MIDDLE SCHOOL MATH

Pre-Algebra and Problem Solving

Bridge from arithmetic to algebra

Global Sovereign University

Building a Bridge to Freedom Through Education

Table of Contents

Section 1: Integers and the Number Line

Integers are whole numbers and their opposites. They include positive numbers, negative numbers, and zero.

The Set of Integers

... -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 ...

• **Positive integers:** Numbers greater than zero (1, 2, 3, ...)

• **Negative integers:** Numbers less than zero (-1, -2, -3, ...)

• **Zero:** Neither positive nor negative

The Number Line

←——|——|——|——|——|——|——|——|——|——|——→

-5 -4 -3 -2 -1 0 1 2 3 4 5

• Numbers increase as you move RIGHT

• Numbers decrease as you move LEFT

Absolute Value

The absolute value of a number is its distance from zero on the number line. Distance is always positive.

Written as |n| (vertical bars)

• |5| = 5 (5 is 5 units from zero)

• |-5| = 5 (-5 is also 5 units from zero)

• |0| = 0

Comparing Integers

• Any positive number > any negative number

• For negative numbers, the one closer to zero is greater: -2 > -5

• For positive numbers, larger is greater: 5 > 2

Opposites

Two numbers that are the same distance from zero but on opposite sides are called opposites.

• The opposite of 7 is -7

• The opposite of -3 is 3

• A number plus its opposite equals zero: 7 + (-7) = 0

Practice: Integers

1. List all integers between -4 and 3: _____

2. Find |–12|: _____

3. Find |8|: _____

4. Compare: -7 ■ -3 (write <, >, or =)

5. Compare: -5 ■ 2

6. What is the opposite of -15? _____

Section 2: Adding and Subtracting Integers

Adding Integers: Same Signs

Rule: Add the absolute values. Keep the common sign.

• 5 + 3 = 8 (both positive → positive)

• (-5) + (-3) = -8 (both negative → negative)

Adding Integers: Different Signs

Rule: Subtract the absolute values. Take the sign of the larger absolute value.

• 7 + (-3) = 4 (|7| > |-3|, so positive)

• (-7) + 3 = -4 (|-7| > |3|, so negative)

• 5 + (-5) = 0 (equal absolute values)

Subtracting Integers

Rule: Add the opposite! Change subtraction to addition and flip the sign of the second number.

• 8 – 3 = 8 + (–3) = 5

• 8 – (–3) = 8 + 3 = 11

• (–8) – 3 = (–8) + (–3) = –11

• (–8) – (–3) = (–8) + 3 = –5

Memory Aid

"Keep-Change-Change" for subtraction:

Keep the first number, Change subtraction to addition, Change the sign of the second number

Practice: Adding and Subtracting Integers

1) 8 + 5 =	2) (-6) + (-4) =	3) 9 + (-5) =
4) (-12) + 7 =	5) 15 + (-15) =	6) (-8) + 12 =
7) 10 – 4 =	8) 6 – 9 =	9) (-7) – 3 =
10) (-5) – (-8) =	11) 12 – (-4) =	12) (-15) – (-6) =

Section 3: Multiplying and Dividing Integers

Sign Rules for Multiplication and Division

Signs	Result	Example
(+) × (+)	Positive	3 × 4 = 12
(–) × (–)	Positive	(–3) × (–4) = 12
(+) × (–)	Negative	3 × (–4) = –12
(–) × (+)	Negative	(–3) × 4 = –12

Quick Rule

• **Same signs → Positive result**

• **Different signs → Negative result**

This works for both multiplication AND division!

Multiple Factors

When multiplying more than two numbers:

• **Even number of negatives → Positive**

• **Odd number of negatives → Negative**

Example: (–2) × (–3) × (–4) = –24 (three negatives = odd = negative)

Division Examples

• 12 ÷ 3 = 4

• (–12) ÷ (–3) = 4

• 12 ÷ (–3) = –4

• (–12) ÷ 3 = –4

Practice: Multiplying and Dividing Integers

1) 6 × 7 =

2) (–5) × (–8) =

3) 9 × (–4) =

4) (–7) × 6 =

5) (–3) × (–4) × (–2) =

6) 2 × (–5) × (–3) =

7) 36 ÷ 6 =

8) (–48) ÷ (–8) =

9) 54 ÷ (–9) =

10) (–72) ÷ 8 =

11) (–100) ÷ (–25) =

12) 81 ÷ (–9) =

Section 4: Order of Operations with Integers

PEMDAS still applies, but now you must also handle negative numbers.

