occasionally offset these with one big win that more than offsets the losses. In the same way, no experienced trader will want to pursue a strategy that leads to multiple small profits but occasionally results in a disastrous loss.[5]

The Gamma

Figure 7-6 shows call and put delta values using the whole-number format. Even though deltas range from 0 to 100 for calls and from −100 to 0 for puts, the graphs are not straight lines. As the underlying price rises or falls, the slope of the graph changes, approaching 0 at both extremes. If this were not true, the delta values of calls could fall below 0 or rise above 100, and the delta values of puts could fall below −100 or rise above 0. The slope appears to be greatest when the underlying price is close to the option's exercise price.

The *gamma* (Γ), sometimes referred to as the option's *curvature*, is the rate of change in the delta as the underlying price changes. The gamma is usually expressed in deltas gained or lost per one-point change in the underlying, with the delta increasing by the amount of the gamma when the underlying rises and falling by the amount of the gamma when the underlying falls. If an option has a gamma of 5, for each point rise (fall) in the price of the underlying, the option will gain (lose) 5 deltas.[6] If the option initially has a delta of 25 and the underlying moves up (down) one full point, the new delta of the option will be 30 (20). If the underlying moves up (down) another point, the new delta will be 35 (15).[7]

From Figure 7-6, we can see that the delta graphs of both calls and puts have essentially the same shape and that the graphs always have a positive slope.

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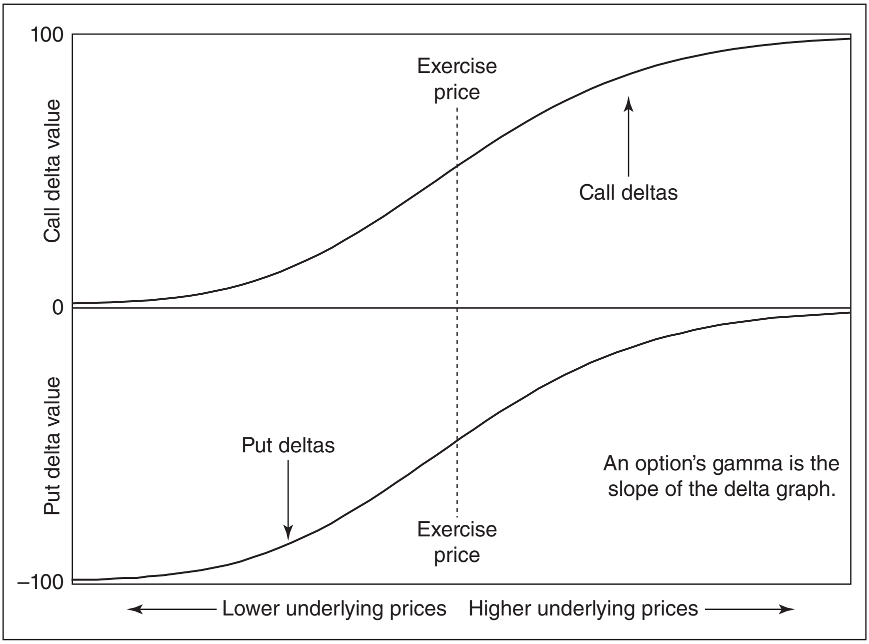
[4] Because option values are based on the forward price of the underlying contract, it is actually the at-the-forward option that tends to have a delta closest to 50. This is one reason why options that are seemingly out of the money can have deltas greater than 50. With a stock at 100, one year to expiration, and an interest rate of 10 percent, the forward price for the stock is 110. Under these conditions, the 110 call will have a delta close to 50, while the 105 call will have a delta greater than 50.

[5] In fact, the delta is only an approximation of the probability that an option will finish in the money. We will see later that the Black-Scholes model generates a number that more precisely reflects this probability.

[6] In fact, the delta is only an approximation of the probability that an option will finish in the money. We will see later that the Black-Scholes model generates a number that more precisely reflects this probability.

[7] For simplicity, we assume here that the gamma is constant. In reality, the gamma, like all risk measures, will change as market conditions change.

Figure 7-6 Delta values.



This suggests that calls and puts with the same exercise price and time to expiration have the same gamma values and that these values are always positive. This may seem strange to a new trader who, because of the delta, tends to associate positive numbers with calls and negative numbers with puts. But regardless of whether we are working with calls or puts, we always add the gamma to the old delta as the underlying price rises and subtract the gamma from the old delta as the underlying price falls. When a trader is long options, whether calls or puts, he has a long gamma position.

For example, consider both an at-the-money call with a delta of 50 and an at-the-money put with a delta of –50. How will the delta change as the underlying price changes if both options have gamma values of 5? If the underlying price rises one full point, we add the gamma of 5 to the call delta of 50 to get the new delta of 55. To get the new put delta if the underlying contract rises one point, we also *add* the gamma of 5 to the put delta of –50 to get the new delta of –45. This is intuitively logical—as the underlying price rises, at-the-money calls move into the money and at-the-money puts move out of the money. If the underlying contract falls one full point, in both cases we *subtract* the gamma, resulting in a call delta of 50 – 5 = 45 and a put delta of –50 – 5 = –55. Now the call is moving out of the money and the put is moving into the money.

Because all options individually have positive gamma values, we can create a positive gamma position by buying options, either calls or puts, and a negative gamma position by selling options. For a complex position consisting of many different options, we use the same interpretation of the gamma as we do for individual options, adding the gamma to the old delta as the underlying contract rises and subtracting the gamma as the market falls. A positive gamma position will gain deltas as the market rises (we are adding a positive number) and lose deltas as the market falls (we are subtracting a positive number). A negative gamma position will behave in just the opposite way, losing deltas as the market rises (we are adding a *negative* number) and gaining deltas as the market falls (we are subtracting a *negative* number). Moreover, the rate of change in the delta will be determined by the size of the gamma position. New traders are often advised to avoid large gamma positions, particularly negative ones, because of the speed with which the directional risk, as reflected by the delta, can change.

While the delta is a measure of how an option's value will change if the underlying price changes, it is important to remember that it represents an instantaneous measure. It is only valid for very small price changes. If the underlying makes a sizable move, any estimate of the option's new value using a constant delta will become less and less reliable. We can, however, improve this estimate if we also take into consideration the gamma.

Suppose that at price S_1 a call has a theoretical value C, a delta Δ, and a gamma Γ. If the price of the underlying changes from S_1 to S_2, what should be the new value of the option? One approach might be to simply multiply the change in price, $S_2 - S_1$, by the delta and add it to the original value C

$$C + [\Delta \times (S_1 - S_2)]$$

But this assumes that the delta is constant, which it is not. As the underlying price moves from S_1 to S_2, the delta of the option is also changing. When the underlying price reaches S_2, the new delta of the option will be

$$\Delta + (S_1 - S_2) \times \Gamma$$

Which delta should we use for our calculation, the original delta (Δ) or the new delta $[\Delta + (S_1 - S_2) \times \Gamma]$? Rather than use either of these delta values, we might logically use the average delta over the price range S_1–S_2

$$\text{Average delta} = [\Delta + \Delta + (S_1 - S_2) \times \Gamma]/2 = \Delta + (S_1 - S_2) \times \Gamma/2$$

This is not a precise solution because the gamma also changes as the underlying price changes, but it will yield a better estimate than using a constant delta. Using the average delta, the new value of the option should be approximately[8]

$$C + (S_1 - S_2) \times [\Delta + (S_1 - S_2) \times \Gamma/2] = C + [(S_1 - S_2) \times \Delta] + [(S_1 - S_2)^2 \times \Gamma/2]$$

This approach applies equally well to puts, as long as we remember that a put will have a negative delta.

For example, suppose that at an underlying price of 97.50, a call option has a theoretical value of 3.65, a delta of 40, and a gamma of 2.5. If the underlying contract rises to 101.50, what should be the option's new value?

At the new underlying price of 101.50, the delta of the option is

$$40 + 4 \times 2.5 = 50$$

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[8] When using the delta to estimate the change in an option's value, we need to remember that it is really a percent value, or a value between 0 and 1.00.

The average delta as the underlying price rises from 97.50 to 101.50 is

$$(40 + 50)/2 = 45$$

Using the average delta, the new option value is approximately

$$3.65 + (4.00 \times 0.45) = 5.45$$

9

Risk Measurement II

Just as an option's theoretical value is sensitive to changes in market conditions, the sensitivities themselves also change as market conditions change. This underscores an important aspect of option trading: nothing remains constant. Depending on market conditions, the same position can exhibit a wide range of risk characteristics. Today's small risk can become tomorrow's big risk.

Although it is impractical to analyze every potential risk, intelligent trading of options still requires us to consider the risk of a position under a wide variety of market conditions. Every serious trader's education must include an understanding of the many different ways in which the risk of a position can change. Having some awareness of how the sensitivities change with changing market conditions is vital if we expect to intelligently manage the very real risks that option trading entails. In this chapter, we will take a closer look at how option risk measures change as market conditions change and how this affects the characteristics of a position.

Delta

We have already looked at the sensitivity of the delta to one possible change in market conditions. In Figure 7-6, we saw that delta changes as the price of the underlying contract changes and that this change is represented by the option's gamma. In addition to changes in the underlying price, the delta is also sensitive to changes in volatility and time.

