# � GRADE 7 �

# MATHEMATICS

**Real-World Problem Solving** Complete Student Edition

**GLOBAL SOVEREIGN UNIVERSITY**

*"Building a Bridge to Freedom Through Education*

**Grade 4 Mathematics: Real-World Problem Solving** *Student Edition*



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# Why This Book Is Different

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## 🏆 NOBEL PRIZE RESEARCH SAYS:

**"Students who score 96% on classroom math tests succeed only 1% of the time in real life."**

— Banerjee & Duflo, *Nature*, February 2025

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## The Research That Proves Traditional Math Education Fails

In February 2025, Nobel Prize-winning economists published a groundbreaking study in *Nature*, the world's most respected scientific journal. They tested over 1,400 children in India and discovered something shocking:

**Children who ace classroom math CANNOT use it in the real world.**

Students who scored 96% on standard school math problems could solve simple real-world market calculations only 1% of the time. Meanwhile, children who worked in markets—without formal education—solved the same problems with 96% accuracy.

The researchers' conclusion was clear: **"Schools need to build a bridge between math learned in the classroom and math encountered in real-life situations."**

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## The Problem: Abstract Doesn't Transfer to Applied

The Nobel laureates found that schoolchildren learned *procedures* but not *understanding*. They could solve "24 × 8" on a worksheet by adding 24 eight times. But give them a real problem—"How much do 8 items cost at $24 each?"—and they froze.

Market children used efficient mental strategies: breaking 11 × 43 into (10 × 43) + 43, rounding for easier calculation, and working with meaningful quantities. Schoolchildren followed rigid, slow methods that worked for tests but failed in life.

**It's not that school kids were less intelligent. It's that their education never connected the math to reality.**

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## The GSU Solution: Real-World Math

This textbook does exactly what the Nobel laureates recommend. Instead of abstract problems, every calculation has a professional context:

## Complete Pre-Algebra Textbook for Seventh Grade

# PART I: THE NUMBER SYSTEM

# CHAPTER 1: RATIONAL NUMBERS

## What You'll Learn

- ★ Understanding what makes a number rational
- ★ Converting between fractions, decimals, and percents
- ★ Adding and subtracting rational numbers
- ★ Multiplying and dividing rational numbers
- ★ Order of operations with rational numbers
- ★ Comparing and ordering rational numbers

## Why This Matters

Rational numbers are everywhere in daily life. When you split a pizza, calculate a discount, measure ingredients, or figure out how much you've saved toward a goal, you're working with rational numbers. Mastering operations with positive and negative fractions and decimals prepares you for algebra and gives you tools you'll use throughout your life.

## LESSON 1.1: What Is a Rational Number?

### The Definition

A **rational number** is any number that can be written as a fraction a/b, where a and b are integers and b ≠ 0.

The word "rational" comes from "ratio"—a rational number is any number that can be expressed as a ratio of two integers.

### Examples of Rational Numbers

**Integers are rational:**

5 = 5/1 ✓

-3 = -3/1 ✓

0 = 0/1 ✓

**Fractions are rational:**

3/4 ✓

-7/8 ✓

15/2 ✓

**Terminating decimals are rational:**

0.75 = 75/100 = 3/4 ✓

2.5 = 25/10 = 5/2 ✓

-0.125 = -125/1000 = -1/8 ✓

**Repeating decimals are rational:**

0.333... = 1/3 ✓

0.666... = 2/3 ✓

0.142857142857... = 1/7 ✓

**What Is NOT Rational?**

Numbers that cannot be written as fractions are called **irrational numbers**.

**Examples of irrational numbers:**

π = 3.14159265358979... (never ends, never repeats)

√2 = 1.41421356237... (never ends, never repeats)

√3 = 1.73205080757... (never ends, never repeats)

**Key Idea:** If a decimal terminates (ends) OR repeats a pattern, it's rational. If it goes on forever without repeating, it's irrational.

**Converting Terminating Decimals to Fractions**

**Example 1:** Convert 0.75 to a fraction.

**Step 1:** Count decimal places. 0.75 has 2 decimal places.

**Step 2:** Write over the appropriate power of 10. 0.75 = 75/100

**Step 3:** Simplify. 75/100 = ¾

**Answer: 0.75 = 3/4**

**Example 2:** Convert 2.125 to a fraction.

**Step 1:** 2.125 has 3 decimal places.

**Step 2:** Write as fraction: 2125/1000

**Step 3:** Simplify by finding GCF (125): 2125 ÷ 125 = 17

1000 ÷ 125 = 8

**Answer: 2.125 = 17/8 or 2 1/8**

**Converting Repeating Decimals to Fractions**

This requires algebra! Let's see how it works.

**Example 3:** Convert 0.333... to a fraction.

**Step 1:** Let x = 0.333...

**Step 2:** Multiply both sides by 10: 10x = 3.333...

**Step 3:** Subtract the original equation: 10x = 3.333...

x = 0.333... ___________________ 9x = 3

**Step 4:** Solve for x: x = 3/9 = 1/3

**Answer: 0.333... = 1/3**

**Example 4:** Convert 0.454545... to a fraction.

**Step 1:** Let x = 0.454545...

**Step 2:** The pattern "45" has 2 digits, so multiply by 100: 100x = 45.454545... **Step 3:** Subtract: 100x = 45.454545...

x = 0.454545... ___________________99x = 45

**Step 4:** Solve: x = 45/99 = 5/11

**Answer: 0.454545... = 5/11**

**Example 5:** Convert 0.1666... to a fraction.

This is trickier because only part of the decimal repeats.

**Step 1:** Let x = 0.1666...

**Step 2:** Multiply by 10 to move past the non-repeating part: 10x = 1.666...

**Step 3:** Multiply by 100: 100x = 16.666...

**Step 4:** Subtract: 100x = 16.666...

10x = 1.666...__________________ 90x = 15

**Step 5:** Solve: x = 15/90 = 1/6

**Answer: 0.1666... = 1/6**

✏️ **Try This**

Convert each to a fraction in simplest form:

1. 0.4
2. 0.625
3. 1.75
4. 0.222...
5. 0.363636...
6. 0.8333...

## LESSON 1.2: Adding and Subtracting Rational Numbers

### The Rules for Signs

When adding numbers with the **same sign:**

- ★ Add the absolute values
- ★ Keep the common sign

When adding numbers with **different signs:**

- ★ Subtract the smaller absolute value from the larger
- ★ Use the sign of the number with the larger absolute value

**Example 6:** Add: -5 + (-3)

Same signs (both negative):

Add absolute values: 5 + 3 = 8

Keep the negative sign

**Answer: -5 + (-3) = -8**

**Example 7:** Add -7 + 4

Different signs:

Subtract: 7 - 4 = 3

The negative number has larger absolute value, so answer is negative **Answer: -7 + 4 = -3**

**Example 8:** Add: 6 + (-2)

Different signs:

Subtract: 6 - 2 = 4

The positive number has larger absolute value, so answer is positive **Answer: 6 + (-2) = 4**

**Subtracting = Adding the Opposite**

**To subtract a number, add its opposite.**

a - b = a + (-b)

**Example 9:** Subtract: 5 - 8

Rewrite: 5 + (-8)

Different signs, larger absolute value is negative: 5 - 8 = 5 + (-8) = **-3**

**Example 10:** Subtract: -4 - 6

Rewrite: -4 + (-6)

Same signs (both negative): -4 + (-6) = **-10**

**Example 11:** Subtract: -3 - (-7)

Rewrite: -3 + 7

Different signs, larger absolute value is positive: -3 + 7 = **4**

**Adding and Subtracting Fractions**

Remember: You need a **common denominator** to add or subtract fractions.

**Example 12:** Add: 2/3 + 1/4

**Step 1:** Find LCD. LCD of 3 and 4 is 12.
**Step 2:** Convert fractions: 2/3 = 8/12 1/4 = 3/12
**Step 3:** Add numerators: 8/12 + 3/12 = 11/12
**Answer: 2/3 + 1/4 = 11/12**

**Example 13:** Add: -2/5 + 3/10
**Step 1:** LCD of 5 and 10 is 10.
**Step 2:** Convert: -2/5 = -4/10 3/10 = 3/10
**Step 3:** Add (different signs): -4/10 + 3/10 = -1/10
**Answer: -2/5 + 3/10 = -1/10**

**Example 14:** Subtract: 5/6 - 2/3

**Step 1:** LCD of 6 and 3 is 6.

**Step 2:** Convert: 5/6 = 5/6 2/3 = 4/6

**Step 3:** Subtract: 5/6 - 4/6 = 1/6

**Answer: 5/6 - 2/3 = 1/6**

**Example 15:** Subtract: -1/4 - 3/8

**Step 1:** Rewrite as addition: -1/4 + (-3/8)

**Step 2:** LCD is 8: -1/4 = -2/8 -3/8 = -3/8

**Step 3:** Add (same signs): -2/8 + (-3/8) = -5/8

**Answer: -1/4 - 3/8 = -5/8**

### Adding and Subtracting Decimals

Line up the decimal points!

**Example 16:** Add: -3.7 + 2.4

Different signs:

Subtract: 3.7 - 2.4 = 1.3

Larger absolute value is negative

**Answer: -3.7 + 2.4 = -1.3**

**Example 17:** Subtract: -2.5 - 1.8

Rewrite: -2.5 + (-1.8)

Same signs:

Add: 2.5 + 1.8 = 4.3

Both negative, so answer is negative

**Answer: -2.5 - 1.8 = -4.3**

### Your Turn

Calculate:

1. -8 + 5 = _____
2. -6 + (-4) = _____
3. 7 - 12 = _____
4. -9 - (-3) = _____

5. 3/4 + (-1/2) = _____

6. -2/3 - 1/6 = _____

7. -4.2 + 1.7 = _____

8. 3.5 - (-2.1) = _____

## LESSON 1.3: Multiplying and Dividing Rational Numbers

**The Sign Rules**

**For multiplication and division:**

**Signs Result**

Positive × Positive = Positive

Negative × Negative = Positive

Positive × Negative Negative

Negative × Positive Negative

**Key Idea:** Same signs = positive result. Different signs = negative result.

This works for both multiplication AND division!

