COLLEGE PREP MATH

Algebra and Beyond

Master the algebra skills essential for college success

Global Sovereign University

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Section 1: The Real Number System

Understanding number types is fundamental to algebra. All numbers we use in everyday math are real numbers.

Types of Numbers

Type	Definition	Examples
Natural Numbers (■)	Counting numbers	1, 2, 3, 4, 5...
Whole Numbers (W)	Natural numbers plus zero	0, 1, 2, 3, 4...
Integers (■)	Whole numbers and negatives	...-2, -1, 0, 1, 2...
Rational Numbers (■)	Can be written as a fraction	½, -3, 0.75, 2■
Irrational Numbers	Cannot be written as a fraction	π, √2, √3, e
Real Numbers (■)	All rational and irrational	All of the above

The Number Line

All real numbers can be placed on a number line. Numbers increase to the right and decrease to the left.

• Every point on the line represents a real number

• Between any two real numbers, there are infinitely many more

Absolute Value

|x| = the distance from x to zero

• |5| = 5

• |-5| = 5

• |0| = 0

Distance is always positive or zero, never negative.

Classifying Numbers

Example: Classify -7

-7 is an integer, a rational number, and a real number

Example: Classify √5

√5 ≈ 2.236... (non-repeating, non-terminating)

√5 is an irrational number and a real number

Practice: Real Numbers

1. Classify 0.333... (repeating): _____

2. Classify √16: _____

3. Is -√9 rational or irrational? _____

4. Find |–12|: _____

5. Find |7 – 10|: _____

6. List all number types that include -5: _____

Section 2: Properties of Real Numbers

These properties are the rules that govern how numbers behave in operations.

The Properties

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \times b = b \times a$
Associative	$(a + b) + c = a + (b + c)$	$(a \times b) \times c = a \times (b \times c)$
Identity	$a + 0 = a$	$a \times 1 = a$
Inverse	$a + (-a) = 0$	$a \times (1/a) = 1, a \neq 0$

Distributive Property

$a(b + c) = ab + ac$

This property connects multiplication and addition.

Example: $5(x + 3) = 5x + 15$

Example: $-2(4x - 7) = -8x + 14$

Zero Product Property

If $ab = 0$, then $a = 0$ or $b = 0$ (or both)

This is crucial for solving quadratic equations.

Example: If $(x - 3)(x + 5) = 0$, then $x = 3$ or $x = -5$

Applying Properties

Simplify: $3(2x + 4) + 5x$

$= 6x + 12 + 5x$ (distributive)

$= 11x + 12$ (combine like terms)

Practice: Properties

1. Name the property: $7 + 3 = 3 + 7$ _____

2. Name the property: $5 \times 1 = 5$ _____

3. Apply distributive: 4(3x - 2) = ______

4. Simplify: 2(x + 5) + 3(x - 1) = ______

5. If (x - 4)(x + 2) = 0, find x: ______

6. What is the additive inverse of -8? ______

Section 3: Exponents and Radicals

Exponent Rules

Rule	Formula	Example
Product Rule	a■ × a■ = a■■■	x^3 × x■ = x■
Quotient Rule	a■ ÷ a■ = a■■■	x■ ÷ $x^2 = x^3$
Power Rule	(a■)■ = a■■	$(x^2)^3$ = x■
Zero Exponent	a■ = 1 ($a \neq 0$)	5■ = 1
Negative Exponent	a■■ = 1/a■	x■² = $1/x^2$
Product to Power	(ab)■ = a■b■	$(2x)^3 = 8x^3$
Quotient to Power	(a/b)■ = a■/b■	$(x/3)^2 = x^2/9$

Working with Radicals

√a is the principal (positive) square root of a

• $\sqrt{25} = 5$ (because $5^2 = 25$)

• $\sqrt{x^2} = |x|$ (absolute value ensures positive result)

Radical Rules

• $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ (Product rule)

• $\sqrt{a/b} = \sqrt{a} / \sqrt{b}$ (Quotient rule)

• $\sqrt{a} \times \sqrt{a} = a$

• Cannot add/subtract unlike radicals: $\sqrt{2} + \sqrt{3} \neq \sqrt{5}$

Simplifying Radicals

Example: Simplify √72

$\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

Example: Simplify $\sqrt{50x^2}$

$\sqrt{50x^2} = \sqrt{25 \times 2 \times x^2} = 5x\sqrt{2}$

Rational Exponents

$a^{(1/n)} = \blacksquare\sqrt{a}$

$a^{(m/n)} = \blacksquare\sqrt{(a\blacksquare)} = (\blacksquare\sqrt{a})\blacksquare$