Figure 9-1 shows what happens to the delta of a call as volatility changes. As volatility increases, the delta of an out-of-the-money call rises and the delta of an in-the-money call falls, with both deltas tending toward 50. This is logical because in a low-volatility market an out-of-the-money call is more likely to remain out of the money and therefore have a delta that is closer to 0, while an in-the-money call is more likely to remain in the money and therefore have a delta that is closer to 100. In a high-volatility market, we have the opposite effect. An out-of-the-money call has a greater likelihood of going into the money;

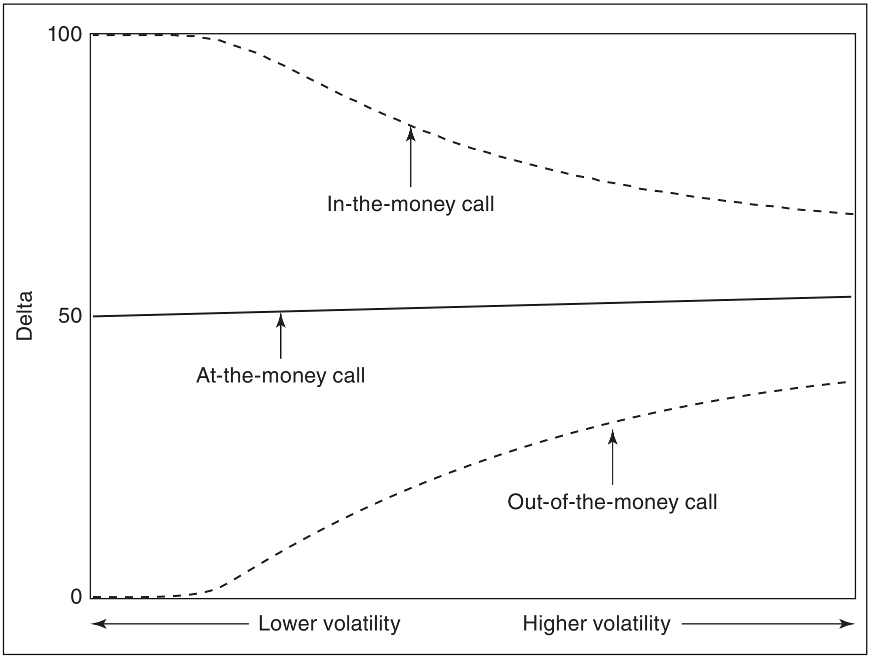
an in-the-money call has a greater likelihood of going out of the money. Consequently, the deltas of both options will move toward 50.

Note that the delta of an at-the-money option tends to remain close to 50 regardless of volatility. This is true in general, although changing interest rates or, in the case of stock options, changing dividends may alter the forward price. Because theoretical pricing models evaluate options in relation to the forward price, the delta of an at-the-money call may in fact be either more or less than 50. Even if the option is exactly at the forward (the exercise price and forward price are the same), a call will still have a delta that is slightly greater than 50 because of the lognormal distribution used to evaluate the option. This is evident in Figure 9-1, where the delta of an at-the-money call tends to increase slightly as volatility increases.

Because an option's delta changes as volatility changes, no trader can be certain that a position is really delta neutral. The delta depends on the volatility of the underlying contract, and this is something that will occur in the future over the life of the option. The volatility we use to calculate the delta is a guess. We might guess right, but we also might guess wrong. And if we guess wrong, our delta values will be wrong.

Rather than try to guess the future volatility, many traders use the *implied delta*, the delta that results from using the implied volatility. Using this approach, the delta will change as implied volatility changes, even if the underlying contract remains the same. Consider a trader who owns 40 call

Figure 9-1 Call delta values as volatility changes.



options with an implied volatility of 32 percent and a corresponding implied delta of 25 each. Because 40 × 25 = 1,000, to hedge the position delta neutral, the trader will sell 10 underlying contracts. If, however, implied volatility rises to 36 percent, the delta of the options will tend toward 50. If the new implied delta is 30, the trader's delta position is now (40 × 30) – (10 × 100) = +200. The trader's position changed from neutral to bullish even though no other market conditions changed.

Because the delta depends on the volatility, but volatility is an unknown factor, calculation of the delta can pose a major problem for a trader, especially for a large option position. Using the implied volatility to calculate the delta is only one possible approach.

Figure 9-2 shows what happens to call deltas as time passes. Note the similarities to Figure 9-1. Delta values move toward 50 if we increase either time to expiration or volatility and move away from 50 if we reduce either of these inputs. In many situations, time and volatility will have a similar effect on options. More time, like higher volatility, increases the likelihood of large price changes. Less time, like lower volatility, reduces the likelihood of large price changes. If a trader cannot immediately determine the effect on an option's value or sensitivity of changing time, he might instead consider the effect of changing volatility. Conversely, if he cannot determine the effect of changing volatility, he might consider the effect of changing time. Both effects are likely to be similar.

Figure 9-2 Call delta values as time passes.

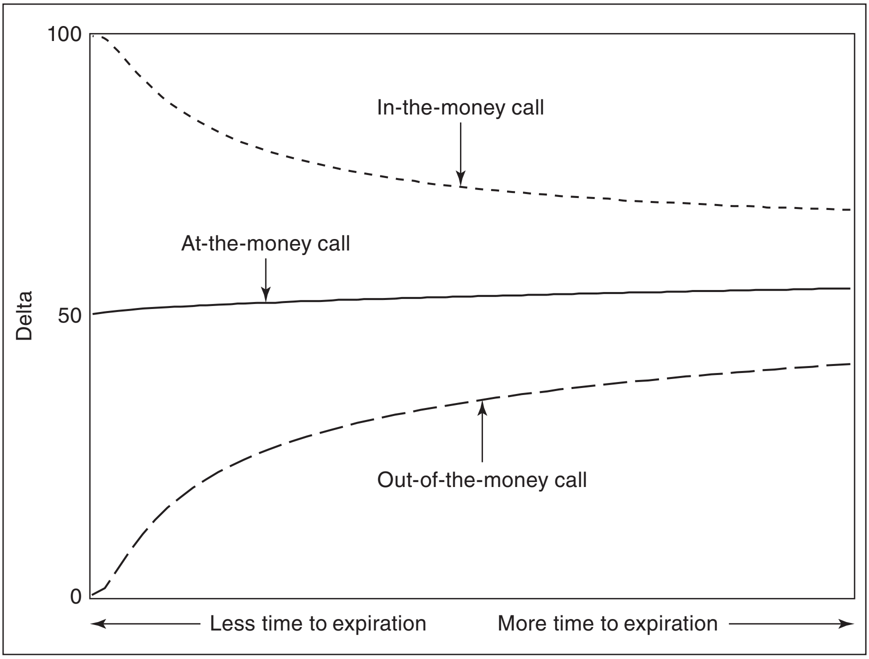
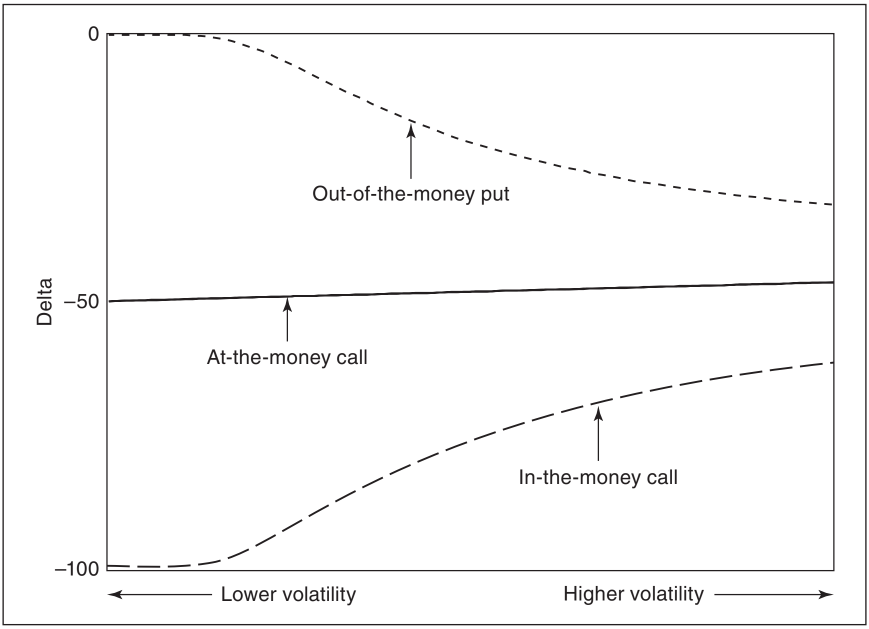
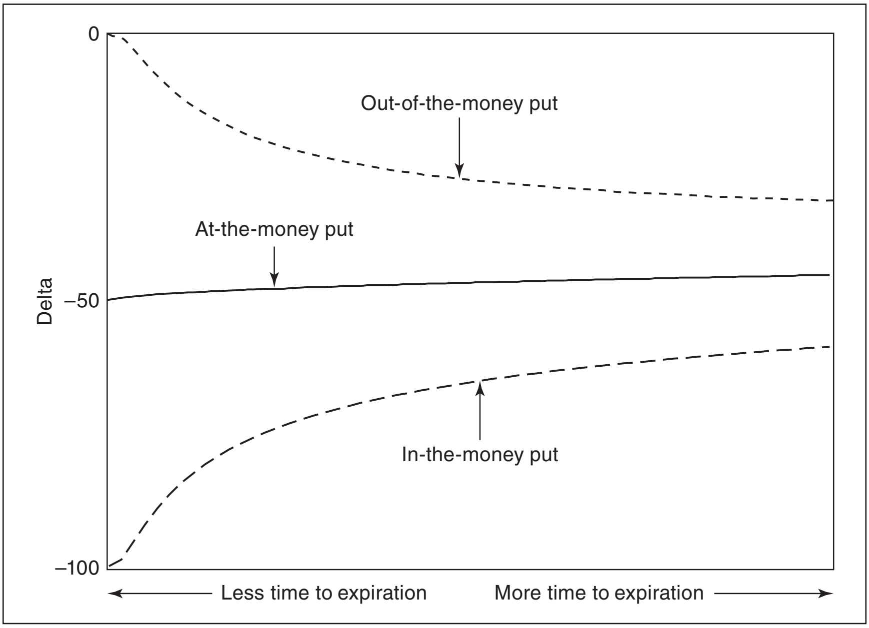


Figure 9-3 Put delta values as volatility changes.



The effects of volatility and time on put deltas are the same as those on call deltas, except that put deltas tend toward 0 and –100 as volatility falls or time passes and toward –50 as volatility rises. This is shown in Figures 9-3 and 9-4.

Figure 9-4 Put delta values as time passes.