**Example 18:** Multiply: (-4)(6)

Different signs → Negative
4 × 6 = 24

**Answer: (-4)(6) = -24**

**Example 19:** Multiply: (-5)(-7)

Same signs → Positive
5 × 7 = 35

**Answer: (-5)(-7) = 35**

**Example 20:** Divide: -20 ÷ 4

Different signs → Negative
20 ÷ 4 = 5

**Answer: -20 ÷ 4 = -5**

**Example 21:** Divide: -36 ÷ (-9)

Same signs → Positive
36 ÷ 9 = 4

**Answer: -36 ÷ (-9) = 4**

**Multiplying Fractions**

Multiply numerators, multiply denominators, then simplify.

**Example 22:** Multiply: (2/3) x (4/5)

Multiply: (2 × 4)/(3 × 5) = 8/15

**Answer: (2/3)(4/5) = 8/15**

**Example 23:** Multiply: (-3/4)(2/5)

Different signs → Negative

Multiply: (3 × 2)/(4 × 5) = 6/20 = 3/10

**Answer: (-3/4)(2/5) = -3/10**

**Example 24:** Multiply: (-2/3)(-9/10)

Same signs → Positive motional cap

Before multiplying, simplify by cross-canceling: 2 and 10 share factor of 2

3 and 9 share factor of 3

(-2/3)(-9/10) = (-1/1)(-3/5) = 3/5

**Answer: (-2/3)(-9/10) = 3/5**

**Dividing Fractions**

**To divide by a fraction, multiply by its reciprocal.** a/b ÷ c/d = a/b × d/c

**Example 25:** Divide: (3/4) ÷ (2/5)

Multiply by reciprocal:
(3/4) × (5/2) = 15/8 = 1 7/8

**Answer: (3/4) ÷ (2/5) = 15/8**

**Example 26:** Divide: (-5/6) ÷ (2/3)

Different signs → Negative

Multiply by reciprocal:
(5/6) × (3/2) = 15/12 = 5/4

**Answer: (-5/6) ÷ (2/3) = -5/4**

**Example 27:** Divide: (-4/9) ÷ (-2/3)

Same signs → Positive

Multiply by reciprocal:
(4/9) × (3/2) = 12/18 = 2/3

**Answer: (-4/9) ÷ (-2/3) = 2/3**

**Multiplying and Dividing Decimals**

**Example 28:** Multiply: (-2.5)(0.4)
Different signs → Negative
2.5 × 0.4 = 1.0

**Answer: (-2.5)(0.4) = -1.0**

**Example 29:** Divide: -7.2 ÷ (-0.9)

Same signs → Positive
7.2 ÷ 0.9 = 8

**Answer: -7.2 ÷ (-0.9) = 8**

**Your Turn**

Calculate:

1. (-8)(7) =______
2. (-6)(-9)  =______
3. 42 ÷ (-7) =______
4. -56 ÷ (-8) =______
5. (3/5)(-2/3) =______
6. (-4/7) ÷ (2/3) =______
7. (-1.5)(0.6) =______
8. -4.8 ÷ (-1.2) =______

## LESSON 1.4: Order of Operations with Rational Numbers

**PEMDAS/GEMDAS Review**

**P/G** - Parentheses/Grouping symbols first

**E** - Exponents

**MD** - Multiplication and Division (left to right)

**AS** - Addition and Subtraction (left to right)

**Example 30:** Evaluate: $-3 + 4 \times 2$

Multiply first: $4 \times 2 = 8$
Then add: $-3 + 8 = 5$

**Answer: $-3 + 4 \times 2 = 5$**

**Example 31:** Evaluate: $(-2)^3 + 5$
Exponent first: $(-2)^3 = (-2)(-2)(-2) = -8$
Then add: $-8 + 5 = -3$

**Answer: $(-2)^3 + 5 = -3$**

⚠️ **Watch Out!** $(-2)^3 = -8$, but $-2^3 = -(2^3) = -8$. In this case they're the same, but be careful with even exponents: $(-2)^2 = 4$, while $-2^2 = -4$.

**Example 32:** Evaluate: 12 ÷ (-3) - 4 × (-2)

Division and multiplication from left to right:
12 ÷ (-3) = -4
4 × (-2) = -8

Now subtract: -4 - (-8) = -4 + 8 = 4

**Answer: 12 ÷ (-3) - 4 × (-2) = 4**

**Example 33:** Evaluate: (1/2 + 1/3) × 6

Parentheses first:
1/2 + 1/3 = 3/6 + 2/6 = 5/6

Then multiply:
5/6 × 6 = 5

**Answer: (1/2 + 1/3) × 6 = 5**

**Example 34:** Evaluate: $-2(3 - 7) + 4^2$

Parentheses: 3 - 7 = -4
Exponent: $4^2 = 16$
Multiply: -2(-4) = 8
Add: 8 + 16 = 24

**Answer: $-2(3 - 7) + 4^2 = 24$**

**Example 35:** Evaluate: [(-6) + 2] ÷ [(-1)(-2)]

Brackets first:
(-6) + 2 = -4
(-1)(-2) = 2

Divide:
-4 ÷ 2 = -2

**Answer: [(-6) + 2] ÷ [(-1)(-2)] = -2**

**Your Turn**

Evaluate:

1. $5 - 3 \times 4 =$______
2. $(-4)^2 - 10 =$______
3. $-6 \div 2 + 8 \div (-4) =$______
4. $(2/3 - 1/2) \times 12 =$______
5. $3(-4 + 6) - 2^3 =$______
6. $[-8 + 2(-3)] \div 7 =$______

## LESSON 1.5: Comparing and Ordering Rational Numbers

**Using a Number Line**

On a number line:

Numbers increase from left to right

Negative numbers are to the left of zero

The further left, the smaller the number

**Comparing Fractions**

To compare fractions, find a common denominator OR convert to decimals.

**Example 36:** Compare: -3/4 and -2/3

**Method 1: Common denominator** $-3/4 = -9/12$

$-2/3 = -8/12$ Since $-9 < -8$, we have $-9/12 < -8/12$

**Answer: $-3/4 < -2/3$**

**Method 2: Convert to decimals.** $-3/4 = -0.75$

$-2/3 = -0.666...$ $-0.75 < -0.666...$

**Answer: $-3/4 < -2/3$**

**Example 37:** Order from least to greatest: 1/2, -3/4, 2/3, -1/3

Convert to decimals:

1/2 = 0.5
-3/4 = -0.75

2/3 ≈ 0.667

-1/3 ≈ -0.333

Order: -0.75, -0.333, 0.5, 0.667

**Answer: -3/4, -1/3, 1/2, 2/3**

**Example 38:** Order from greatest to least: -2.5, -2.05, -2.55, -2.1

All negative, so smaller absolute value = greater number.

Absolute values: 2.5, 2.05, 2.55, 2.1

Order of absolute values (least to greatest): 2.05, 2.1, 2.5, 2.55

So original numbers (greatest to least): **-2.05, -2.1, -2.5, -2.55**

**Your Turn**

1. Compare: -5/8 ___ -3/5 (use < or >)
2. Compare: -0.45 ___ -0.405
3. Order from least to greatest: -0.7, -3/4, -0.65, -7/10 ____________
4. Order from greatest to least: 2/5, -1/2, 0.35, -0.6 ____________

# Chapter 1 Practice Problems

### Section A: Converting Between Forms

Convert each decimal to a fraction in simplest form: 1. 0.8

2. 0.35 ____________
3. 1.25 ____________
4. 0.125 ____________
5. 0.555... ____________
6. 0.272727... ____________
7. 0.4166... ____________
8. 2.333…____________

### Section B: Adding and Subtracting

Calculate:

9. -15 + 8 ____________
10. -7 + (-12) ____________
11. 6 - 14 ____________
12. -8 - (-5) ____________
13. 3/4 + (-1/2) ____________
14. -5/6 + 2/3 ____________
15. -2/5 - 3/10 ____________
16. 7/8 - (-1/4) ____________
17. -3.6 + 2.1 ____________
18. -5.2 - 3.8 ____________
19. 4.5 + (-7.3) ____________
20. -0.75 - (-1.25) ____________

**Section C: Multiplying and Dividing**

Calculate:

21. (-9)(8) _____________

22. (-7)(-11) _____________

23. -48 ÷ 6 _____________

24. -63 ÷ (-9) _____________

25. (2/3)(-3/4) _____________

26. (-5/8)(-4/15) _____________

27. (3/5) ÷ (-2/3) _____________

28. (-7/10) ÷ (-14/15) _____________

29. (-2.4)(0.5) _____________

30. (-0.8)(-1.5) _____________

31. 5.4 ÷ (-0.9) _____________

32. -3.6 ÷ (-0.4)_____________

**Section D: Order of Operations**

Evaluate:

33. 8 - 12 ÷ 4 _____________

34. -5 + 3 × (-2) _____________

35. $(-3)^2$ - 4 × 2 _____________

36. 15 ÷ (-3) + $(-2)^3$ _____________

37. (4 - 10) ÷ 2 + 5 _____________

38. -2(5 - 8) + 4 _____________

39. [(-12) ÷ 4] × [(-2)(-3)] _____________

40. $(1/2)^2$ + 1/4 _____________

41. 2/3 × (3/4 - 1/2) _____________

42. $(-1/2)^3 + 3/8$ ____________

**Section E: Comparing and Ordering**

Compare using < or >:

43. -7/8 ___ -5/6

44. -0.625 ___ -5/8

45. 3/5 ___ 0.59

46. -1.333... ___ -4/3

Order from least to greatest:

47. 0.5, -0.6, 1/3, -1/2 ____________

48. -3/4, -0.8, -7/10, -0.72 ____________

49. 2/3, 0.65, 5/8, 0.7 ____________

50. -1.25, -5/4, -1.2, -6/5 ____________

**Section F: Word Problems**

51. The temperature was -4°F in the morning. It rose 12°F by noon, then dropped 7°F by evening. What was the evening temperature? ____________

52. A submarine was at -250 feet. It rose 75 feet, then dove 120 feet. What is its new depth? ____________

53. Maria has $45.50 in her account. She writes checks for $23.75 and $18.00, then deposits $15.25. What is her new balance? ____________

54. A recipe calls for 2/3 cup of sugar. If you want to make 1 1/2 times the recipe, how much sugar do you need? ____________

55. A stock lost 3/4 point on Monday, gained 1 1/2 points on Tuesday, and lost 5/8 point on Wednesday. What was the total change? ____________

## �� Challenge Problems

1. If a × b = -12 and a + b = 1, find two possible pairs of values for a and b. ____________

2. The average of five numbers is -3. Four of the numbers are 2, -7, 4, and -6. What is the fifth number? ____________

3. Simplify: (-1/2)(-2/3)(-3/4)(-4/5)(-5/6) ____________

4. If x = -2/3 and y = 3/4, evaluate: (x + y)/(x - y) ____________

5. The temperature drops 2.5°F every hour. If it's currently 18°F, what will the temperature be in 10 hours? When will it reach 0°F? ____________

## Chapter 1 Summary

**Key Concepts**

**Rational Numbers:**

Can be written as a/b, where a and b are integers and b ≠ 0

Include integers, fractions, terminating decimals, repeating decimals

**Adding/Subtracting:**

Same signs: Add absolute values, keep the sign

Different signs: Subtract absolute values, use sign of larger absolute value

Subtracting = adding the opposite

**Multiplying/Dividing:**

Same signs → Positive result

Different signs → Negative result

**Order of Operations:** PEMDAS (Parentheses, Exponents, Multiplication/Division, Addition/Subtraction)

**Key Formulas and Rules**

**Operation Same Signs Different Signs**

Add Add, keep sign Subtract, larger sign

**Operation Same Signs Different Signs**

| |
|---|
| Add opposite |
| Positive |

Subtract Add the opposite.

Multiply Negative

Divide Positive Negative

*This chapter provides the foundation for all algebraic work with rational numbers.*

# CHAPTER 2: COMPLEX EQUATIONS AND INEQUALITIES

## What You'll Learn

Solving multi-step equations with complex terms

Working with equations involving fractions and decimals

Solving compound inequalities

Solving absolute value equations

Graphing solutions on number lines

## Why This Matters

As mathematical problems become more complex, so do the equations we need to solve. These advanced equation-solving skills are essential for algebra and beyond. Inequalities help us describe ranges of solutions—like budget limits, acceptable measurements, or qualifying scores—that occur constantly in real life.

## LESSON 2.1: Multi-Step Equations with Fractions

### Strategy

When equations contain fractions, you can:

1. Work directly with fractions, OR
2. Multiply both sides by the LCD to eliminate fractions first

### Method 1: Working with Fractions

**Example 1:** Solve: $(2/3)x + 1/4 = 5/6$

**Step 1:** Subtract 1/4 from both sides. $(2/3)x = 5/6 - 1/4$

**Step 2:** Find common denominator (12): $5/6 = 10/12$ $1/4 = 3/12$ $(2/3)x = 10/12 - 3/12 = 7/12$

**Step 3:** Multiply both sides by reciprocal of 2/3 (which is 3/2): $x = (7/12) \times (3/2) = 21/24 =$ **7/8**

**Check:** $(2/3)(7/8) + 1/4 = 14/24 + 6/24 = 20/24 = 5/6$ ✓

**Method 2: Clearing Fractions**

Example 2: Solve: (2/3)x + 1/4 = 5/6 (same equation, different method)

**Step 1:** Find LCD of 3, 4, and 6. LCD = 12

**Step 2:** Multiply every term by 12: 12 × (2/3)x + 12 × (1/4) = 12 × (5/6)

8x + 3 = 10 **Step 3:** Solve the simpler equation: 8x = 7 x = **7/8**

**Example 3:** Solve: (x/2) - (x/5) = 3

LCD = 10. Multiply every term by 10: 10(x/2) - 10 (x/5) = 10 (3) 5x - 2x = 30 3x = 30 x = 10

**Check:** 10/2 - 10/5 = 5 - 2 = 3 ✓

**Example 4:** Solve: (x + 3)/4 = (x - 1)/3

Cross multiply:
3(x + 3) = 4(x - 1)
3x + 9 = 4x - 4
9 + 4 = 4x - 3x
13 = x

**x = 13**

**Check:** (13 + 3)/4 = 16/4 = 4 and (13 - 1)/3 = 12/3 = 4 ✓

✏️ **Try This**

Solve:

1. (1/2)x + (1/3) = 5/6 _____________
2. (x/3) + (x/4) = 7 _____________
3. (2x - 1)/5 = (x + 2)/3 _____________
4. (3/4)x - 2 = (1/2)x + 1 _____________

## LESSON 2.2: Equations with Decimals

**Strategy**

Multiply by a power of 10 to eliminate decimals.

**Decimal places Multiply by**

1 (tenths) 10

2 (hundredths) 100

3 (thousandths) 1000

**Example 5:** Solve: $0.3x + 1.2 = 2.7$

**Method 1: Direct** $0.3x = 2.7 - 1.2$ $0.3x = 1.5$ $x = 1.5 \div 0.3 = \mathbf{5}$

**Method 2: Clear decimals** Multiply by 10: $3x + 12 = 27$ $3x = 15$ $x = \mathbf{5}$

**Example 6:** Solve: $0.05x + 0.25 = 0.45$

Multiply by 100 (2 decimal places): $5x + 25 = 45$ $5x = 20$ $x = \mathbf{4}$

**Check:** $0.05(4) + 0.25 = 0.20 + 0.25 = 0.45$ ✓

**Example 7:** Solve: $0.4(x - 3) = 0.2x + 1.8$

Distribute: $0.4x - 1.2 = 0.2x + 1.8$

Multiply by 10: $4x - 12 = 2x + 18$ $2x = 30$ $x = \mathbf{15}$

**Check:** $0.4(15 - 3) = 0.4(12) = 4.8$ $0.2(15) + 1.8 = 3 + 1.8 = 4.8$ ✓

### ✏️ Try This

Solve:

1. $0.5x - 0.3 = 1.7$ ____________
2. $0.08x + 0.04 = 0.2$ ____________
3. $0.25(x + 4) = 0.5x - 1$ ____________

# LESSON 2.3: More Complex Multi-Step Equations

**Equations with Variables on Both Sides and Grouping**

**Example 8:** Solve: $3(2x - 4) - 2(x + 5) = 2(x - 1)$

**Step 1:** Distribute on both sides. $6x - 12 - 2x - 10 = 2x - 2$

**Step 2:** Combine like terms on the left side. $4x - 22 = 2x - 2$

**Step 3:** Move variables to one side. $4x - 2x = -2 + 22$ $2x = 20$

**Step 4:** Solve. $x =$ **10**

**Check:** $3(20 - 4) - 2(10 + 5) = 3(16) - 2(15) = 48 - 30 = 18$ $2(10 - 1) = 2(9) = 18$ ✓

**Example 9:** Solve: $5 - 2(3x - 1) = 4(x + 2) - 3x$

Distribute:
$5 - 6x + 2 = 4x + 8 - 3x$

Combine like terms:
$7 - 6x = x + 8$

Move variables: $-6x - x = 8 - 7$ $-7x = 1$ $x =$ **-1/7**

**Example 10:** Solve: $(1/2)(4x + 6) = (2/3)(6x - 9)$

Distribute:
$2x + 3 = 4x - 6$

Move variables: $3 + 6 = 4x - 2x$ $9 = 2x$ $x =$ **9/2** or **4.5**

**Special Cases Review**

**No Solution:** Variables cancel, leaving a false statement.

**Example 11:** Solve: $2(x + 3) = 2x + 8$

$2x + 6 = 2x + 8$
$6 = 8$ ✗ (False!)
**No solution**

**Infinite Solutions:** Variables cancel, leaving a true statement.

**Example 12:** Solve: 3(x - 2) + 6 = 3x

3x - 6 + 6 = 3x

3x = 3x

0 = 0 ✓ (Always true!)