- $8^{(1/3)} = \sqrt[3]{8} = 2$
- $16^{(3/4)} = (\blacksquare\sqrt{16})^3 = 2^3 = 8$

Practice: Exponents and Radicals

1. Simplify: $x\blacksquare \times x\blacksquare$ = ______
2. Simplify: $(y^3)\blacksquare$ = ______
3. Simplify: $2\blacksquare^3$ = ______
4. Simplify: $\sqrt{98}$ = ______
5. Simplify: $\sqrt{(18x\blacksquare)}$ = ______
6. Evaluate: $27^{(2/3)}$ = ______

Section 4: Polynomials

A polynomial is an expression with variables and coefficients, using only addition, subtraction, multiplication, and non-negative integer exponents.

Vocabulary

• **Monomial:** One term ($5x^2$, $-3y$, 7)

• **Binomial:** Two terms ($x + 5$, $3x^2 - 2$)

• **Trinomial:** Three terms ($x^2 + 3x + 2$)

• **Degree:** Highest exponent (degree of $4x^3 + 2x$ is 3)

• **Leading coefficient:** Coefficient of highest degree term

Adding and Subtracting Polynomials

Combine like terms (same variable and exponent)

Example: $(3x^2 + 5x - 2) + (2x^2 - 3x + 7)$

$= 3x^2 + 2x^2 + 5x - 3x - 2 + 7$

$= 5x^2 + 2x + 5$

Example: $(4x^2 - 3x + 1) - (2x^2 + 5x - 3)$

$= 4x^2 - 3x + 1 - 2x^2 - 5x + 3$ (distribute the negative)

$= 2x^2 - 8x + 4$

Multiplying Polynomials

Multiply each term in the first by each term in the second.

Example: $3x(2x^2 - 5x + 4)$

$= 6x^3 - 15x^2 + 12x$

Example: $(x + 3)(x + 5)$ using FOIL

F: $x \times x = x^2$

O: $x \times 5 = 5x$

I: $3 \times x = 3x$

L: $3 \times 5 = 15$

$= x^2 + 5x + 3x + 15 = x^2 + 8x + 15$

Special Products

Square of a Binomial:

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Difference of Squares:

$(a + b)(a - b) = a^2 - b^2$

Practice: Polynomials

1. Add: $(4x^2 - 3x + 2) + (2x^2 + 5x - 7) =$ ______

2. Subtract: $(5x^2 + 2x - 1) - (3x^2 - 4x + 2) =$ ______

3. Multiply: $4x(3x^2 - 2x + 5) =$ ______

4. FOIL: $(x + 4)(x - 2) =$ ______

5. Expand: $(x + 5)^2 =$ ______

6. Multiply: $(3x - 2)(3x + 2) =$ ______

Section 5: Factoring

Factoring is the reverse of multiplying—breaking an expression into factors that multiply to give the original.

Greatest Common Factor (GCF)

Always look for the GCF first!

Example: Factor $6x^3 + 9x^2$

GCF = $3x^2$

= $3x^2(2x + 3)$

Factoring Trinomials ($x^2 + bx + c$)

Find two numbers that multiply to c and add to b.

Example: Factor $x^2 + 7x + 12$

Need: __ × __ = 12 and __ + __ = 7

Numbers: 3 and 4

= $(x + 3)(x + 4)$

Example: Factor $x^2 - 5x + 6$

Need: __ × __ = 6 and __ + __ = -5

Numbers: -2 and -3

= $(x - 2)(x - 3)$

Factoring $ax^2 + bx + c$ ($a \neq 1$)

AC Method:

Example: Factor $2x^2 + 7x + 3$

1. Multiply a × c: 2 × 3 = 6

2. Find factors of 6 that add to 7: 1 and 6

3. Rewrite: $2x^2 + x + 6x + 3$

4. Group: $(2x^2 + x) + (6x + 3)$

5. Factor each: $x(2x + 1) + 3(2x + 1)$

6. Final: $(2x + 1)(x + 3)$

Special Factoring Patterns

Difference of Squares: $a^2 - b^2 = (a + b)(a - b)$

Example: $x^2 - 16 = (x + 4)(x - 4)$

Perfect Square Trinomial:

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Example: $x^2 - 10x + 25 = (x - 5)^2$

Practice: Factoring

1. Factor: $8x^2 + 12x =$ ______

2. Factor: $x^2 + 9x + 20 =$ ______

3. Factor: $x^2 - 7x + 12 =$ ______

4. Factor: $x^2 - 25 =$ ______

5. Factor: $x^2 + 6x + 9 =$ ______

6. Factor: $3x^2 + 10x + 8 =$ ______

Section 6: Solving Linear Equations

A linear equation has variables with exponent 1. The goal: isolate the variable on one side.