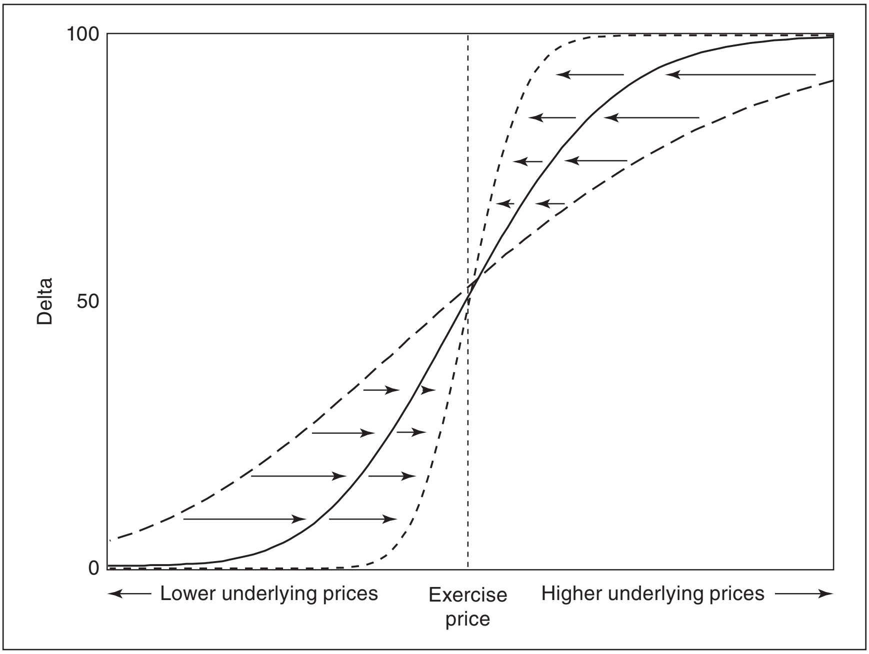
An alternative method of displaying the effects of changing time and volatility on delta values is shown in Figure 9-5. This is similar to Figure 7-6 except that we have varied time and volatility. As we lower time or reduce volatility, delta values for calls move very quickly toward either 0 for out-of-the-money options or 100 for in-the-money options.

Because delta values are affected by the passage of time, a position that is delta neutral today may not be delta neutral tomorrow, even if all other market conditions remain unchanged. Of course, with many months remaining to expiration, the passage of even several days may have little effect on the delta. If, however, expiration is quickly approaching, the passage of just one day, because it represents a large portion of the option's remaining life, can have a dramatic effect on the delta.

As option traders have become more aware of the importance of risk management, they have begun to pay closer attention to changes in the sensitivities themselves as market conditions change. In some cases, they have also begun to attach names (although not necessarily Greek letters) to these higher-order sensitivities. The sensitivity of the delta to a change in volatility is sometimes referred to as the option's *vanna*. The sensitivity of the delta to the passage of time is sometimes referred to as the option's *delta decay* or its *charm*.[1]

Which delta values are the most sensitive to changes in volatility (vanna) and time (charm)? We know that delta values will tend either toward 50 as

Figure 9-5 Call delta values as time passes or volatility declines.



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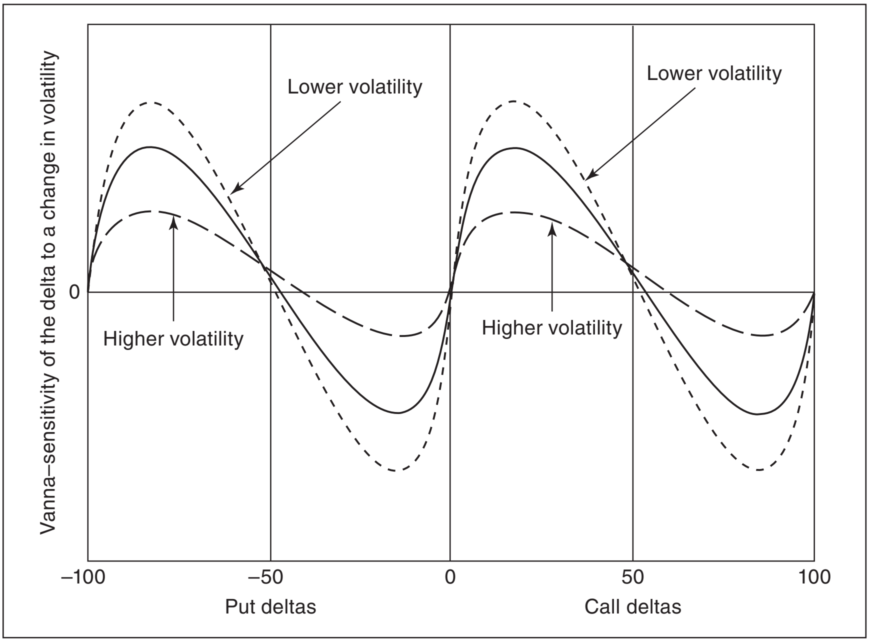
[1] In mathematics, the "sensitivity of a sensitivity" is a second-order sensitivity. The gamma, vanna, and charm are all second-order sensitivities (the sensitivity of the delta to a change in underlying price, volatility, and time to expiration, respectively).

we increase volatility or time, or away from 50 (toward 0 or 100) as we reduce volatility or time. Logically, delta values that are already close to 0, 50, or 100 are the least likely to change. At the same time, delta values that are approximately midway between these numbers are most likely to change. This is borne out by Figures 9-6 and 9-7, the vanna and charm for options with different deltas. Note that the shapes of the graphs are identical for calls and puts, with the vanna and charm approximately 0 around a delta of 50 or –50.[2] We can also see that vanna and charm are greatest for call delta values close to 20 and 80 and put delta values close to –20 and –80. Options with these deltas will move the most quickly toward 50 if we raise volatility or away from 50 if we lower volatility or reduce time to expiration.

The three vanna graphs also show that the vanna moves in the opposite direction of volatility, falling as we raise volatility and rising as we reduce volatility. The graphs of the charm exhibit similar characteristics with respect to the passage of time, falling with more time to expiration and rising with less time to expiration.

In Figures 9-6 and 9-7, we have ignored the effect of changing time on the vanna and the effect of changing volatility on the charm. From previous discussions, we might expect time and volatility to have the same effect on both these values. However, whereas vanna values are affected by changes in volatility, they are not significantly affected by changes in time to expiration. Whereas charm values are affected by time to expiration, they are not significantly affected by changes in volatility.

Figure 9-6 Vanna of an option.



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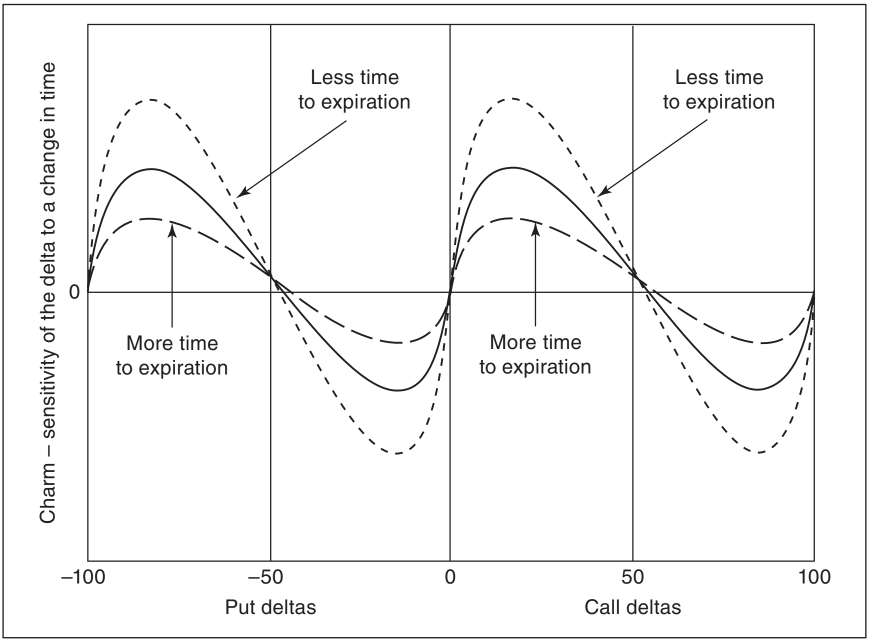
[2] The vanna is actually 0 for delta values slightly larger than 50 and smaller than –50. This is due to the nonsymmetrical characteristic of the lognormal distribution.

Figure 9-7 Charm of an option.



Theta

The theta of an option, the rate at which it decays, will vary depending not only on market conditions but also on whether an option is in the money, at the money, or out of the money. In Figure 9-8, we can see that the theta of an option is greatest when it is at the money. As the option moves either into or out of the money, its theta declines. Because the theta of an option is a function of its time value, and because very deeply in the money options and very far out of the money options have very little time value, it is logical that such options have a very low theta.

Note also that when all other conditions are the same, an at-the-money option at a higher underlying price has a greater theta value than an at-the-money option with lower underlying price. To understand why, consider two calls, one with an exercise price of 10 and one with an exercise price of 1,000, where both options are at the money and both calls have the same amount of time to expiration and the same implied volatility. Which option will be worth more? Clearly, the 1,000 call will be worth more because it represents the right to buy a more valuable asset.[3] Because both options are at the money and therefore consist solely of time value, the theta of the 1,000 call must be greater than the theta of the 10 call.

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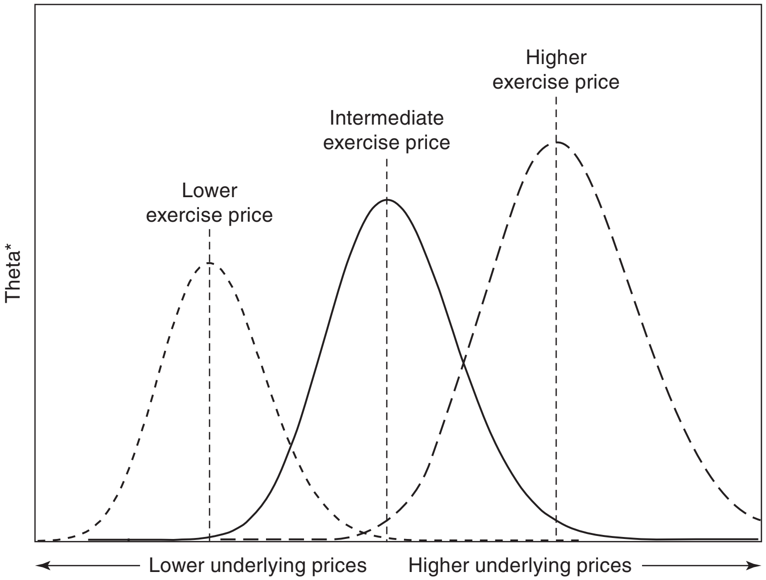
[3] In fact, the theoretical value and theta of two otherwise identical at-the-money options are proportional to their exercise prices. In this example, the 1,000 call will be worth exactly 100 times more than the 10 call, and its theta will be exactly 100 times greater.