**All real numbers are solutions (identity)**

✏️ **Try This**

Solve:

1. 4(x - 2) - 3(x + 1) = x - 5____________
2. 2(3x + 1) = 3(2x - 4) + 14 ____________
3. (1/3)(6x - 9) = 2(x + 1) - 5 ____________

## LESSON 2.4: Solving Inequalities Review

### Inequality Symbols

**Symbol Meaning Graph**

| |
|---|
| Less than |
| Greater than |
| Less than or equal to |

< Open circle > Open circle ≤ Closed circle

≥ Greater than or equal to Closed circle

**Solving One-Step Inequalities**

**Same rules as equations, BUT:**

⚠️ **When you multiply or divide by a negative number, FLIP the inequality sign!**

**Example 13:** Solve and graph: $x + 5 < 12$

$x < 12 - 5$
$x < 7$

Graph: Open circle at 7, arrow pointing left
←————————○
7

**Example 14:** Solve and graph: $-3x \geq 15$

Divide by -3 (FLIP the sign!):
$x \leq -5$

Graph: Closed circle at -5, arrow pointing left
←————————●
-5

**Solving Multi-Step Inequalities**

**Example 15:** Solve: $4x - 7 > 13$

$4x > 20$ $x >$ **5**

**Example 16:** Solve: $-2x + 5 \leq 11$

$-2x \leq 6$ $x \geq$ **-3** (flip the sign!)

**Example 17:** Solve: $3(x - 2) + 4 \geq 2x - 5$

$3x - 6 + 4 \geq 2x - 5$ $3x - 2 \geq 2x - 5$ $x \geq$ **-3**

### ✏️ Try This

Solve and graph:

1. $x - 4 > 2$ ____________
2. $-5x < 20$ ____________
3. $2x + 3 \leq 11$ ____________
4. $-3(x - 1) \geq 9$ ____________

# LESSON 2.5: Compound Inequalities

### "And" Compound Inequalities

"And" means BOTH conditions must be true. The solution is the intersection (overlap).

**Example 18:** Solve: $x > 2$ AND $x < 7$

This can be written as: $\mathbf{2 < x < 7}$

Graph:

○————————○

2 7

All numbers between 2 and 7 (not including 2 and 7).

**Example 19:** Solve: $-3 \leq x + 2 < 5$

Solve as a "sandwich" inequality—do the same thing to all

three parts: Subtract 2 from all parts: $-3 - 2 \leq x < 5 - 2$ $\mathbf{-5 \leq}$

$\mathbf{x < 3}$

Graph:

●————————○

-5 3

**Example 20:** Solve: $1 < 2x - 3 \leq 7$
Add 3 to all parts:
$4 < 2x \leq 10$

Divide all parts by 2: $\mathbf{2 < x \leq 5}$

### "Or" Compound Inequalities

**"Or"** means AT LEAST ONE condition must be true.

The solution is the **union** (combined).

**Example 21:** Solve: $x < -2$ OR $x > 3$

Graph:

←——○ ○——→

-2 3

Two separate regions: everything less than -2 OR everything greater than 3.

**Example 22:** Solve: $3x + 1 < -5$ OR $2x - 3 > 7$

Solve each separately:

**First inequality:** $3x + 1 < -5$ $3x < -6$ $x < -2$

**Second inequality:** $2x - 3 > 7$ $2x > 10$ $x > 5$

**Solution: $x < -2$ OR $x > 5$**

✏️ **Try This**

Solve and graph:

1. $x > -1$ AND $x \leq 4$ ____________
2. $-2 < x + 3 < 6$ ____________
3. $2x - 1 < 5$ OR $2x - 1 > 9$ ____________
4. $-4 \leq 3x + 2 \leq 11$ ____________

## LESSON 2.6: Absolute Value Equations

### Understanding Absolute Value

**Absolute value** is the distance from zero on a number line.

$|a| = a$ if $a \geq 0$
$|a| = -a$ if $a < 0$

$|5| = 5$ and $|-5| = 5$ (both are 5 units from zero)

**Solving $|x| = c$**

If $|x| = c$ (where $c > 0$), then $x = c$ OR $x = -c$

**Example 23:** Solve: $|x| = 7$

$x = 7$ OR $x = -7$

**Solutions: $x = 7, x = -7$**

**Solving $|ax + b| = c$**

**Step 1:** Set up two equations:

$ax + b = c$

$ax + b = -c$

**Step 2:** Solve each equation.

**Example 24:** Solve: $|x - 3| = 5$

**Equation 1:** $x - 3 = 5 \rightarrow x = 8$ **Equation 2:** $x - 3 = -5 \rightarrow x = -2$

**Solutions: $x = 8, x = -2$**

**Check:** $|8 - 3| = |5| = 5$ ✓ and $|-2 - 3| = |-5| = 5$ ✓

**Example 25:** Solve: $|2x + 1| = 9$

**Equation 1:** $2x + 1 = 9 \rightarrow 2x = 8 \rightarrow x = 4$ **Equation 2:** $2x + 1 = -9 \rightarrow 2x = -10 \rightarrow x = -5$ **Solutions: $x = 4, x = -5$**

**Example 26:** Solve: $|3x - 4| + 2 = 11$

First, isolate the absolute value:
$|3x - 4| = 9$

**Equation 1:** $3x - 4 = 9 \rightarrow 3x = 13 \rightarrow x = 13/3$

**Equation 2:** $3x - 4 = -9 \rightarrow 3x = -5 \rightarrow x = -5/3$

**Solutions: $x = 13/3, x = -5/3$**

**Special Cases**

**Example 27:** Solve: |x + 2| = -3

Absolute value is NEVER negative!

**No solution**

**Example 28:** Solve: |4x - 8| = 0

Absolute value equals zero only when the expression inside equals zero:

4x - 8 = 0 4x = 8 x = **2**

Only one solution.

✏️ **Try This**

Solve:

1. |x| = 12 _____________
2. |x + 4| = 6 _____________
3. |2x - 5| = 11 _____________
4. |x - 1| + 3 = 8 _____________
5. |3x + 6| = -2 _____________

## LESSON 10.7: Writing and Solving Real-World Inequalities

**Setting Up Inequalities**

**Phrase Symbol** At most, no more than, maximum ≤ At least, no less than, minimum ≥ More than, greater than >

Less than, fewer than <

**Example 29:** You need at least 80 points to pass. You have 53 points. How many more points (p) do you need?

$53 + p \geq 80$
$p \geq 27$

**You need at least 27 more points.**

**Example 30:** A phone plan costs $35 plus $0.10 per text. You can spend at most $60. How many texts can you send?

$35 + 0.10t \leq 60$
$0.10t \leq 25$
$t \leq 250$

**You can send at most 250 texts.**

**Example 31:** The temperature must stay between 65°F and 75°F. Write and solve an inequality if the current temperature is 60°F and it rises r degrees.

$65 \leq 60 + r \leq 75$ $65 - 60 \leq r \leq 75 - 60$ $\mathbf{5 \leq r \leq 15}$

The temperature must rise between 5 and 15 degrees.

✏️ **Try This**
1. A roller coaster requires a height of at least 48 inches. Write an inequality for acceptable heights (h).

2. You have $25 and want to buy notebooks costing $3 each. How many can you buy?

3. A quiz has 10 questions worth 10 points each. You need more than 70 points to pass. How many must you get right?

## Chapter 2 Practice Problems

### Section A: Equations with Fractions

Solve:

1. (1/2)x + (2/3) = 7/6 ____________
2. (x/4) - (x/6) = 2 ____________

3. $(x + 2)/3 = (x - 4)/2$ _____________

4. $(2x + 1)/5 = (3x - 1)/4$ _____________

**Section B: Equations with Decimals**

Solve:

5. $0.6x + 0.4 = 2.2$ _____________

6. $0.15x - 0.05 = 0.4$ _____________

7. $0.3(x - 2) = 0.5x + 1.4$ _____________

**Section C: Complex Multi-Step Equations**

Solve:

8. $5(x - 3) - 2(x + 4) = 10$ _____________

9. $4(2x - 1) = 3(x + 2) + 5x$_____________

10. $(1/2)(6x + 4) = (1/3)(9x - 6)$ _____________

Identify: one solution, no solution, or all real numbers?

11. $3(x + 2) = 3x + 6$ _____________

12. $2(x - 4) = 2x - 5$ _____________

13. $5x - (2x + 3) = 3x - 3$ _____________

**Section D: Solving Inequalities**

Solve and graph:

14. $x + 7 > 3$ _____________

15. $-4x \leq 12$ _____________

16. $3x - 5 < 10$ _____________

17. $-2(x + 3) \geq 8$ _____________

**Section E: Compound Inequalities**

Solve:

18. $x > 1$ AND $x < 6$ ____________

19. $-5 < 2x + 1 < 9$ ____________

20. $x \leq -3$ OR $x \geq 2$ ____________

21. $3x - 2 < 4$ OR $3x - 2 > 10$ ____________

22. $-1 \leq (x - 3)/2 < 4$ ____________

**Section F: Absolute Value Equations**

Solve:

23. $|x| = 15$ ____________

24. $|x - 5| = 8$ ____________

25. $|3x + 2| = 14$ ____________

26. $|2x - 1| - 3 = 6$ ____________

27. $|4x + 8| = 0$ ____________

28. $|x - 2| = -5$ ____________

**Section G: Word Problems**

29. A student has test scores of 78, 85, and 82. What score is needed on the fourth test to have an average of at least 85?