The Balance Principle

Whatever you do to one side, do to the other side.

Multi-Step Equations

Strategy:

1. Simplify each side (distribute, combine like terms)

2. Get all variable terms on one side

3. Get all constants on the other side

4. Divide to isolate the variable

Example: Solve $3(x - 2) + 4 = 2x + 7$

$3x - 6 + 4 = 2x + 7$ (distribute)

$3x - 2 = 2x + 7$ (combine)

$3x - 2x = 7 + 2$ (move terms)

$x = 9$

Check: $3(9 - 2) + 4 = 3(7) + 4 = 25$; $2(9) + 7 = 25$ ✓

Equations with Fractions

Multiply every term by the LCD to clear fractions.

Example: Solve $x/2 + x/3 = 10$

LCD = 6

$6(x/2) + 6(x/3) = 6(10)$

$3x + 2x = 60$

$5x = 60$

$x = 12$

Variables on Both Sides

Example: Solve 5x + 3 = 2x + 15

5x - 2x = 15 - 3

3x = 12

x = 4

Special Cases

• **No solution:** Variables cancel, false statement (3 = 5)

• **Infinite solutions:** Variables cancel, true statement (4 = 4)

Practice: Linear Equations

1. Solve: 4x - 7 = 2x + 9

2. Solve: 3(2x - 1) = 5x + 4

3. Solve: x/4 + 5 = x/2 - 1

4. Solve: 2(x + 3) - 5 = 3(x - 1)

5. Solve: 5x - (2x + 3) = 3x - 3

6. Solve: 0.5x + 1.2 = 0.3x + 2.8

Section 7: Graphing Linear Equations

Linear equations graph as straight lines. Understanding slope and intercepts is key.

Slope-Intercept Form

$y = mx + b$

• m = slope (rise/run)

• b = y-intercept (where line crosses y-axis)

Example: $y = 2x + 3$ has slope 2 and y-intercept (0, 3)

Understanding Slope

Slope = rise/run = (y■ - y■)/(x■ - x■)

• Positive slope: line goes up from left to right

• Negative slope: line goes down from left to right

• Zero slope: horizontal line

• Undefined slope: vertical line

Finding Slope from Two Points

Example: Find slope between (1, 3) and (4, 9)

$m = (9 - 3)/(4 - 1) = 6/3 = 2$

Graphing a Line

Method 1: Using slope-intercept form

1. Plot the y-intercept (0, b)

2. Use slope to find another point: rise over run

3. Draw the line through both points

Example: Graph $y = -3x + 4$

1. Plot (0, 4)

2. Slope = -3 = -3/1: down 3, right 1 → (1, 1)

3. Connect the points

Other Forms of Linear Equations

Standard Form: Ax + By = C

Point-Slope Form: y - y■ = m(x - x■)

Converting to Slope-Intercept Form

Example: Convert 2x + 3y = 12 to slope-intercept form

3y = -2x + 12

y = -2/3 x + 4

Slope = -2/3, y-intercept = 4

Practice: Graphing

1. Find slope and y-intercept: y = 4x - 5

2. Find slope between (2, 5) and (6, 13): _____

3. Convert to slope-intercept: 4x - 2y = 8

4. Write equation with slope 3 and y-int (0, -2): _____

5. What is the slope of a horizontal line? _____

6. Write equation through (1, 4) with slope 2: _____

Section 8: Systems of Equations

A system is two or more equations with the same variables. The solution is the point(s) where all equations are true.

Solving by Substitution

Solve one equation for a variable, substitute into the other.

Example:

$y = 2x + 1$

$3x + y = 11$

Substitute: $3x + (2x + 1) = 11$

$5x + 1 = 11$

$5x = 10$

$x = 2$

Back-substitute: $y = 2(2) + 1 = 5$

Solution: (2, 5)

Solving by Elimination

Add or subtract equations to eliminate a variable.

Example:

$2x + 3y = 13$

$4x - 3y = 5$

Add equations (y terms cancel):

$6x = 18$

$x = 3$

Substitute: $2(3) + 3y = 13 \rightarrow y = 7/3$

Solution: (3, 7/3)

Types of Solutions

• **One solution:** Lines intersect at one point

• **No solution:** Lines are parallel (same slope, different intercepts)

• **Infinite solutions:** Lines are the same (equations are equivalent)

Practice: Systems

Solve each system:

1. $y = 3x - 2$ and $x + y = 6$

2. $2x + y = 10$ and $x - y = 2$

3. $3x + 2y = 12$ and $x - y = 1$

4. $y = x + 4$ and $y = 2x + 1$

Section 9: Inequalities

Inequalities compare expressions using <, >, ≤, or ≥ instead of =.