PEMDAS Review

Parentheses → **E**xponents → **M**ultiplication/**D**ivision → **A**ddition/**S**ubtraction

Examples with Integers

Example 1: $-3 + 4 \times (-2)$

Multiply first: $4 \times (-2) = -8$

Then add: $-3 + (-8) = -11$

Example 2: $(-6)^2 - 10$

Exponent first: $(-6)^2 = (-6) \times (-6) = 36$

Then subtract: $36 - 10 = 26$

Example 3: -4^2 vs $(-4)^2$

$-4^2 = -(4^2) = -16$ (only the 4 is squared)

$(-4)^2 = (-4) \times (-4) = 16$ (the entire -4 is squared)

Watch the parentheses!

Example 4: $20 \div (-4) + 3 \times (-2)$

Left to right for M/D and A/S:

$20 \div (-4) = -5$

$3 \times (-2) = -6$

$-5 + (-6) = -11$

Practice: Order of Operations

1) $5 + 3 \times (-4) =$

2) $(-8) + 12 \div (-3) =$

3) $(-5)^2 + 10 =$

4) $-5^2 + 10 =$

5) $6 \times (-2) - 8 \div 4 =$

6) $(10 - 15) \times (-3) =$

7) $-20 \div 4 + (-3)^2 =$

8) $(-2)^3 + 4 \times 5 =$

Section 5: Ratios and Proportions

What Is a Ratio?

A ratio compares two quantities. It can be written three ways:

• With a colon: 3:4

• As a fraction: 3/4

• With the word 'to': 3 to 4

Simplifying Ratios

Simplify like fractions—divide both numbers by their GCF.

Example: 12:18 → GCF is 6 → 12÷6 : 18÷6 = 2:3

Unit Rates

A unit rate has a denominator of 1. Divide to find it.

Example: 240 miles in 4 hours = 240÷4 = 60 miles per hour

Example: $15 for 5 pounds = $15÷5 = $3 per pound

What Is a Proportion?

A proportion states that two ratios are equal.

Example: 2/3 = 8/12 (both simplify to 2/3)

Solving Proportions: Cross Multiply

If a/b = c/d, then a × d = b × c

Example: Solve 3/5 = x/20

Cross multiply: 3 × 20 = 5 × x

60 = 5x

x = 60 ÷ 5 = 12

Example: Solve 4/x = 12/15

Cross multiply: 4 × 15 = x × 12

60 = 12x

x = 60 ÷ 12 = 5

Real-World Applications

Example: If 3 apples cost $2.40, how much do 7 apples cost?

Set up proportion: 3/2.40 = 7/x

Cross multiply: 3x = 7 × 2.40 = 16.80

x = 16.80 ÷ 3 = $5.60

Practice: Ratios and Proportions

1. Simplify the ratio 24:36: _____

2. Find the unit rate: 150 miles in 3 hours = _____ mph

3. Solve: 4/7 = x/21

4. Solve: 5/8 = 35/x

5. If 4 tickets cost $30, how much do 10 tickets cost?

6. A recipe uses 2 cups of flour for 3 dozen cookies. How much flour for 9 dozen?

Section 6: Percentages

'Percent' means 'per hundred.' 50% means 50 out of 100, or 50/100 = 0.5

Converting Between Percents, Decimals, and Fractions

From	To	Method	Example
Percent	Decimal	Divide by 100 (move decimal 2 left)	45% = 0.45
Decimal	Percent	Multiply by 100 (move decimal 2 right)	0.75 = 75%
Percent	Fraction	Put over 100, simplify	60% = 60/100 = 3/5
Fraction	Percent	Divide, then × 100	3/4 = 0.75 = 75%

Three Types of Percent Problems

Type 1: Find the Part (What is 25% of 80?)