Figure 9-8 Theta of an option as the underlying price changes.



*Although it is common to express theta as a negative number, the theta values in Figures 9-8, 9-10, and 9-11 are expressed in terms of absolute value.

Figure 9-9 shows the theoretical value of an in-the-money, at-the-money, and out-of-the-money option as time passes. Early in the option's life, the rate of decay (the slope of the theoretical-value graph) is similar for each option. But late in the option's life, as expiration approaches, the rate of decay slows for in-the-money and out-of-the-money options, whereas it accelerates for an at-the-money option, approaching infinity at the moment of expiration. These characteristics, which apply to both calls and puts, are shown in Figure 9-10.[4]

The effect on the theta of changing volatility is shown in Figure 9-11. If we ignore interest, with a 0 volatility, the theta of any option will be 0. As we increase volatility, we increase the time premium, at the same time increasing the theta.

Note that the graph of the at-the-money option is essentially a straight line, with the theta being directly proportional to the volatility. For an at-the-money option, the theta at a volatility of 20 percent is exactly double the theta at a volatility of 10 percent. The same is not necessarily true for higher exercise prices (out-of-the-money calls and in-the-money puts) or lower exercise prices (in-the-money calls and out-of-the-money puts). The theta tends to decline as volatility declines but may become 0 well before the volatility is 0.

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[4] The theta values for in-the-money and out-of-the-money options are actually slightly different. However, the values are so close that in Figure 9-10 we use one line to represent both options.

Figure 9-9 Theoretical value of an option as time passes.

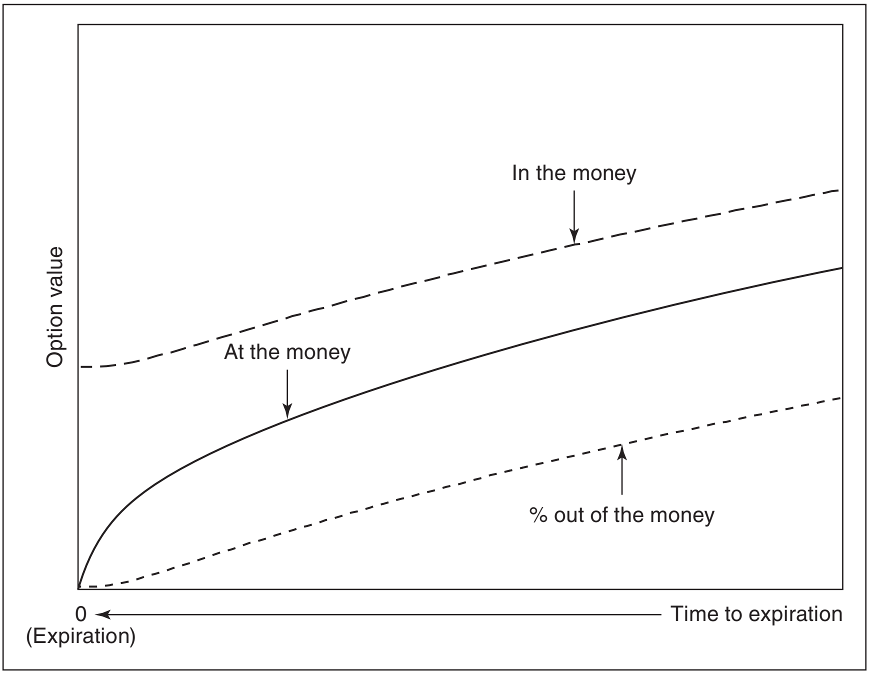


Figure 9-10 Theta of an option as time passes.

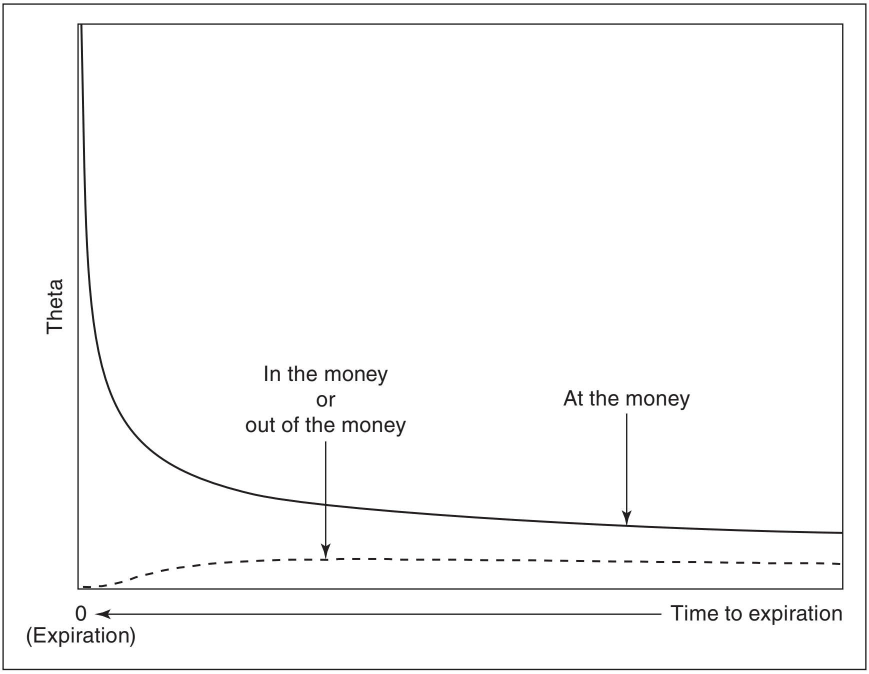


Figure 9-11 Theta of an option as volatility changes.

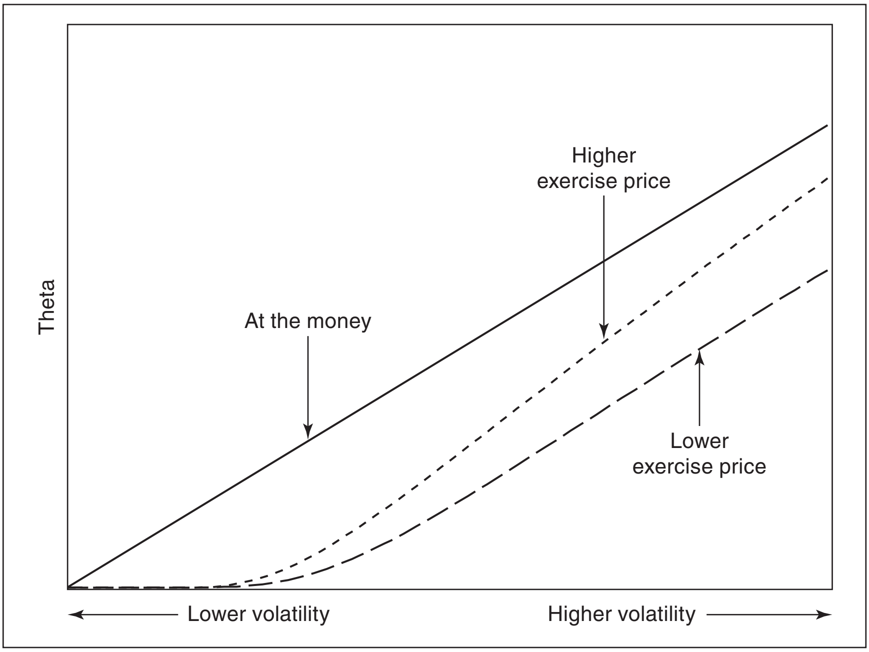


Figure 9-11 was constructed with the higher and lower exercise prices equally far away from the current underlying price. Note that the higher exercise price has a greater theta than the lower exercise price, with the difference increasing with increasing volatility. We touched on the explanation for this in Chapter 6. If a call and a put are both equally out of the money, under the assumptions of a lognormal distribution, the out-of-the-money call (the higher exercise price) will carry greater time premium than the out-of-the-money put (the lower exercise price). If there is no movement in the price of the underlying contract, the option with more time premium (the higher exercise price) must necessarily decay more quickly than the option with less time premium (the lower exercise price).

If we know the value of an option today, is there any way to estimate the option's theta? There is no convenient method for estimating the theta of in-the-money and out-of-the-money options, but for an at-the-money option, we know that theta is directly proportional to volatility (Figure 9-11). We also know from Chapter 6 that volatility is proportional to the square root of time

$$\text{volatility}_t = \text{volatility}_{\text{annual}} \times \sqrt{t}$$

The theta of an at-the-money option must therefore be proportional to the square root of time. If TV_t is an option's theoretical value at time t (in days to expiration), then the theoretical value one day later TV_{t-1} is

$$TV_{t-1} = TV_t \times \sqrt{(t-1)/t}$$

The theta is therefore

$$TV_t - TV_t \times \sqrt{(t-1)/t} = TV_t \times \left[1 - \sqrt{(t-1)/t}\right]$$

As time passes, the value of $1-\sqrt{(t-1)/t}$ becomes increasingly large. Consequently, the theta of an at-the-money option will also become increasingly large (Figure 9-7).