30. A rental car costs \$40/day plus \$0.25/mile. You have \$100. If you rent for 1 day, how many miles can you drive?

31. The sum of three consecutive odd integers is between 50 and 70. Find all possible sets of integers.

## �� Challenge Problems

1. Solve: |2x - 3| = |x + 4| ____________
2. Solve: (2x + 1)/3 - (x - 2)/4 = (x + 5)/6 ____________
3. For what values of k does |x - 3| = k have exactly one solution? ____________
4. Solve the compound inequality: 2 ≤ |x - 1| ≤ 5 ____________
5. A store sells items for $12 each with a $5 shipping fee OR items for $14 each with free shipping. For how many items is the first option cheaper? ____________

## Chapter 2 Summary

**Strategies for Complex Equations**

**Fractions:** Multiply by LCD to clear fractions

**Decimals:** Multiply by power of 10 to clear decimals

**Grouping:** Distribute first, then combine like terms

**Compound Inequalities**

**Type Meaning Graph**

Both true

AND Overlap (bounded region)

OR At least one true Union (two regions)

**Absolute Value Equations**

|ax + b| = c (where c > 0)

Solve: ax + b = c AND ax + b = -c

Two solutions (usually)

|ax + b| = 0 → One solution (ax + b = 0)

|ax + b| = negative → No solution

**Key Reminders**

⚠️ Flip inequality sign when multiplying/dividing by negative
⚠️Absolute value is never negative
⚠️Always check solutions in original equation

*Mastering complex equations and inequalities is essential preparation for algebra and advanced mathematics.*

# CHAPTER 3: INTRODUCTION TO LINEAR EQUATIONS

## What You'll Learn

Understanding linear equations and their graphs

Finding and interpreting slope

Writing equations in slope-intercept form

Graphing linear equations

Understanding parallel and perpendicular lines

Writing equations from graphs and points

## Why This Matters

Linear relationships are everywhere—from phone bills to distance traveled to temperature conversions. Understanding how to graph and write linear equations helps you analyze trends, make predictions, and solve real-world problems. This chapter builds the foundation for all of algebra.

## LESSON 3.1: Understanding Linear Equations

### What Is a Linear Equation?

A **linear equation** in two variables creates a straight line when graphed.

**Standard forms:**

$y = mx + b$ (slope-intercept form)

$Ax + By = C$ (standard form)

**Identifying Linear Equations**

Linear equations:

Have variables only to the first power (no $x^2$, $x^3$, $\sqrt{x}$, etc.)

Have no products of variables (no xy)

| Linear | Not Linear |
| --- | --- |
| $y = 3x + 2$ | $y = x^2 + 1$ |
| $2x + 5y = 10$ | $xy = 6$ |
| $y = -4$ | $y = \sqrt{x}$ |
| $x = 7$ | $y = 1/x$ |

**Solutions to Linear Equations**

A **solution** is an ordered pair (x, y) that makes the equation true.

**Example 1:** Is (2, 5) a solution to $y = 2x + 1$?

Substitute: $5 = 2(2) + 1 = 4 + 1 = 5$ ✓

**Yes, (2, 5) is a solution.**

**Example 2:** Is (3, -1) a solution to $2x + 3y = 3$?

Substitute: $2(3) + 3(-1) = 6 - 3 = 3$ ✓

**Yes, (3, -1) is a solution.**

**Example 3:** Find three solutions to y = 2x - 1.

Choose x values and calculate y:

**x y = 2x - 1 Point**

| |
|---|
| 2(0) - 1 = -1 |
| 2(1) - 1 = 1 |

0 (0, -1)

1 (1, 1)

2 2(2) - 1 = 3 (2, 3)

✏️ **Try This**

1. Is (1, 4) a solution to y = 3x + 1? ____________
2. Is (-2, 5) a solution to 3x + y = -1? ____________
3. Find three solutions to y = -x + 4.____________

## LESSON 3.2: Understanding Slope

### What Is Slope?

**Slope** measures the steepness of a line—how much y changes for each unit change in x.

**Slope = Rise / Run = Change in y / Change in x**

### The Slope Formula

For two points $(x_1, y_1)$ and $(x_2, y_2)$:

**$m = (y_2 - y_1) / (x_2 - x_1)$**

**Example 4:** Find the slope of the line through (1, 2) and (4, 8).

m = (8 - 2) / (4 - 1) = 6/3 = **2**

The line rises 2 units for every 1 unit to the right.

**Example 5:** Find the slope through (-2, 5) and (3, -5).

m = (-5 - 5) / (3 - (-2)) = -10/5 = **-2**

The line falls 2 units for every 1 unit to the right.

**Example 6:** Find the slope through (1, 4) and (5, 4).

m = (4 - 4) / (5 - 1) = 0/4 = **0**

A horizontal line has slope 0.

**Example 7:** Find the slope through (3, 1) and (3, 6).

m = (6 - 1) / (3 - 3) = 5/0 = **undefined**

A vertical line has an undefined slope.

**Types of Slope**

**Slope Line Direction**

| |
|---|
| Goes up left to right |
| Goes down left to right |
| Horizontal |

Positive (m > 0) / Negative (m < 0) \ Zero (m = 0) —

Undefined Vertical

**Interpreting Slope**

**Example 8:** A car travels 240 miles in 4 hours. What does the slope represent? slope = 240 miles / 4 hours = 60 miles per hour

**The slope (60) represents the speed—60 miles per hour.**

✏️ **Try This**

Find the slope:

1. Through (2, 3) and (6, 11) ____________
2. Through (0, 5) and (4, -3) ____________
3. Through (-1, 2) and (4, 2) ____________
4. Through (5, -1) and (5, 7) ____________

## LESSON 3.3: Slope-Intercept Form

### The Form

**y = mx + b**

Where:

**m** = slope

**b** = y-intercept (where the line crosses the y-axis)

### Identifying m and b

**Example 9:** Identify the slope and y-intercept: y = 3x - 4

**Slope (m) = 3 Y-intercept (b) = -4** → Point (0, -4)

**Example 10:** Identify slope and y-intercept: y = -2x + 5

**Slope (m) = -2 Y-intercept (b) = 5** → Point (0, 5)

**Example 11:** Identify slope and y-intercept: y = (1/2)x

**Slope (m) = 1/2 Y-intercept (b) = 0** → Point (0, 0)

### Converting to Slope-Intercept Form

**Example 12:** Rewrite 2x + 3y = 12 in slope-intercept form.

Solve for y:

3y = -2x + 12

y = (-2/3)x + 4

**Slope = -2/3, y-intercept = 4**

**Example 13:** Rewrite 4x - y = 7 in slope-intercept form.

-y = -4x + 7
y = 4x - 7

**Slope = 4, y-intercept = -7**

✏️ **Try This**
Identify slope and y-intercept:

1. y = 5x + 2
2. y = -x - 3
3. y = (3/4)x

Convert to slope-intercept form:

4. 3x + y = 9
5. 2x - 4y = 8

## LESSON 3.4: Graphing Linear Equations

**Method 1: Using Slope and Y-Intercept**
**Steps:**

1. Plot the y-intercept (0, b)
2. Use the slope (rise/run) to find another point
3. Draw the line through both points

**Example 14:** Graph y = 2x - 1

**Step 1:** Y-intercept is -1. Plot (0, -1).

**Step 2:** Slope is 2 = 2/1 (rise 2, run 1). From (0, -1): go up 2, right 1 → (1, 1)

**Step 3:** Draw a line through (0, -1) and (1, 1).

**Example 15:** Graph y = -(2/3)x + 4

**Step 1:** Y-intercept is 4. Plot (0, 4).

**Step 2:** Slope is -2/3 (down 2, right 3). From (0, 4): go down 2, right 3 → (3, 2)

**Step 3:** Draw a line through (0, 4) and (3, 2).

**Method 2: Using a Table**

**Example 16:** Graph y = -x + 3 using a table.

**x y = -x + 3 Point**

| |
|---|
| -(-1) + 3 = 4 |
| -(0) + 3 = 3 |
| -(1) + 3 = 2 |

-1 (-1, 4)

0 (0, 3)

1 (1, 2)

2 -(2) + 3 = 1 (2, 1)

Plot these points and connect them with a line.

**Method 3: Using Intercepts**

**Example 17:** Graph 3x + 2y = 6 using intercepts.

**X-intercept** (set y = 0): 3x + 0 = 6 → x = 2 → Point (2, 0)

**Y-intercept** (set x = 0): 0 + 2y = 6 → y = 3 → Point (0, 3)

Plot (2, 0) and (0, 3), then connect.

✏️ **Try This**

Graph each equation:

1. $y = 3x + 1$ (use slope and y-intercept)
2. $y = -(1/2)x + 2$ (use slope and y-intercept)
3. $2x + y = 4$ (use intercepts)

## LESSON 3.5: Writing Equations of Lines

### Given Slope and Y-Intercept

Just substitute into $y = mx + b$.

**Example 18:** Write the equation of a line with slope 4 and

y-intercept -2. $\mathbf{y = 4x - 2}$

### Given Slope and a Point

Use point-slope form, then convert to slope-intercept.

**Point-slope form:** $y - y_1 = m(x - x_1)$

**Example 19:** Write the equation of a line with slope 3 passing through (2, 5). $y - 5 = 3(x - 2)$ $y - 5 = 3x - 6$ $y = 3x - 6 + 5$ $y = \mathbf{3x - 1}$

**Example 20:** Write the equation of a line with slope -2 passing through (-1, 4). $y - 4 = -2(x - (-1))$ $y - 4 = -2(x + 1)$ $y - 4 = -2x - 2$ $y = -2x - 2 + 4$ $y = \mathbf{-2x + 2}$

### Given Two Points

**Step 1:** Find the slope using $m = (y_2 - y_1)/(x_2 - x_1)$

**Step 2:** Use point-slope form with either point

**Example 21:** Write the equation through (1, 3) and (3, 9).

**Step 1:** $m = (9 - 3)/(3 - 1) = 6/2 = 3$

**Step 2:** Using point (1, 3): $y - 3 = 3(x - 1)$ $y - 3 = 3x - 3$ $\mathbf{y = 3x}$

**Example 22:** Write the equation through (-2, 5) and (4, -1).

**Step 1:** $m = (-1 - 5)/(4 - (-2)) = -6/6 = -1$

**Step 2:** Using point (4, -1): $y - (-1) = -1(x - 4)$ $y + 1 = -x + 4$ $\mathbf{y = -x + 3}$

✏️ **Try This**

Write the equation:

1. Slope 2, y-intercept 5 ____________
2. Slope -3, through (1, 4) ____________
3. Through (2, 1) and (4, 5)____________
4. Through (0, 3) and (6, 0) ____________

## LESSON 3.6: Parallel and Perpendicular Lines

**Parallel Lines**

**Parallel lines** never intersect and have the **same slope**.

If line 1 has slope m, a parallel line also has slope m.