Formula: Part = Percent × Whole

Solution: 0.25 × 80 = 20

Type 2: Find the Percent (15 is what percent of 60?)

Formula: Percent = Part ÷ Whole

Solution: 15 ÷ 60 = 0.25 = 25%

Type 3: Find the Whole (30 is 20% of what number?)

Formula: Whole = Part ÷ Percent

Solution: 30 ÷ 0.20 = 150

Percent Increase and Decrease

Percent Change = (Amount of Change ÷ Original) × 100

Example: Price went from $40 to $50. What's the percent increase?

Change = $50 – $40 = $10

Percent increase = (10 ÷ 40) × 100 = 25%

Common Percent Benchmarks

Percent	Fraction	Decimal
10%	1/10	0.1
20%	1/5	0.2
25%	1/4	0.25
50%	1/2	0.5
75%	3/4	0.75
100%	1	1.0

Practice: Percentages

1. Convert 35% to a decimal: _____

2. Convert 0.08 to a percent: _____

3. What is 30% of 250? _____

4. 18 is what percent of 90? _____

5. 45 is 15% of what number? _____

6. A $80 item is on sale for $60. What is the percent decrease? _____

7. Calculate a 15% tip on a $45 meal: _____

8. An item costs $50. With 8% tax, what is the total? _____

Section 7: Introduction to Variables

A variable is a letter (like x, y, or n) that represents an unknown number.

Algebraic Expressions

An expression combines numbers, variables, and operations.

• 3x means 3 × x (3 times some number)

• x + 5 means some number plus 5

• 2x + 7 means 2 times some number, plus 7

Vocabulary

• **Coefficient:** The number in front of a variable (in 3x, the coefficient is 3)

• **Constant:** A number without a variable (in 2x + 5, the constant is 5)

• **Term:** Parts separated by + or – (in 3x + 2y – 5, there are 3 terms)

• **Like terms:** Terms with the same variable (3x and 5x are like terms)

Evaluating Expressions

To evaluate, substitute the given value for the variable.

Example: Evaluate 3x + 7 when x = 4

3(4) + 7 = 12 + 7 = 19

Example: Evaluate 2a – b when a = 5 and b = 3

2(5) – 3 = 10 – 3 = 7

Combining Like Terms

Add or subtract the coefficients of like terms.

• 5x + 3x = 8x

• 7y – 2y = 5y

• 4x + 3 + 2x + 5 = 6x + 8 (combine x terms, combine constants)

Translating Words to Algebra

Words	Algebra
a number plus 5	x + 5
7 less than a number	x – 7
twice a number	2x
a number divided by 3	x/3 or x ÷ 3
5 more than twice a number	2x + 5

Practice: Variables

1. Evaluate 4x – 3 when x = 6: _____

2. Evaluate x^2 + 2x when x = 5: _____

3. Simplify: 8y + 3y – 2y = _____

4. Simplify: 5x + 4 + 3x – 7 = _____

5. Write as an expression: 'six less than a number' _____

6. Write as an expression: 'three times a number, increased by 8' _____

Section 8: Solving One-Step Equations

An equation states that two expressions are equal. Solving means finding the value that makes it true.

The Golden Rule

Whatever you do to one side, you must do to the other side.

This keeps the equation balanced.

Addition Equations (use subtraction to solve)

Example: x + 7 = 15

Subtract 7 from both sides: x + 7 – 7 = 15 – 7

x = 8

Check: 8 + 7 = 15 ✓

Subtraction Equations (use addition to solve)

Example: x – 5 = 12

Add 5 to both sides: x – 5 + 5 = 12 + 5

x = 17

Check: 17 – 5 = 12 ✓

Multiplication Equations (use division to solve)

Example: 4x = 28

Divide both sides by 4: 4x ÷ 4 = 28 ÷ 4

x = 7

Check: 4(7) = 28 ✓

Division Equations (use multiplication to solve)

Example: x/3 = 9

Multiply both sides by 3: (x/3) × 3 = 9 × 3

x = 27

Check: 27/3 = 9 ✓

Practice: One-Step Equations

1) x + 9 = 14

2) y – 8 = 12

3) n + 15 = 7

4) 5x = 45

5) 7y = –49

6) –3n = 24

7) x/4 = 8

8) y/6 = –5

9) n/–2 = 10

Section 9: Solving Two-Step Equations

Two-step equations require two operations to isolate the variable.