For example, consider an at-the-money option with a theoretical value of 2.50 and 30 days remaining to expiration. The option's theta will be approximately

$$2.50 \times \left(1 - \sqrt{29/30}\right) = 2.50 \times (1 - 0.9832) \approx 0.042$$

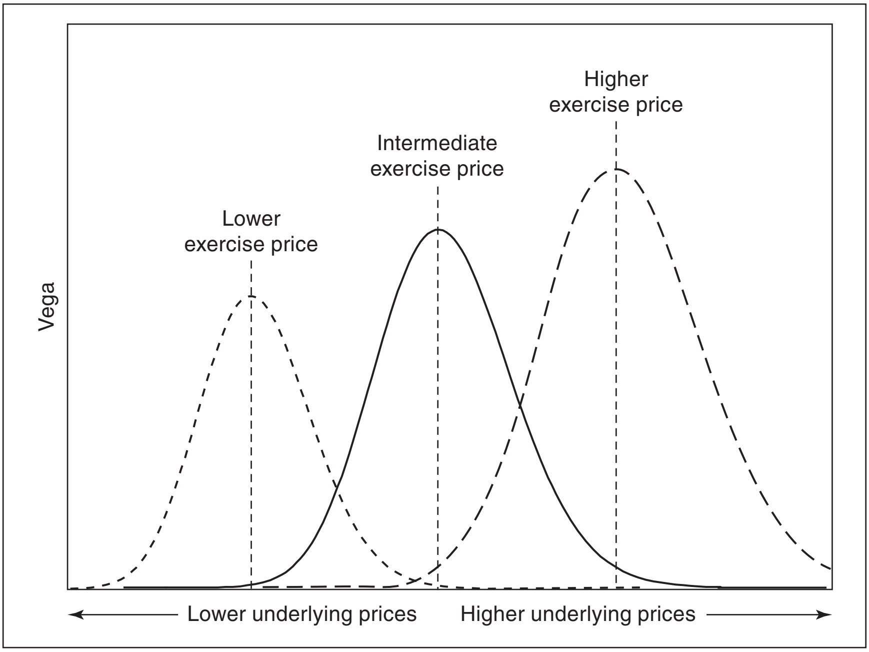
One day later, with 29 days remaining to expiration, the theta will be

$$(2.50 - 0.042) \times \left(1 - \sqrt{28/29}\right) = 2.458 \times (1 - 0.9826) \approx 0.043$$

Vega

Figure 9-12 shows the vega of an option as we change the underlying price. Note that this figure is almost identical to Figure 9-8. As with the theta, the vega is greatest when an option is at the money, and an at-the-money option with

Figure 9-12 Vega of an option as the underlying price changes.

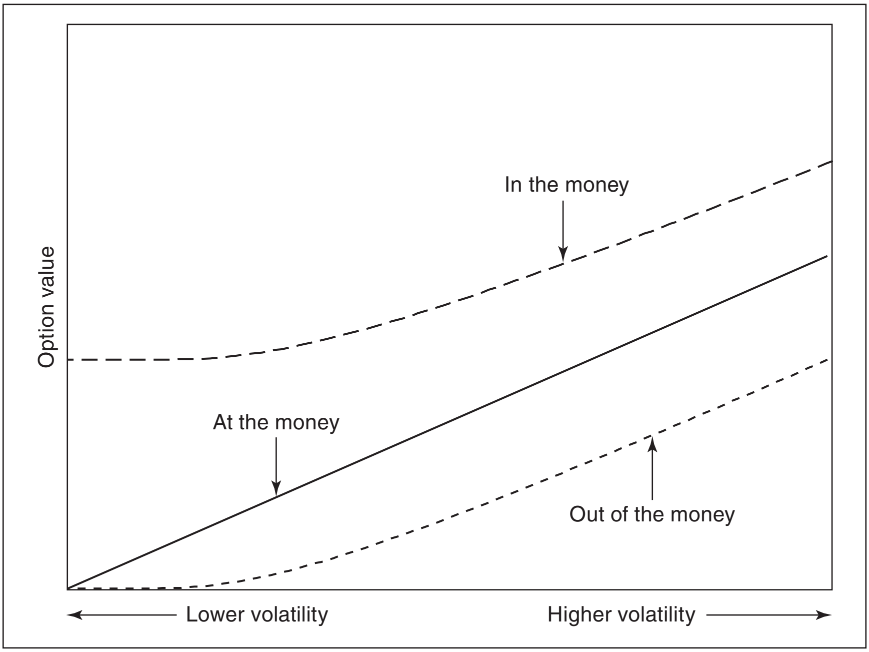


a higher exercise price has a greater vega than an at-the-money option with a lower exercise price. Moreover, the vega of an at-the-money option is proportional to its exercise price. Assuming that all other conditions are the same, an at-the-money option with an exercise price of 100 will have a vega that is twice that of an option with an exercise price of 50. Note that the term *vanna*, which previously referred to the sensitivity of delta to a change in volatility, can also refer to the sensitivity of the vega to a change in the underlying price. Both interpretations are mathematically identical.

Figure 9-13 shows the theoretical value of an in-the-money, at-the-money, and out-of-the-money option as we change volatility. Of particular note is the fact that the value of an at-the-money option is essentially a straight line. Because the vega is the slope of the graph, we can conclude that the vega of an at-the-money option is relatively constant with respect to changes in volatility. Whether volatility is 20 percent, 30 percent, or some higher value, the vega of an at-the-money option will be the same.

The effect on the vega of changing volatility is shown in Figure 9-14. While the vega of the at-the-money option is relatively constant, the vega values of in-the-money and out-of-the-money options tend to rise with higher volatility.[5] This is logical when we recall that as we raise volatility, the deltas of in-the-money and out-of-the-money options tend toward 50, causing the options to act more and more as if they are at the money. Because at-the-money options have the greatest vega (see Figure 9-12), we would expect the

Figure 9-13 Theoretical value of an option as volatility changes.



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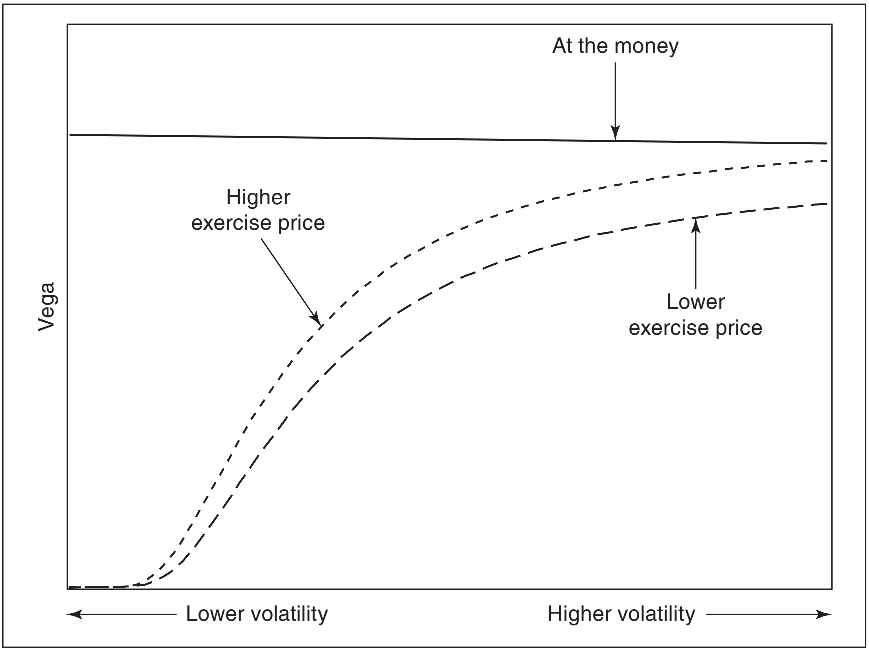
[5] In fact, we can see from Figure 9-14 that the vega of an at-the-money option declines very slightly as we raise volatility. This will be discussed in greater detail in Chapter 18.

Figure 9-14 Vega of an option as volatility changes.



vega values to rise. The sensitivity of vega to a change in volatility is sometimes referred to as either the *volga* or the *vomma* (both terms are a contraction of volatility and gamma—either *vol*atility *ga*mma or *vo*latility ga*mma*).

Figure 9-15 shows volga values for calls and puts with varying deltas. We have already noted that an at-the-money option with a delta of approximately 50 has a relatively constant vega and, consequently, a volga close to 0. However, as an option moves either into the money or out of the money, the volga begins to increase, reaching its maximum for calls with deltas of approximately 10 and 90 and puts with deltas of approximately –10 and –90. Additionally, as we increase time, volga values for in-the-money and out-of-the-money options become more sensitive to the passage of time, with long-term options having greater volga values than short-term options.

In Figure 9-16, we can see how vega values change as time changes, rising as we increase time to expiration and falling as we reduce time. This characteristic, that long-term options are always more sensitive to changes in volatility than short-term options, was introduced in Chapter 6 (see Figures 6-11 and 6-12).

The sensitivity of the vega to changes in time to expiration, sometimes referred to as either *vega decay* or *DvegaDtime*, is shown in Figure 9-17. The vega of options with delta values between 10 and 90 tends to be the most sensitive to the passage of time. This sensitivity increases as we reduce time to expiration; as time passes, the vega of short-term options will change more quickly than the vega of long-term options.

Figure 9-15 Volga (vomma) of an option.

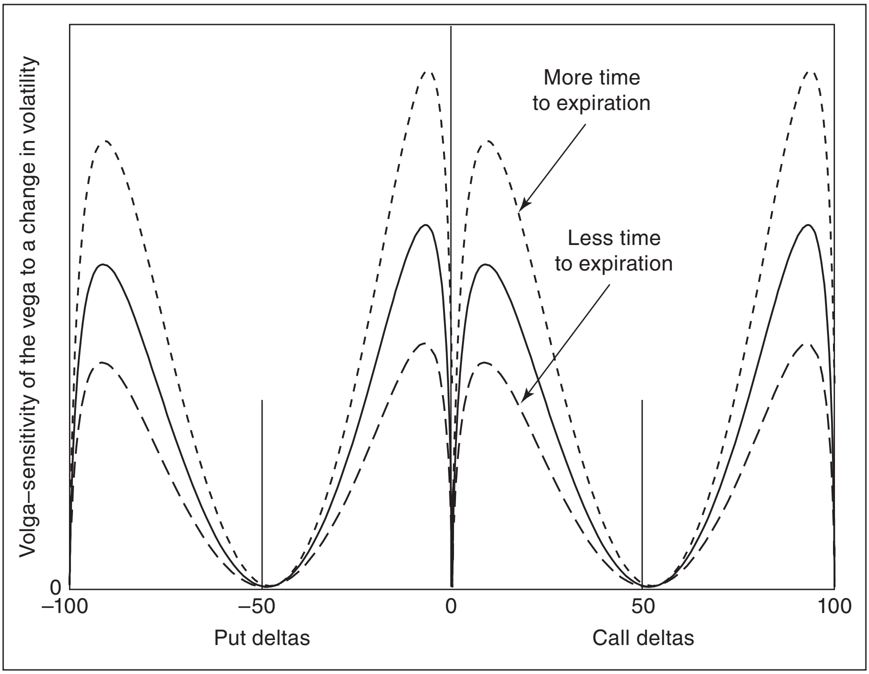


Figure 9-16 Vega of an option as time passes.