**Example 23:** Line A has equation y = 2x + 3. Line B is parallel to A and passes through (1, 5). Find the equation of Line B.

Parallel means same slope: m = 2

Using point (1, 5): y - 5 = 2(x - 1) y - 5 = 2x - 2 **y = 2x + 3**

Wait—that's the same line! Let me reconsider.

y - 5 = 2(x - 1)
y - 5 = 2x - 2
y = 2x + 3

Actually, (1, 5) IS on line A: 2(1) + 3 = 5 ✓

**Let's redo with point (1, 7):** y - 7 = 2(x - 1) y - 7 = 2x - 2 **y = 2x + 5**

**Example 24:** Write an equation parallel to y = -3x + 1 through (2, 4).

Same slope: m = -3

y - 4 = -3(x - 2) y - 4 = -3x + 6 **y = -3x + 10**

### Perpendicular Lines

**Perpendicular lines** intersect at 90° angles. Their slopes are **negative reciprocals**.

If line 1 has slope m, a perpendicular line has slope **-1/m**.

| Original slope | Perpendicular slope |
|---|---|
| 2 | -1/2 |
| -3 | 1/3 |
| 1/4 | -4 |
| -2/5 | 5/2 |

**Example 25:** Line A has equation y = 4x - 2. Find the slope of a perpendicular line.

Original slope: 4

Perpendicular slope: **-1/4**

**Example 26:** Write an equation perpendicular to y = (2/3)x + 1 through (4, 5).

Original slope: 2/3
Perpendicular slope: -3/2

y - 5 = -(3/2)(x - 4) y - 5 = -(3/2)x + 6 **y = -(3/2)x + 11**

### Checking If Lines Are Parallel or Perpendicular

Example 27: Are y = 3x + 2 and y = 3x - 5 parallel, perpendicular, or neither?

Both have slope 3. Same slope → **Parallel**

**Example 28:** Are y = 2x + 1 and y = -(1/2)x + 3 parallel, perpendicular, or neither?

Slopes: 2 and -1/2
Product: 2 × (-1/2) = -1

Slopes multiply to -1 → **Perpendicular**

✏️ **Try This**

1. Write an equation parallel to y = 4x - 3 through (1, 2).
2. Write an equation perpendicular to y = 2x + 5 through (6, 3). 3.

Are y = -2x + 4 and y = (1/2)x - 1 parallel, perpendicular, or neither? ____________

## Chapter 3 Practice Problems

**Section A: Solutions and Linear Equations**

1. Is (3, 7) a solution to y = 2x + 1? ____________

2. Is (-1, 4) a solution to 3x + 2y = 5? ____________

3. Find three solutions to y = -2x + 5. ____________

**Section B: Finding Slope**

Find the slope:

4. Through (1, 4) and (5, 12) ____________

5. Through (-3, 2) and (1, -6)____________

6. Through (0, 5) and (4, 5) ____________

7. Through (2, -1) and (2, 8) ____________

**Section C: Slope-Intercept Form**

Identify slope and y-intercept:

8. y = 4x - 7 ____________

9. y = -(2/3)x + 1 ____________

10. y = 5x ____________

Convert to slope-intercept form:

11. 5x + y = 10 ____________

12. 2x - 3y = 9 ____________

13. x - 2y = 6 ____________

**Section D: Graphing**

Graph each equation:

14. $y = x + 2$ ____________

15. $y = -2x + 4$ ____________

16. $y = (1/3)x - 1$ ____________

17. $3x + y = 6$ (use intercepts) ____________

**Section E: Writing Equations**

Write the equation:

18. Slope 5, y-intercept -2 ____________

19. Slope -1, through (3, 2) ____________

20. Through (0, 4) and (2, 10) ____________

21. Through (-1, 5) and (3, -3) ____________

22. Horizontal line through (2, -4) ____________

23. Vertical line through (-3, 5) ____________

**Section F: Parallel and Perpendicular**

24. Write an equation parallel to $y = 3x - 2$ through (2, 1). ____________

25. Write an equation perpendicular to $y = 5x + 1$ through (0, 3). ____________

26. Are $y = 4x - 1$ and $8x - 2y = 6$ parallel, perpendicular, or neither? ____________

27. Are $y = 3x + 2$ and $y = -(1/3)x - 1$ parallel, perpendicular, or neither? ____________

**Section G: Applications**

28. A plumber charges $50 plus $40 per hour. Write an equation for the total cost C in terms of hours h. What is the slope, and what does it represent? ____________

29. A candle is 12 inches tall and burns 2 inches per hour. Write an equation for height H in terms of time t. When will the candle burn out? ____________

30. Two points on a linear graph are (2, 150) and (5, 240), representing distance in miles after hours of driving. Find the equation and interpret the slope. ____________

## �� Challenge Problems

1. Find the equation of a line perpendicular to y = (3/4)x - 2 that passes through the point where the line crosses the x-axis.____________

2. A line passes through (1, k) and (k, 5) with slope 2. Find k. ____________

3. Find the area of the triangle formed by the line 2x + 3y = 12 and the coordinate axes. ____________

4. Line A passes through (0, 3) and (4, 0). Line B passes through (0, -1) and (2, 5). Find where the lines intersect. ____________

5. Write the equation of a line that passes through (3, 4) and is equidistant from (1, 2) and (5, 6). ____________

## Chapter 3 Summary

### Key Formulas

**Slope formula:** $m = (y_2 - y_1)/(x_2 - x_1)$

**Slope-intercept form:** $y = mx + b$

**Point-slope form:** $y - y_1 = m(x - x_1)$

### Slope Relationships

| Relationship | Slopes |
|---|---|
| Parallel | Equal ($m_1 = m_2$) |
| Perpendicular | Negative reciprocals ($m_1 \times m_2 = -1$) |

### Graphing Methods

1. **Slope and y-intercept:** Plot b, then use slope
2. **Table of values:** Calculate several (x, y) pairs
3. **Intercepts:** Find where line crosses each axis

**Key Concepts**

Slope represents rate of change

Y-intercept represents starting value

Horizontal lines: $m = 0$

Vertical lines: $m$ = undefined

Linear equations are the foundation of algebra and essential for understanding more complex relationships.

# CHAPTER 4: SYSTEMS OF EQUATIONS What You'll Learn

Understanding systems of linear equations

Solving systems by graphing

Identifying types of solutions (one, none, infinitely many)

Interpreting solutions in context

Introduction to algebraic methods

## Why This Matters

Many real-world problems involve multiple conditions that must be satisfied simultaneously. When will two phone plans cost the same? Where do two roads intersect? How many of each item should a store stock? Systems of equations help solve problems with multiple unknowns and multiple constraints.

## LESSON 4.1: What Is a System of Equations?

**Definition**

A **system of equations** is a set of two or more equations with the same variables.

**Example:** $y = 2x + 1$ $y = -x + 7$

We want to find values of x and y that make BOTH equations true simultaneously.

**Solutions to Systems**

A **solution** to a system is an ordered pair (x, y) that satisfies ALL equations in the system.

**Example 1:** Is (2, 5) a solution to

the system? $y = 2x + 1$

$y = -x + 7$

Check equation 1: $5 = 2(2) + 1 = 5$ ✓

Check equation 2: $5 = -(2) + 7 = 5$ ✓

**Yes, (2, 5) is the solution.**

**Example 2:** Is (3, 4) a solution to

the system? $y = x + 1$

$y = 2x - 1$

Check equation 1: $4 = 3 + 1 = 4$ ✓

Check equation 2: $4 = 2(3) - 1 = 5$ ✗

**No, (3, 4) is NOT a solution** (fails the second equation).

**Example 3:** Is (1, 3) a solution to

the system? $2x + y = 5$

$x - y = -2$

Check equation 1: $2(1) + 3 = 5$ ✓

Check equation 2: $1 - 3 = -2$ ✓

**Yes, (1, 3) is the solution.**

✏️ **Try This**

Is the given point a solution to the system?

1. (4, 1): y = x - 3 and y = -x + 5
2. (2, 3): y = 2x - 1 and y = x + 2
3. (-1, 2): 3x + y = -1 and x - 2y = -5

## LESSON 4.2: Solving Systems by Graphing

**The Method**

1. Graph both equations on the same coordinate plane
2. Find the point of intersection
3. Check the solution in both equations

**Example 4:** Solve by graphing:

y = x + 1

y = -x + 5

**Graph** y **= x + 1:** Y-intercept: 1, Slope: 1 Points: (0, 1), (1, 2), (2, 3)

**Graph y = -x + 5:** Y-intercept: 5, Slope: -1 Points: (0, 5), (1, 4), (2, 3) The lines intersect at **(2, 3)**.

**Check: y = x + 1:** 3 = 2 + 1 ✓ y = -x + 5: 3 = -2 + 5 ✓

**Example 5:** Solve by graphing:

y = 2x - 1 ____________

y = (1/2)x + 2 ____________

**Graph y = 2x - 1:** Y-intercept: -1, Slope: 2 Points: (0, -1), (1, 1), (2, 3)

**Graph y = (1/2)x + 2:** Y-intercept: 2, Slope: 1/2 Points: (0, 2), (2, 3), (4, 4)

The lines intersect at **(2, 3)**.