Inequality Symbols

- < less than
- > greater than
- ≤ less than or equal to
- ≥ greater than or equal to

Solving Inequalities

Same rules as equations, with one exception:

When you multiply or divide by a NEGATIVE number, FLIP the inequality sign!

Example: Solve $3x + 5 > 14$

$3x > 9$

$x > 3$

Example: Solve $-2x + 6 \leq 12$

$-2x \leq 6$

$x \geq -3$ (flip the sign!)

Graphing Inequalities on a Number Line

- < or > : open circle (not including)
- ≤ or ≥ : closed circle (including)
- Arrow points in direction of solutions

Compound Inequalities

AND (intersection): Both conditions must be true

Example: $-2 < x < 5$ (x is between -2 and 5)

OR (union): At least one condition must be true

Example: x < -1 or x > 3

Solving Compound Inequalities

Example: Solve -3 ≤ 2x + 1 < 7

Subtract 1 from all parts: -4 ≤ 2x < 6

Divide all parts by 2: -2 ≤ x < 3

Practice: Inequalities

1. Solve: 5x - 3 > 17

2. Solve: -4x + 2 ≤ 18

3. Solve: 2(x - 3) < x + 4

4. Solve: -1 < 3x + 2 ≤ 11

5. Write inequality: x is at least 5 ______

6. Write inequality: x is no more than 10 ______

Section 10: Quadratic Equations

A quadratic equation has the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Solving by Factoring

Use the Zero Product Property: if ab = 0, then a = 0 or b = 0

Example: Solve $x^2 + 5x + 6 = 0$

Factor: $(x + 2)(x + 3) = 0$

$x + 2 = 0$ or $x + 3 = 0$

$x = -2$ or $x = -3$

Solving by Square Roots

When equation is in form $x^2 = k$

Example: Solve $x^2 = 49$

$x = \pm\sqrt{49} = \pm 7$

Example: Solve $(x - 3)^2 = 16$

$x - 3 = \pm 4$

$x = 3 + 4 = 7$ or $x = 3 - 4 = -1$

The Quadratic Formula

For $ax^2 + bx + c = 0$:

$x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$

Example: Solve $2x^2 + 5x - 3 = 0$

$a = 2, b = 5, c = -3$

$x = (-5 \pm \sqrt{(25 + 24)}) / 4$

$x = (-5 \pm \sqrt{49}) / 4$

$x = (-5 \pm 7) / 4$

$x = 2/4 = 1/2$ or $x = -12/4 = -3$

The Discriminant

$b^2 - 4ac$ tells us about the solutions:

• Positive: Two real solutions

• Zero: One real solution (repeated)

• Negative: No real solutions (complex)

Practice: Quadratics

1. Solve by factoring: $x^2 - x - 12 = 0$

2. Solve by factoring: $x^2 + 4x = 0$

3. Solve: $x^2 = 81$

4. Solve: $(x + 2)^2 = 25$

5. Use quadratic formula: $x^2 + 6x + 5 = 0$

6. Find discriminant: $2x^2 + 3x + 5 = 0$. How many real solutions?

Section 11: Functions

A function is a rule that assigns exactly one output to each input.

Function Notation

f(x) is read "f of x"

$f(x) = 2x + 3$ means: take x, multiply by 2, add 3

Example: If $f(x) = 2x + 3$, find $f(5)$

$f(5) = 2(5) + 3 = 13$

Domain and Range

• **Domain:** All possible input (x) values

• **Range:** All possible output (y or f(x)) values

Example: $f(x) = \sqrt{x}$

Domain: $x \geq 0$ (can't take square root of negative)

Range: $y \geq 0$ (square root is never negative)

The Vertical Line Test

A graph represents a function if every vertical line crosses it at most once.