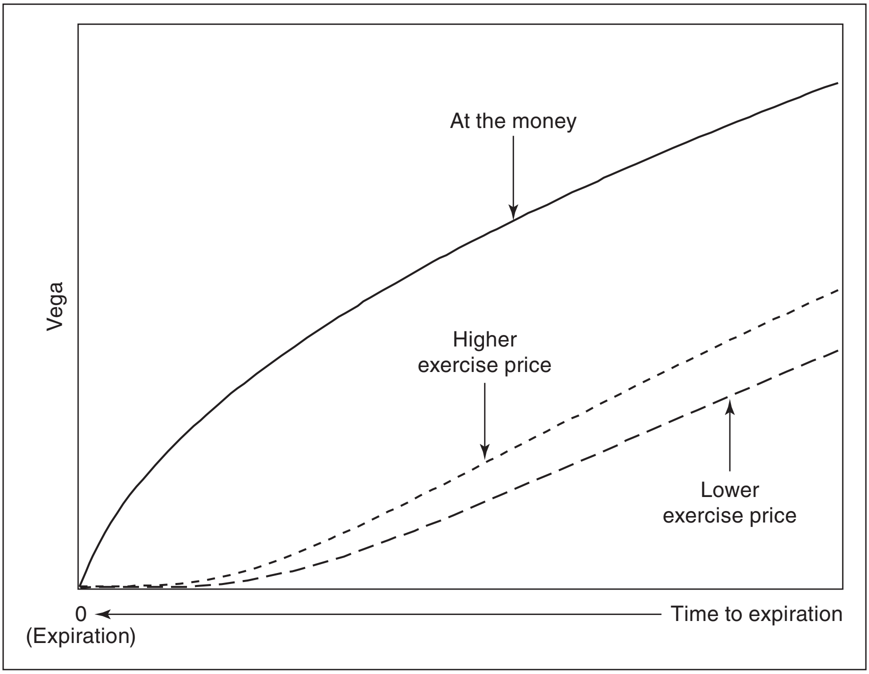
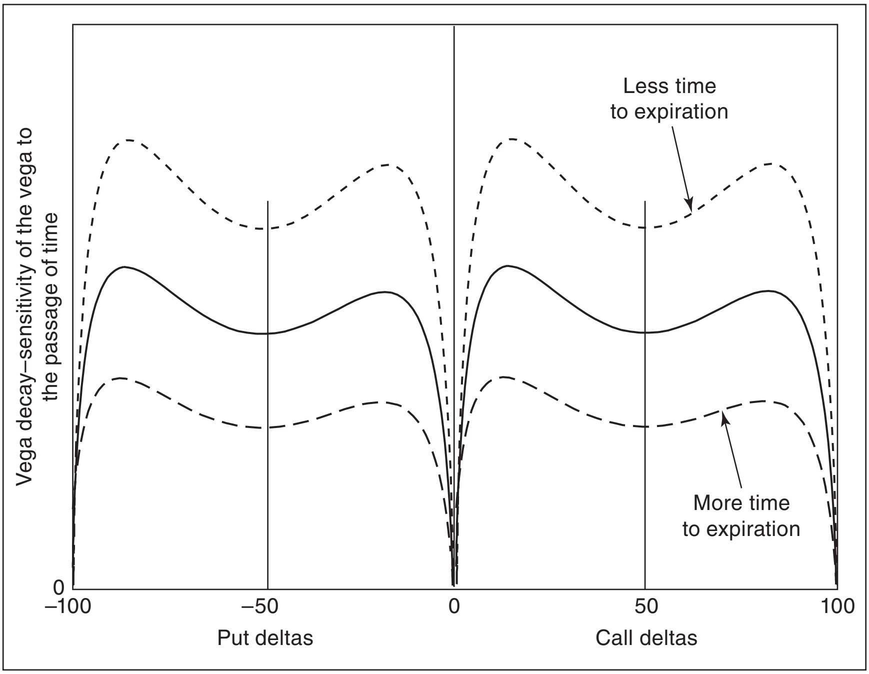


Figure 9-17 Vega decay of an option.



Gamma

The gamma measures the sensitivity of the delta to a change in the underlying price. But the gamma itself is sensitive to changes in market conditions.[6]

In Figure 9-18, we can see that the gamma is greatest when an option is at the money. This is similar to theta and vega, which are also greatest when an option is at the money, and leads to an important principle of option trading: *gamma, theta, and vega are greatest when an option is at the money*. Because of this, at-the-money options tend to be the most actively traded in most option markets. Such options have the characteristics that traders are looking for when they go into an option market.

Unlike the theta and vega of at-the-money options, which increase at higher exercise prices, the gamma of an at-the-money option declines at higher exercise prices. To understand why, recall that the gamma is the change in the delta per one-point change in the underlying price. But theoretical pricing models measure change in percentage terms. By this measure, a one-point price change with the underlying at 50 (a 2 percent change) is greater than

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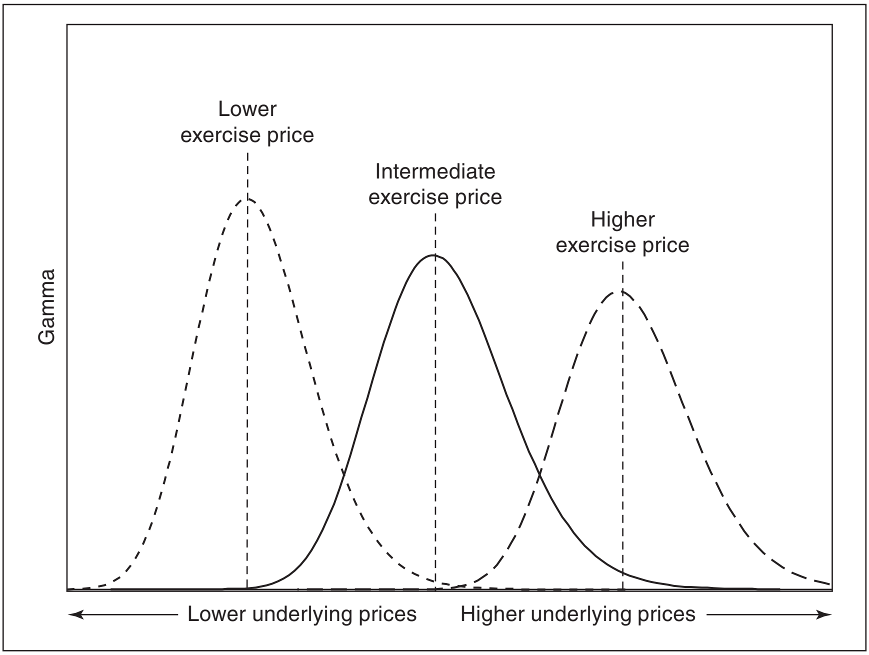
[6] Because the gamma is a second-order sensitivity—the sensitivity of the delta to a change in the underlying price—the sensitivity of gamma to a change in market conditions is a third-order sensitivity. For a discussion of some of the higher-order sensitivities, see Espen Gaarder Haug, *The Complete Guide to Option Pricing Formulas* (New York: McGraw-Hill, 2007); Espen Gaarder Haug, "Know Your Weapon, Part 1," *Wilmott Magazine*, May 2003: 49–57, also available at http://www.wilmott.com/pdfs/050527_haug.pdf; and Espen Gaarder Haug, "Know Your Weapon, Part 2," *Wilmott Magazine*, July–August 2003:50–56, also available at http://www.nuclearphynance.com/User percent20Files/2552/0307_haug.pdf.

Figure 9-18 Gamma of an option as the underlying price changes.

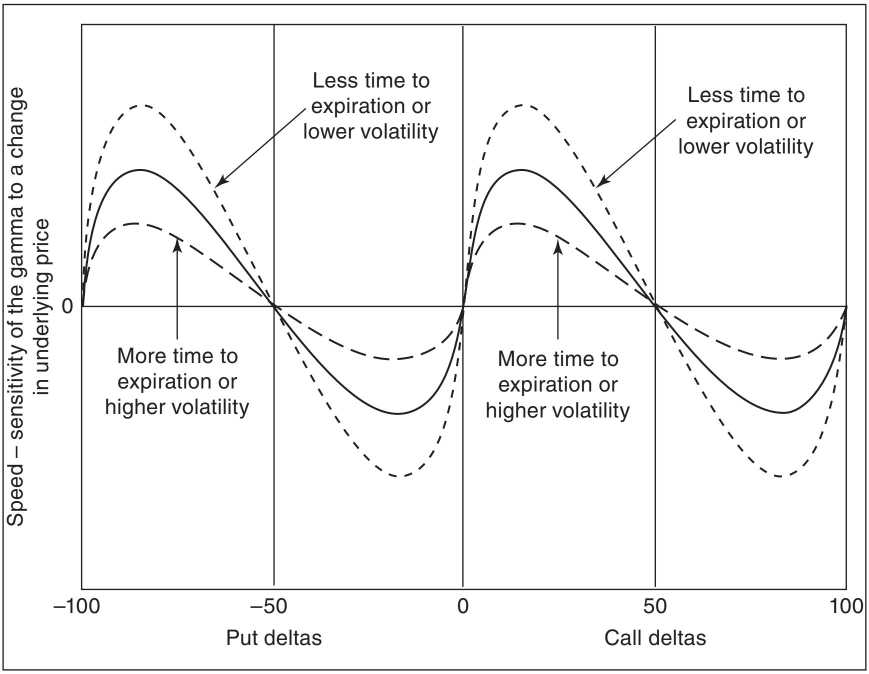


a one-point price change with the underlying at 100 (a 1 percent change). Although the theta and vega of at-the-money options are proportional to their exercise prices, the gamma is inversely proportional. The gamma of an option with an exercise price of 50 will be twice as large as the gamma of an option with an exercise price of 100.