The Strategy: Reverse Order of Operations

1. First, undo addition or subtraction

2. Then, undo multiplication or division

(This is PEMDAS backwards!)

Examples

Example 1: $2x + 5 = 13$

Step 1: Subtract 5 from both sides: $2x = 8$

Step 2: Divide both sides by 2: $x = 4$

Check: $2(4) + 5 = 8 + 5 = 13$ ✓

Example 2: $3x – 7 = 14$

Step 1: Add 7 to both sides: $3x = 21$

Step 2: Divide both sides by 3: $x = 7$

Check: $3(7) – 7 = 21 – 7 = 14$ ✓

Example 3: $x/4 + 3 = 10$

Step 1: Subtract 3 from both sides: $x/4 = 7$

Step 2: Multiply both sides by 4: $x = 28$

Check: $28/4 + 3 = 7 + 3 = 10$ ✓

Example 4: $–5x + 12 = –8$

Step 1: Subtract 12 from both sides: $–5x = –20$

Step 2: Divide both sides by –5: $x = 4$

Check: $–5(4) + 12 = –20 + 12 = –8$ ✓

Practice: Two-Step Equations

1) $2x + 4 = 18$

2) $3y - 5 = 16$

3) $4n + 9 = 1$

4) $5x - 8 = 27$

5) $x/2 + 6 = 11$

6) $y/3 - 4 = 2$

7) $-2x + 10 = 4$

8) $6y - 15 = -3$

9) $x/5 + 8 = 3$

Section 10: The Coordinate Plane

The coordinate plane is a grid used to locate points using two numbers.

Parts of the Coordinate Plane

• **x-axis:** The horizontal number line

• **y-axis:** The vertical number line

• **Origin:** The point where the axes cross (0, 0)

• **Quadrants:** The four sections created by the axes

The Four Quadrants

Quadrant	x-values	y-values	Example
I (upper right)	Positive	Positive	(3, 4)
II (upper left)	Negative	Positive	(–3, 4)
III (lower left)	Negative	Negative	(–3, –4)
IV (lower right)	Positive	Negative	(3, –4)

Ordered Pairs

A point is written as (x, y) — always x first, then y.

• x tells how far to go LEFT or RIGHT from the origin

• y tells how far to go UP or DOWN

Memory aid: 'x comes before y' just like in the alphabet

Plotting Points

To plot (3, –2):

1. Start at the origin (0, 0)

2. Move 3 units RIGHT (positive x)

3. Move 2 units DOWN (negative y)

4. Mark the point

Practice: Coordinate Plane

1. In which quadrant is (–5, 7)? _____

2. In which quadrant is (4, –3)? _____

3. What are the coordinates of the origin? _____

4. If a point has x = 0, where is it located? _____

5. Plot these points (draw your own grid): A(2, 4), B(–3, 1), C(–2, –4), D(4, –2)

6. What is the distance from (0, 0) to (3, 0)? _____

Section 11: Mean, Median, Mode, and Range

These are measures of central tendency—ways to describe a data set with a single number.

Mean (Average)

Add all values, then divide by the count.

Example: Find the mean of 12, 15, 18, 20, 25

Sum: 12 + 15 + 18 + 20 + 25 = 90

Count: 5 values

Mean: 90 ÷ 5 = 18

Median (Middle Value)

Arrange in order, then find the middle value.