**Check:** y = 2x - 1: 3 = 2(2) - 1 = 3 ✓ y = (1/2)x + 2: 3 = (1/2)(2) + 2 = 3 ✓

**Example 6:** Solve by graphing:

x + y = 4 ________________________________

2x - y = 2 ________________________________

First, convert to slope-intercept form:

Equation 1: y = -x + 4 (slope -1, y-int 4) ________________________________
Equation 2: y = 2x - 2 (slope 2, y-int -2) ________________________________

**Graph both lines.**

Intersection: **(2, 2)**

**Check:** x + y = 4: 2 + 2 = 4 ✓ 2x - y = 2: 2(2) - 2 = 2 ✓

✏️ **Try This**

Solve by graphing:

1. y = x + 2 and y = -x + 4 ________________________________
2. y = 3x - 1 and y = x + 3 ________________________________
3. x + y = 5 and x - y = 1 ________________________________

## LESSON 12.3: Types of Solutions

### Three Possibilities

**Graph Lines Solutions Slopes**

| | |
|---|---|
| Cross at one point | **One solution** |
| Never cross | **No solution** |

Intersecting Different slopes

Parallel: Same slope, different y-intercepts

Same line overlaps completely

**Infinite solutions** Same slope, same y-intercept

**One Solution (Most Common)**

The lines intersect at exactly one point.

**Example 7:**

y = 2x + 1 (slope 2)

y = -x + 4 (slope -1)

Different slopes → lines must intersect → **One solution**

**No Solution (Parallel Lines)**

The lines never intersect.

**Example 8:** Solve:

y = 3x + 2 ____________

y = 3x - 1 ____________

Both have slope 3, but different y-intercepts (2 and -1).

These are **parallel lines → No solution**

The system is called **inconsistent**.

**Example 9:** Determine if there's a solution:

2x + y = 5____________

4x + 2y = 7 ____________

Rewrite in slope-intercept form:

y = -2x + 5 ____________

y = -2x + 3.5 ____________

Same slope (-2), different y-intercepts → **Parallel, no solution**

**Infinite Solutions (Same Line)**

The equations describe the same line.

**Example 10:** Solve:

$y = 2x + 3$ ____________

$4x - 2y = -6$ ____________

Rewrite the second equation:

$-2y = -4x - 6$ ____________

$y = 2x + 3$ ____________

**Both equations are the same line!**

Every point on the line is a solution → **Infinitely many solutions** The system is called **dependent**.

**Example 11:** Determine the number of solutions:

$3x + 6y = 12$ ____________

$x + 2y = 4$ ____________

Rewrite both:

$y = -(1/2)x + 2$ ____________

$y = -(1/2)x + 2$ ____________

Same equation → **Infinitely many solutions**

Notice: The first equation is 3 times the second equation.

**Identifying Without Graphing**

Compare slopes and y-intercepts:

**Example 12:** How many solutions?

$y = 4x - 5$ (slope 4, y-int -5) ____________

$y = 4x + 2$ (slope 4, y-int 2) ____________

Same slope, different y-intercepts → Parallel → **No solution**

**Example 13:** How many solutions?

y = -2x + 7 (slope -2) ____________

y = 3x - 1 (slope 3) ____________

Different slopes → Lines intersect → **One solution**

✏️ **Try This**

How many solutions? (Don't solve—just determine the number.)

1. y = 5x + 3 and y = 5x - 2 ____________
2. y = -x + 4 and y = 2x + 1 ____________
3. 2x + y = 8 and 4x + 2y = 16 ____________
4. y = 3x + 1 and y = 3x + 1 ____________

## LESSON 4.4: Applications of Systems

**Setting Up Systems from Word Problems**

Example 14: Movie tickets cost $12 for adults and $8 for children. A group buys 9 tickets totaling $84. How many of each type? ____________

**Define variables:** Let a = number of adult tickets Let c = number of child tickets

**Write equations:** Total tickets: a + c = 9 Total cost: 12a + 8c = 84

**Solve by graphing** (or substitution—we'll preview this):

From equation 1: c = 9 - a
Substitute: 12a + 8(9 - a) = 84
12a + 72 - 8a = 84
4a = 12
a = 3

Then c = 9 - 3 = 6

**3 adult tickets and 6 child tickets**

Check: 3 + 6 = 9 ✓ and 12(3) + 8(6) = 36 + 48 = 84 ✓

**Example 15:** Two phone plans:

Plan A: $30/month + $0.10 per text

Plan B: $40/month + $0.05 per text

When do they cost the same? ____________

Let t = number of texts
Let C = total cost

Plan A: $C = 30 + 0.10t$
Plan B: $C = 40 + 0.05t$

Set equal (find intersection):
$30 + 0.10t = 40 + 0.05t$
$0.05t = 10$
$t = 200$

**At 200 texts, both plans cost $50.**

For fewer than 200 texts, Plan A is cheaper.
For more than 200 texts, Plan B is cheaper.

**Example 16:** A rectangle's perimeter is 36 cm. Its length is 3 cm more than its width. Find the dimensions. Let l = length, w = width

Perimeter: $2l + 2w = 36$
Relationship: $l = w + 3$

Substitute:
$2(w + 3) + 2w = 36$
$2w + 6 + 2w = 36$
$4w = 30$
$w = 7.5$

Then $l = 7.5 + 3 = 10.5$

**Length = 10.5 cm, Width = 7.5 cm**

✏️ **Try This**

1. Concert tickets: \$15 for students, \$25 for adults. 200 tickets were sold for \$3,500. How many of each? ____________

2. A number is 5 more than twice another number. Their sum is 26. Find both numbers. ____________

3. Gym A: \$100 joining fee + \$30/month. Gym B: \$50 joining fee + \$40/month. When are costs equal? ____________

## LESSON 4.5: Preview of Algebraic Methods

**Why Use Algebraic Methods?**

Graphing has limitations:

- Hard to read exact solutions that aren't integers
- Time-consuming for complex equations
- Not precise enough for many applications

Algebraic methods (substitution and elimination) give exact answers.

**Substitution Method (Preview)**

If one variable is isolated, substitute it into the other equation.

**Example 17:** Solve:

$y = 3x - 5$

$2x + y = 15$

**Step 1:** y is already isolated in equation 1.

**Step 2:** Substitute into equation 2: $2x + (3x - 5) = 15$ $5x - 5 = 15$ $5x = 20$ $x = 4$

**Step 3:** Find y: $y = 3(4) - 5 = 7$

**Solution: (4, 7)**

**Example 18:** Solve:

$x = 2y + 1$ _____________

$3x - 4y = 7$ _____________

Substitute $x = 2y + 1$ into equation 2:
$3(2y + 1) - 4y = 7$ _____________
$6y + 3 - 4y = 7$ _____________
$2y = 4$
$y = 2$

Find x: $x = 2(2) + 1 = 5$ _____________

**Solution: (5, 2)**

**Elimination Method (Preview)**

Add or subtract equations to eliminate one variable

**Example 19:** Solve:

$x + y = 10$ _____________

$x - y = 4$ _____________

**Add the equations:** $(x + y) + (x - y) = 10 + 4$ $2x = 14$ $x = 7$

**Substitute to find y:** $7 + y = 10$ $y = 3$

**Solution: (7, 3)**

**Example 20:** Solve:

$2x + 3y = 12$

$2x + y = 8$

**Subtract to eliminate x:** $(2x + 3y) - (2x + y) = 12 - 8$ $2y = 4$ $y = 2$

**Substitute:** $2x + 2 = 8$, $2x = 6$ $x = 3$

**Solution: (3, 2)**

✏️ **Try This**

Solve using substitution:

1. y = x + 3 and 2x + y = 12 ____________
2. x = 4y and x + y = 15 ____________

Solve using elimination:

3. x + y = 9 and x - y = 3 ____________
4. 3x + 2y = 19 and 3x - 2y = 5 ____________

## Chapter 4 Practice Problems

### Section A: Checking Solutions

Is the point a solution?

1. (3, 5): y = 2x - 1 and y = -x + 8 ____________
2. (4, 2): y = x - 2 and y = 3x - 10____________
3. (-2, 1): x + y = -1 and 2x - y = -5 ____________

### Section B: Solving by Graphing

Solve by graphing:

4. y = x + 3 and y = -x + 1 ____________
5. y = 2x and y = -x + 6 ____________
6. y = (1/2)x + 1 and y = -x + 4 ____________
7. x + y = 6 and x - y = 2 ____________

### Section C: Types of Solutions

Determine: one solution, no solution, or infinitely many?

8. y = 4x + 1 and y = 4x - 3 ____________
9. y = -2x + 5 and y = 3x - 5 ____________
10. 2x + 4y = 8 and x + 2y = 4 ____________
11. y = 5x - 2 and 10x - 2y = 4 ____________

12. 3x - y = 6 and y = 3x + 2 ____________

**Section D: Applications**

13. Admission: $10 adults, $6 children. Total 120 people, $960 collected. How many adults?

14. A number is 12 more than another. Their sum is 48. Find both.

15. Two cars leave a city. Car A goes 60 mph, and Car B goes 50 mph (starts 1 hour later, same direction). When does B catch A?