Because at-the-money options have the greatest gamma, as the underlying price moves toward the exercise price, the gamma of an option will rise, and as the underlying price moves away from the exercise price, the gamma will fall. The sensitivity of the gamma to a change in the underlying price, sometimes referred to as the *speed*, is shown in Figure 9-19. The speed is greatest for out-of-the-money options with deltas close to 15 for calls and –15 for puts and for in-the-money options with deltas close to 85 for calls and –85 for puts. As we increase time to expiration or volatility, the speed of an option declines; as we reduce time to expiration or volatility, the speed rises. The gamma is least sensitive to changes in the underlying price for at-the-money options (a delta close to 50 for calls or –50 for puts) or for very deeply in-the-money or very far out-of-the-money options (deltas close to 0 and close to 100 for calls or –100 for puts).

The gamma will also be sensitive to changes in time to expiration and volatility. This is shown in Figure 9-20. We know that gamma is greatest when an option is at the money and declines as the option moves either into the money or out of the money. Of particular importance is the fact that the gamma of an at-the-money option rises as time passes or as we reduce volatility and falls as we increase volatility. To see why, consider a 100 call with the market at 97.50.

Figure 9-19 Speed of an option.



Because the option is currently out of the money, its delta is less than 50. We also know that as time passes or we reduce volatility, delta values move away from 50. If we are close to expiration or in a very low-volatility market, the delta of the option will be well below 50, perhaps 25. If the underlying market should then rise 5 points to 102.50, the delta of the option will be greater than 50, perhaps 75. With the underlying market rising from 97.50 to 102.50 and the delta rising from 25 to 75, the approximate gamma should be

$$\frac{75-25}{102.50-97.50}=\frac{50}{5}=\mathbf{10}$$

If, however, expiration is far in the future or we are in a high-volatility market, the delta of the 100 call will stay close to 50. With the underlying market at 97.50, the delta of the option may be 45. If the market then rises to 102.50, the delta may be only 55. The approximate gamma is then

$$\frac{55-45}{102.50-97.50}=\frac{50}{5}=\mathbf{2}$$

The effect is just the opposite for in-the-money and out-of-the-money options. The gamma will fall if we reduce volatility and rise if we increase volatility.[7] Because gamma and theta are closely related, if we were to graph

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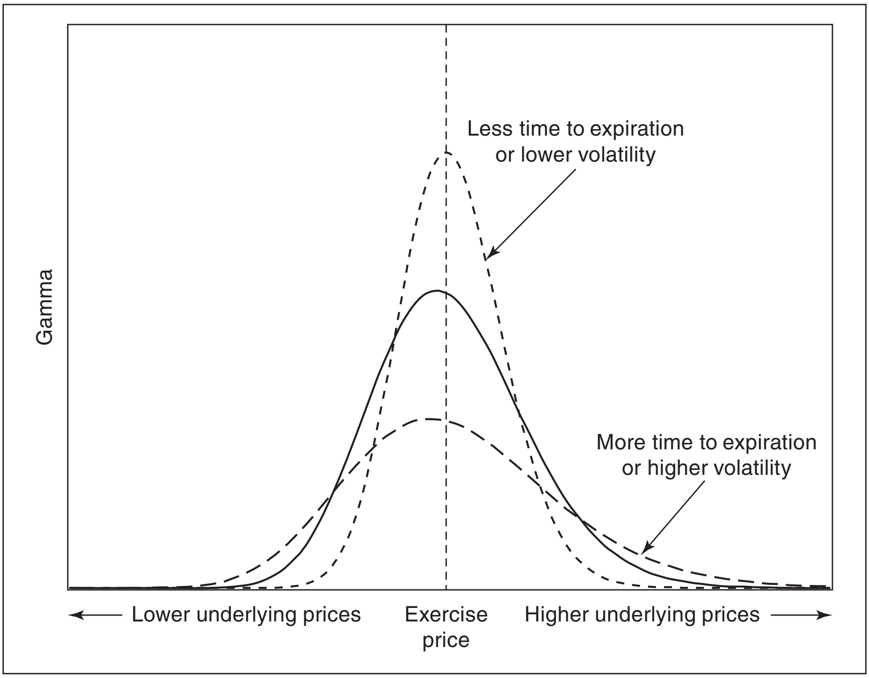
[7] This is a general rule. Sometimes an option that is only slightly in the money or out of the money will act like an at-the-money option. Whether an option's characteristics will resemble those of an at-the-money, in-the-money, or out-of-the-money option will depend on a variety of factors, including volatility and time to expiration.

Figure 9-20 Gamma of an option as time passes or volatility changes.



the gamma of an option as time passes, the result would be very similar to Figure 9-10, with the gamma instead of the theta along the *y*-axis.

The sensitivity of the gamma to the passage of time, sometimes referred to as its *color*, is shown in Figure 9-21. The color is greatest for at-the-money calls and puts, with gamma values becoming smaller as we increase time to expiration and larger as we reduce time (hence a negative color value). Calls with deltas close to 5 or 95 and puts with deltas close to –5 or –95 also have large color values. Here, however, an increase in time causes gamma values to rise, whereas the passage of time causes gamma values to fall (a positive color). Moreover, reducing time or volatility will increase color values, making an option's gamma more sensitive to changes in the passage of time. Increasing time or volatility will reduce color values, making an option's gamma less sensitive to the passage of time. Calls with deltas close to 15 or 85 and puts with deltas close to –15 and –85 tend to have color values close to 0. The gamma values of such options will be relatively insensitive to the passage of time.

The sensitivity of an option's gamma to a change in volatility, sometimes referred to as its *zomma*, is shown in Figure 9-22. Zomma characteristics are similar to color characteristics. The zomma is large for at-the-money calls and puts, with gamma values becoming smaller as volatility rises and larger as volatility falls (a negative zomma). Calls with deltas close to 5 or 95 and puts with deltas close to –5 or –95 also have large zomma values. Here, however, an

Figure 9-21 Color of an option.

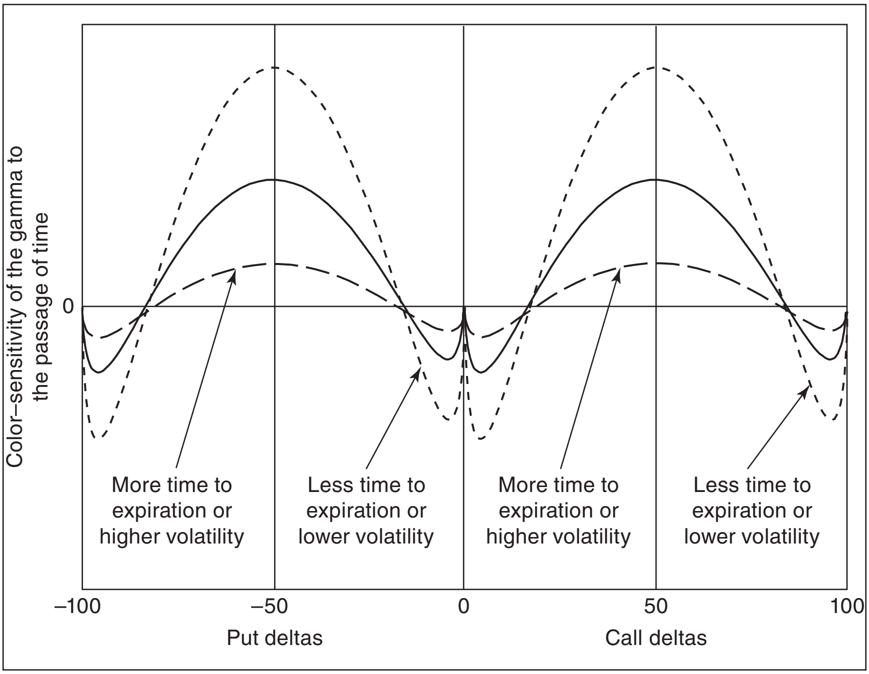
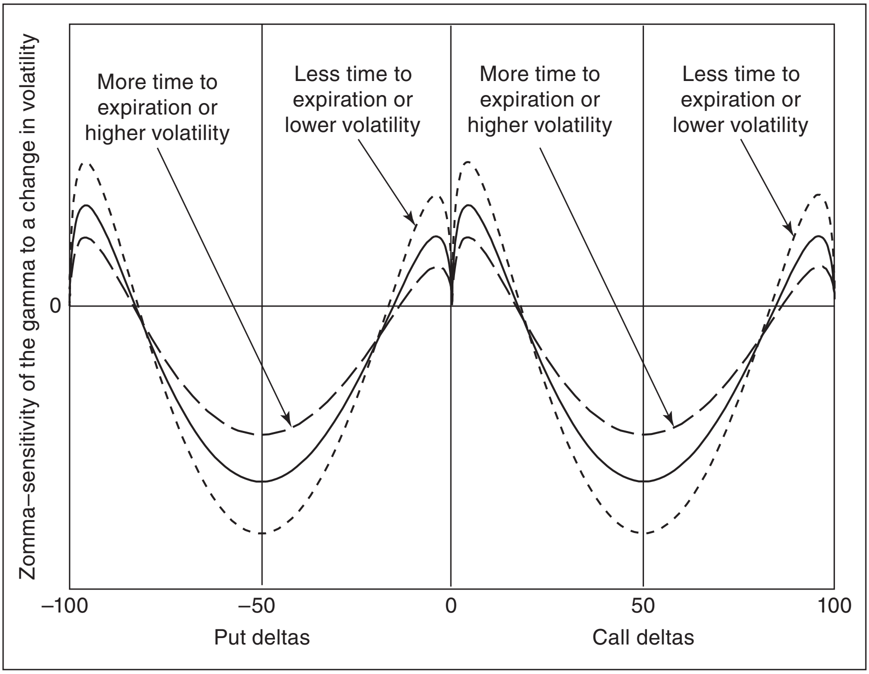


Figure 9-22 Zomma of an option.



increase in volatility causes gamma values to rise and a decline in volatility causes gamma values to fall (a positive zomma). Moreover, reducing time or volatility will increase the zomma, making an option's gamma more sensitive to changes in volatility. Increasing time or volatility will reduce the zomma, making an option's gamma less sensitive to changes in volatility. Calls with deltas close to 15 or 85 and puts with deltas close to –15 and –85 tend to have zomma values close to 0. The gamma values of such options will be relatively insensitive to changes in volatility.