Types of Functions

Type	Form	Graph Shape
Linear	$f(x) = mx + b$	Straight line
Quadratic	$f(x) = ax^2 + bx + c$	Parabola
Absolute Value	$f(x) = \|x\|$	V-shape
Square Root	$f(x) = \sqrt{x}$	Half parabola

Evaluating Functions

Example: $f(x) = x^2 - 3x + 2$. Find $f(-2)$

$f(-2) = (-2)^2 - 3(-2) + 2 = 4 + 6 + 2 = 12$

Practice: Functions

1. If $f(x) = 3x - 7$, find $f(4)$: ______

2. If $g(x) = x^2 + 2x$, find $g(-3)$: ______

3. If $h(x) = |x - 5|$, find $h(2)$: ______

4. What is the domain of $f(x) = 1/(x-3)$? ______

5. What is the domain of $f(x) = \sqrt{(x-4)}$? ______

6. Is $y = x^2$ a function? Why? ______

Section 12: Word Problem Strategies

The Problem-Solving Process

1. **Read** the problem carefully—multiple times
2. **Define variables**—what does x represent?
3. **Write an equation** from the problem
4. **Solve** the equation
5. **Check** your answer in the original problem
6. **Answer** in a complete sentence

Common Problem Types

Number Problems:

"The sum of two consecutive integers is 47. Find them."

Let x = first integer, $x + 1$ = second

$x + (x + 1) = 47 \rightarrow 2x + 1 = 47 \rightarrow x = 23$

The integers are 23 and 24.

Age Problems:

"Maria is 4 years older than twice her son's age. The sum of their ages is 52. Find their ages."

Let x = son's age, $2x + 4$ = Maria's age

$x + (2x + 4) = 52 \rightarrow 3x + 4 = 52 \rightarrow x = 16$

Son is 16, Maria is 36.

Distance Problems ($d = rt$):

"A car travels 180 miles at 60 mph. How long does it take?"

$180 = 60t \rightarrow t = 3$ hours

Mixture Problems:

"How much 20% solution and 50% solution to make 15 liters of 30% solution?"

Let x = liters of 20%, (15 - x) = liters of 50%

0.20x + 0.50(15 - x) = 0.30(15)

0.20x + 7.5 - 0.50x = 4.5

-0.30x = -3 → x = 10 liters of 20%

Practice: Word Problems

1. The sum of three consecutive even integers is 78. Find them.

2. A rectangle's length is 3 more than twice its width. Perimeter is 48 cm. Find dimensions.

3. How long to travel 240 miles at 55 mph? (Round to nearest tenth)

4. A number increased by 15% equals 92. Find the original number.

Answer Key

Section 1: Real Numbers

1) Rational 2) Natural, Whole, Integer, Rational, Real (=4) 3) Rational (-3) 4) 12 5) 3 6) Integer, Rational, Real

Section 2: Properties

1) Commutative 2) Identity 3) $12x - 8$ 4) $5x + 7$ 5) $x = 4$ or $x = -2$ 6) 8

Section 3: Exponents and Radicals

1) x■ 2) y^{12} 3) 1/8 4) $7\sqrt{2}$ 5) $3x^2\sqrt{2}$ 6) 9

Section 4: Polynomials

1) $6x^2 + 2x - 5$ 2) $2x^2 + 6x - 3$ 3) $12x^3 - 8x^2 + 20x$ 4) $x^2 + 2x - 8$ 5) $x^2 + 10x + 25$ 6) $9x^2 - 4$

Section 5: Factoring

1) $4x(2x + 3)$ 2) $(x + 4)(x + 5)$ 3) $(x - 3)(x - 4)$ 4) $(x + 5)(x - 5)$ 5) $(x + 3)^2$ 6) $(3x + 4)(x + 2)$

Section 6: Linear Equations

1) $x = 8$ 2) $x = 7$ 3) $x = 24$ 4) $x = 4$ 5) Infinite solutions 6) $x = 8$

Section 7: Graphing

1) $m = 4$, $b = -5$ 2) $m = 2$ 3) $y = 2x - 4$ 4) $y = 3x - 2$ 5) 0 6) $y = 2x + 2$

Section 8: Systems

1) (2, 4) 2) (4, 2) 3) (2, 3) 4) (3, 7)

Section 9: Inequalities

1) $x > 4$ 2) $x \geq -4$ 3) $x < 10$ 4) $-1 < x \leq 3$ 5) $x \geq 5$ 6) $x \leq 10$

Section 10: Quadratics

1) $x = 4$ or $x = -3$ 2) $x = 0$ or $x = -4$ 3) $x = \pm 9$ 4) $x = 3$ or $x = -7$ 5) $x = -1$ or $x = -5$ 6) Discriminant = -31, no real solutions

Section 11: Functions

1) 5 2) 3 3) 3 4) $x \neq 3$ (all real except 3) 5) $x \geq 4$ 6) Yes, passes vertical line test

Section 12: Word Problems

1) 24, 26, 28 2) Width = 7 cm, Length = 17 cm 3) 4.4 hours 4) 80

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