Odd count: Middle number is the median

Even count: Average the two middle numbers

Example: Find the median of 7, 12, 5, 18, 9

Order: 5, 7, 9, 12, 18

Median: 9 (the middle value)

Example: Find the median of 4, 8, 12, 16

Order: 4, 8, 12, 16 (already in order)

Median: (8 + 12) ÷ 2 = 10

Mode (Most Frequent)

The value that appears most often.

• A data set can have no mode, one mode, or multiple modes

Example: Find the mode of 3, 5, 5, 7, 8, 5, 9

Mode: 5 (appears 3 times)

Range (Spread)

Largest value minus smallest value.

Example: Find the range of 12, 5, 18, 9, 25

Range: 25 – 5 = 20

Practice: Statistics

Use this data set for problems 1-4: 15, 22, 18, 22, 13, 25, 18

1. Find the mean: _____

2. Find the median: _____

3. Find the mode(s): _____

4. Find the range: _____

Use this data set for problems 5-8: 45, 52, 48, 55, 50, 47, 53, 49

5. Find the mean: _____

6. Find the median: _____

7. Find the mode: _____

8. Find the range: _____

Section 12: Problem Solving Strategies

The 4-Step Problem Solving Process

1. UNDERSTAND — Read carefully. What do you know? What do you need to find?

2. PLAN — Choose a strategy. Set up an equation or draw a diagram.

3. SOLVE — Execute your plan. Show your work.

4. CHECK — Does your answer make sense? Verify it.

Problem Solving Strategies

• Draw a picture or diagram

• Make a table or organized list

• Look for a pattern

• Work backwards

• Guess and check (systematically)

• Write an equation

• Break into smaller parts

Practice Problems

1. A phone plan costs $25 per month plus $0.10 per text. If Maria's bill was $37, how many texts did she send?

_____

2. The sum of three consecutive integers is 66. Find the three integers.

_____

3. A store is having a 20% off sale. If an item's sale price is $48, what was the original price?

_____

4. Two numbers have a sum of 50 and a difference of 14. Find both numbers.

_____

5. A rectangle's length is 5 more than twice its width. If the perimeter is 46 cm, find the dimensions.

_____

Answer Key

Section 1: Integers

1) –3, –2, –1, 0, 1, 2 2) 12 3) 8 4) < 5) < 6) 15

Section 2: Adding/Subtracting Integers

1) 13 2) –10 3) 4 4) –5 5) 0 6) 4 7) 6 8) –3 9) –10 10) 3 11) 16 12) –9

Section 3: Multiplying/Dividing Integers

1) 42 2) 40 3) –36 4) –42 5) –24 6) 30 7) 6 8) 6 9) –6 10) –9 11) 4 12) –9

Section 4: Order of Operations

1) –7 2) –12 3) 35 4) –15 5) –14 6) 15 7) 4 8) 12

Section 5: Ratios and Proportions

1) 2:3 2) 50 mph 3) x = 12 4) x = 56 5) $75 6) 6 cups

Section 6: Percentages

1) 0.35 2) 8% 3) 75 4) 20% 5) 300 6) 25% 7) $6.75 8) $54

Section 7: Variables

1) 21 2) 35 3) 9y 4) 8x – 3 5) x – 6 6) 3x + 8

Section 8: One-Step Equations

1) x = 5 2) y = 20 3) n = –8 4) x = 9 5) y = –7 6) n = –8 7) x = 32 8) y = –30 9) n = –20

Section 9: Two-Step Equations

1) x = 7 2) y = 7 3) n = –2 4) x = 7 5) x = 10 6) y = 18 7) x = 3 8) y = 2 9) x = –25

Section 10: Coordinate Plane

1) II 2) IV 3) (0, 0) 4) On the y-axis 5) [Plot points] 6) 3 units

Section 11: Statistics

1) 19 2) 18 3) 18 and 22 4) 12 5) 49.875 6) 49.5 7) No mode 8) 10

Section 12: Problem Solving

1) 120 texts 2) 21, 22, 23 3) $60 4) 32 and 18 5) Width = 6 cm, Length = 17 cm

GLOBAL SOVEREIGN UNIVERSITY

Building a Bridge to Freedom Through Education — Not Handouts

www.globalsovereignuniversity.org