Given the fact that the gamma is greatest for at-the-money options and that the gamma of an at-the-money option increases as time passes or volatility declines, experienced traders know that at-the-money options close to expiration in a low-volatility environment are among the riskiest options that one can trade. Although these *gamma options* initially have delta values close to 50, their deltas can change dramatically with only small moves in the price of the underlying contract, moving very quickly toward 0 or 100.

Lambda (Λ)

The delta tells us the point change in an option's value for a given point change in the price of the underlying contract. But we might also ask how an option's value changes in percentage terms for a given percentage change in the underlying price.

Consider a call option with a theoretical value of 4.00 and a delta of 20, with the underlying contract trading at a price of 100. If the underlying contract rises one point to 101, the new delta of the option (ignoring the gamma) should be approximately 4.20. But how much are these changes in percentage terms? The underlying changed by 1 percent (1/100), whereas the option changed by 5 percent (0.20/4.00). The option has a *lambda*, or *elasticity*, of 5. In percentage terms, it will change at five times the rate of the underlying contract.

We can see that the lambda is simply the option's delta (using the decimal format) multiplied by the ratio of the underlying price S to the option's theoretical value

$$\Lambda = \Delta \times (S/TV)$$

In our example, lambda is

$$0.20 \times 100/4.00 = \mathbf{5}$$

Traders sometimes refer to the lambda as the option's *leverage value*. Although lambda is not a widely used risk measure, it may still be worth looking at some basic lambda characteristics. These are shown in Figures 9-23 (call lambda values) and 9-24 (put lambda values). Logically, because the lambda is calculated from the delta, calls have positive lambda values and puts have negative lambda values. We can see that the lambda is greatest for out-of-the-money options—as the underlying price rises, call lambda values decline

Figure 9-23 Lambda of a call as time passes or volatility changes.

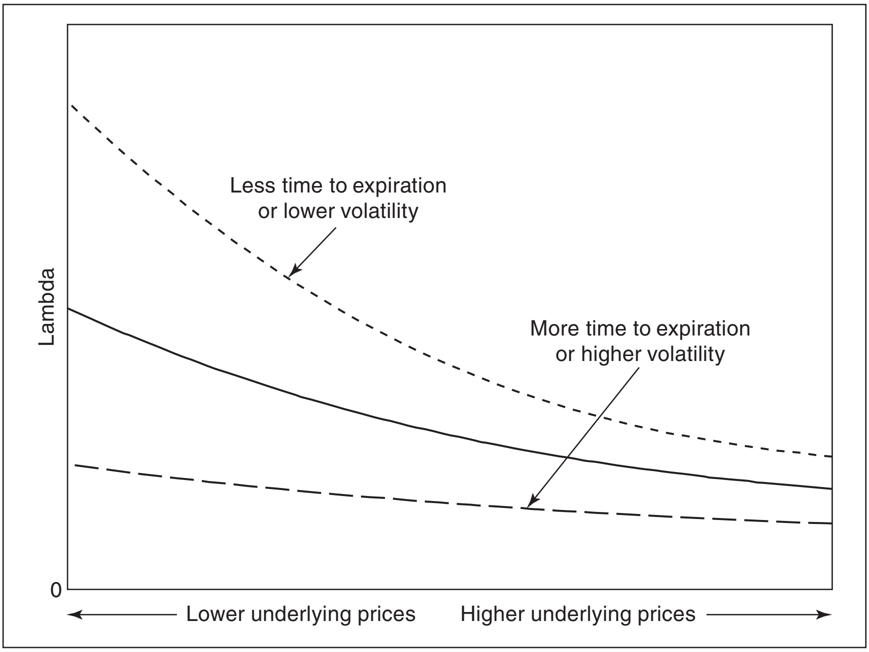
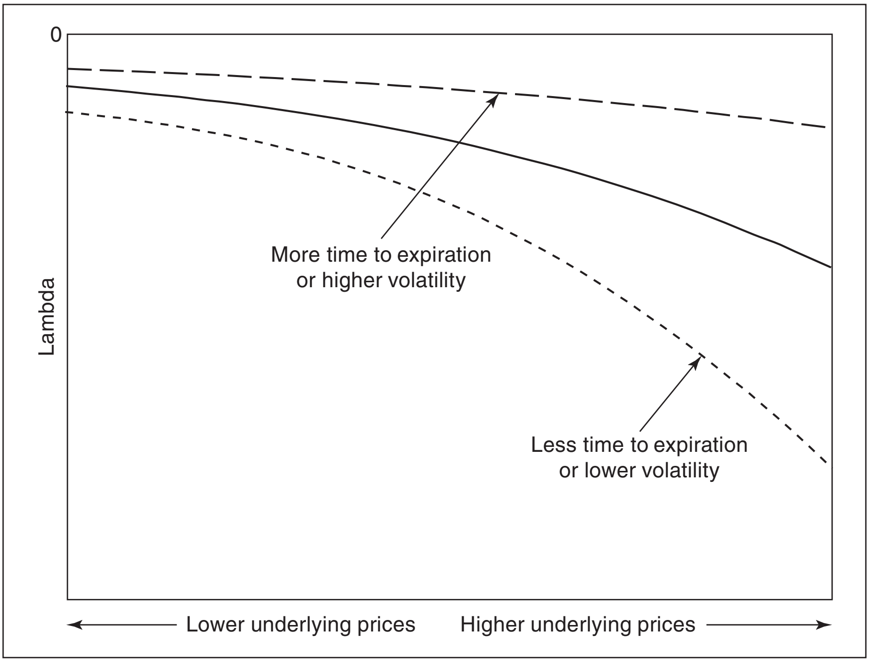


Figure 9-24 Lambda of a put as time passes or volatility changes.



and put lambda values rise (they take on large negative values). Lambda values are also sensitive to changes in time and volatility. If we increase volatility, lambda values for both calls and puts fall. If we reduce volatility or as time passes, lambda values for both calls and puts rise.

A trader who wants the biggest possible return on his investment, in percentage terms, compared with an equal investment in the underlying contract can maximize his lambda by trading out-of-the-money options close to expiration in a low-volatility environment. Of course, this is true only in theory. There may be other considerations, such as the bid-ask spread and liquidity of the option market, that might make a large lambda position impractical compared with a similar position in the underlying market.

It may seem that we have gone into undue detail in our examination of the option risk measures. Although it is certainly true that not every risk is important in every situation, experienced traders have learned that it is almost impossible to overemphasize the importance of risk management in option trading. Because options are affected by so many different market forces, unless a trader is aware of and understands the many ways in which option values change, he cannot hope to successfully manage the very real risks that option trading entails.

A summary of the primary risk characteristics discussed in this chapter is given in Figures 9-25 and 9-26.

Figure 9-25 Traditional risk measures.

C = call theoretical value P = put theoretical value S = underlying price or spot price
t = time to expiration σ = annual volatility r = domestic interest rate rf = foreign interest rate

Risk Name	Sensitivity of the	To a Change in	Math	Maximized
Delta (Δ)	Theoretical value (in points)	Underlying price (in points)	$\partial C/\partial S \approx \partial P/\partial S + 1$	Deeply in the money
Lambda (Λ) [omega (Ω)] elasticity	Theoretical value (in percent)	Underlying price (in percent)	$\Delta c^*(S/C)$ $\Delta p^*(S/P)$	Out of the money Close to expiration Low volatility
Gamma (Γ) curvature	Delta	Underlying price	$\partial^2 C/\partial S^2 =$ $\partial^2 P/\partial S^2$ $\partial \Delta/\partial S$	At the money Close to expiration Low volatility
Theta (Θ) time decay	Theoretical value	Time to expiration	$\partial C/\partial t$ $\partial P/\partial t$	At the money Close to expiration Low volatility
Vega	Theoretical value	Volatility	$\partial C/\partial \sigma = \partial P/\partial \sigma$	At the money Long term
Rho (P)	Theoretical value	Interest rate	$\partial C/\partial r$ $\partial P/\partial r$	Deeply in the money Long term
Rhof or phi (Φ)	Theoretical value	Foreign interest rate or dividend yield	$\partial C/\partial rf$ $\partial P/\partial rf$	Deeply in the money Long term

Figure 9-26 Nontraditional higher-order risk measures.

Risk Name	Sensitivity of the	To a Change in	Math	Maximized
Vanna	Delta Vega	Volatility Underlying price	$\partial^2 C/\partial S\partial\sigma$ $\partial^2 P/\partial S\partial\sigma$	15-20, 80-85 delta Low volatility
Charm delta decay	Delta Theta	Time Underlying price	$\partial^2 C/\partial S\partial t$ $\partial^2 P/\partial S\partial t$	15-20, 80-85 delta Close to expiration
Speed	Gamma	Underlying price	$\partial^3 C/\partial S^3 = \partial^3 P/\partial S$ $\partial^2 \Delta/\partial S^2 \quad \partial\Gamma/\partial S$	15-20, 80-85 delta Low volatility Close to expiration
Color gamma decay	Gamma Charm	Time to expiration Underlying price	$\partial^3 C/\partial S^2\partial t$ $\partial^3 P/\partial S^2\partial t$ $\partial\Gamma/\partial t$	At the money Close to expiration Low volatility
Volga (vomma)	Vega	Volatility	$\partial^2 C/\partial\sigma^2 =$ $\partial^2 P/\partial\sigma^2$	10, 90 delta Long term Low volatility
Vega decay	Vega	Time	$\partial^2 C/\partial\sigma\partial t$ $\partial^2 P/\partial\sigma\partial t$	20, 80 delta Close to expiration
Zomma	Gamma Vanna	Volatility Underlying price	$\partial^3 C/\partial S^2\partial\sigma =$ $\partial^3 P/\partial S^2\partial\sigma$ $\partial\Gamma/\partial\sigma$	At the money Close to expiration Low volatility