16. A 30-foot rope is cut into two pieces. One piece is 6 feet longer than the other. Find each length. ____________

17. Plan A: $25 + $0.15/mile. Plan B: $40 + $0.10/mile. When are they equal? ____________

**Section E: Algebraic Methods (Preview)**

Solve using substitution:

18. y = 2x + 4 and x + y = 10 ____________

19. x = y - 5 and 2x + 3y = 15 ____________

20. y = -x + 7 and 3x + 2y = 11 ____________

Solve using elimination:

21. x + y = 14 and x - y = 6 ____________

22. 2x + y = 11 and x + y = 7____________

23. 4x + 3y = 25 and 2x + 3y = 17 ____________

## �� Challenge Problems

1. Find values of k so that the system has no solution:

   y = 3x + 5 ____________

   y = kx + 2 ____________

2. A grocer mixes nuts costing $4/lb with nuts costing $7/lb to make 12 pounds of mixture worth $5.50/lb. How many pounds of each?____________

3. A boat travels 30 km downstream in 2 hours and 30 km upstream in 3 hours. Find the boat's speed in still water and the current's speed. ____________

4. Three consecutive integers sum to 72. Write this as a system of equations and solve. ____________

5. Find the equation of a line that passes through the intersection of y = 2x - 1 and y = -x + 5, and is perpendicular to y = 3x + 4. ____________

## Chapter 4 Summary

**Key Concepts**

**System of equations:** Two or more equations with the same variables

**Solution:** An ordered pair that satisfies ALL equations

**Types of Systems**

**Type Lines Solutions: How to Identify**

| | |
|---|---|
| Intersect | One |
| Parallel | None |

Independent Different slopes

Inconsistent: Same slope, different y-intercepts

Dependent: Same line Infinite Same slope AND y-intercept

**Solving Methods**

1. Graphing: Plot both lines, find intersection
2. Substitution: Solve one equation for a variable, substitute
3. Elimination: Add/subtract equations to eliminate a variable

**Problem-Solving Steps**

1. Define variables
2. Write equations from given information
3. Solve the system
4. Check the solution
5. Answer in context

*Systems of equations are essential tools for solving real-world problems with multiple constraints.*

# CHAPTER 5: REVIEW AND APPLICATIONS

## What You'll Learn

- Connecting concepts across all chapters
- Solving multi-step problems that combine skills
- Applying mathematical concepts to real-world situations
- Building problem-solving strategies

## Why This Matters

Mathematics isn't learned in isolation—real problems require combining multiple skills. This chapter brings together everything you've learned this year, showing how different concepts connect and how to tackle complex problems systematically.

## LESSON 5.1: Number Sense and Operations Review

### Rational Number Operations

**Example 1:** Simplify: -3/4 + 2/3 × (-6/5)

Follow the order of operations (multiply first):
= -3/4 + (2/3 × -6/5)
= -3/4 + (-12/15)
= -3/4 + (-4/5)

Find common denominator (20): = -15/20 + (-16/20) = **-31/20** or **-1 11/20**

**Example 2:** A submarine is at -250 feet. It rises 75 feet, then dives 120 feet. What is its final depth?

-250 + 75 + (-120) = -250 + 75 - 120 = -175 - 120 = **-295 feet**

### Proportional Relationships

**Example 3:** A car travels 195 miles on 6.5 gallons of gas. At this rate, how far can it travel on 10 gallons?

Find unit rate: 195 ÷ 6.5 = 30 miles per gallon

Distance on 10 gallons: 30 × 10 = **300 miles**

**Percent Applications**

**Example 4:** A \$450 television is on sale for 25% off. Sales tax is 8%. What is the final price?

Discount: \$450 × 0.25 = \$112.50 Sale price: \$450 - \$112.50 = \$337.50 Tax: \$337.50 × 0.08 = \$27 Final price: \$337.50 + \$27 = **\$364.50**

Example 5: You invest \$2,000 at 4.5% simple interest. How much will you have after 3 years?

I = Prt = \$2,000 × 0.045 × 3 = \$270 Total = \$2,000 + \$270 = \$2,270

✏️ **Practice Problems**

1. Simplify: -5/6 - 1/3 + (-1/2) ____________
2. A recipe for 8 servings uses 2.5 cups of flour. How much flour for 12 servings? ____________
3. A price increased from \$80 to \$92. What was the percent increase? ____________
4. Find the simple interest on \$3,500 at 6% for 2.5 years. ____________

## LESSON 5.2: Expressions, Equations, and Inequalities Review

**Simplifying Expressions**

**Example 6:** Simplify: 4(2x - 3) - 3(x + 5)

= 8x - 12 - 3x - 15 = 8x - 3x - 12 - 15 = **5x - 27**

**Solving Multi-Step Equations**

**Example 7:** Solve: 5(x - 2) = 3(x + 4) - 2

5x - 10 = 3x + 12 - 2 5x - 10 = 3x + 10 2x = 20 x = **10**

**Example 8:** Solve: (2x + 1)/3 = (x - 1)/2

Cross multiply: 2(2x + 1) = 3(x - 1) 4x + 2 = 3x - 3 x = **-5**

**Solving Inequalities**

**Example 9:** Solve and graph: $-3x + 7 < 16$
$-3x < 9$

$x > -3$ (flip sign when dividing by negative)

Graph: Open circle at -3, arrow pointing right.

**Absolute Value**

**Example 10:** Solve: $|2x - 5| = 9$

Case 1: $2x - 5 = 9 \rightarrow 2x = 14 \rightarrow x = 7$

Case 2: $2x - 5 = -9 \rightarrow 2x = -4 \rightarrow x = -2$

**Solutions: $x = 7$, $x = -2$**

**Word Problems with Equations**

**Example 11:** The sum of three consecutive even integers is 150. Find them.

Let n = first integer

$n + (n + 2) + (n + 4) = 150$

$3n + 6 = 150$

$3n = 144$

$n = 48$

**The integers are 48, 50, and 52.**

✏️ **Practice Problems**

5. Simplify: $2(3a + 4) - 5(a - 2)$ ___________________________

6. Solve: $4x - 3(x + 2) = 2(x - 5)$ ___________________________

7. Solve: $-2(x + 1) \geq 4x - 8$ ___________________________

8. Solve: $|x + 3| = 7$ ___________________________

9. The perimeter of a rectangle is 84 cm. The length is 6 cm more than the width. Find the dimensions. ___________________________

# LESSON 5.3: Geometry Review

**Angle Relationships**

**Example 12:** Two parallel lines are cut by a transversal. One angle measures 65°. Find all eight angles. If one angle is 65°:

Corresponding angles: 65°

Alternate interior angles: 65°

Same-side interior angles: 180° - 65° = 115°

**Angles: 65°, 65°, 65°, 65°, 115°, 115°, 115°, 115°**

**Triangle Properties**

**Example 13:** In a triangle, one angle is 20° more than the second, and the third is twice the second. Find all angles.

Let $x$ = second angle
Then $x + 20$ = first angle
And $2x$ = third angle

$x + (x + 20) + 2x = 180$
$4x + 20 = 180$
$4x = 160$
$x = 40$

**Angles: 60°, 40°, and 80°**

**Circle Measurements**

**Example 14:** A circular pool has a diameter of 24 feet. Find the circumference and area.

Circumference: $C = \pi d = 3.14 \times 24 =$ **75.36 feet** Area: $A = \pi r^2 = 3.14 \times 12^2 = 3.14 \times 144 =$ **452.16 square feet**

**Cylinder Volume and Surface Area**

Example 15: A cylindrical can has a radius of 4 cm and a height of 10 cm. Find the volume and total surface area.

Volume: $V = \pi r^2 h = 3.14 \times 16 \times 10 = 502.4\ cm^3$

**Example 15:** A cylindrical can has a radius of 4 cm and a height of 10 cm. Find the volume and total surface area. Volume: $V = \pi r^2 h = 3.14 \times 16 \times 10 =$ **$502.4\ cm^3$**

Surface Area: $SA = 2\pi r^2 + 2\pi rh = 2(3.14)(16) + 2(3.14)(4)(10) = 100.48 + 251.2 =$ **$351.68\ cm^2$**

**Similar Figures**

**Example 16:** Two similar triangles have perimeters of 15 cm and 25 cm. If the area of the smaller is 18 $cm^2$, find the area of the larger.

Scale factor (linear): 25/15 = 5/3

Area scale factor: $(5/3)^2 = 25/9$

Larger area: $18 \times (25/9) = 450/9 =$ **$50\ cm^2$**

✏️ **Practice Problems**

10. An exterior angle of a triangle is 120°. One remote interior angle is 50°. Find the other remote interior angle and the third angle of the triangle. ____________

11. Find the area and circumference of a circle with radius 7 cm (use $\pi \approx 22/7$). ____________

12. A cylinder has a diameter of 10 m and a height of 8 m. Find the volume. ____________

13. Can sides 5, 9, and 15 form a triangle? Explain. ____________

14. Two similar rectangles have widths of 6 and 9. If the smaller has area 48, find the area of the larger. ____________

# LESSON 5.4: Statistics and Probability Review

**Statistical Analysis**

**Example 17:** Compare two data sets:

Class A scores: 72, 78, 80, 85, 85

Class B scores: 65, 75, 80, 85, 95

**Class A:** Mean: (72 + 78 + 80 + 85 + 85) ÷ 5 = 400 ÷ 5 = 80 Median: 80 (middle value)

Class B: Mean: (65 + 75 + 80 + 85 + 95) ÷ 5 = 400 ÷ 5 = 80

**Class A:** Mean: (72 + 78 + 80 + 85 + 85) ÷ 5 = 400 ÷ 5 = 80 Median: 80 (middle value)

**Class B:** Mean: (65 + 75 + 80 + 85 + 95) ÷ 5 = 400 ÷ 5 = 80 Median: 80

Both have the same mean and median, but Class B has more spread (range: 30 vs. 13).

**Making Inferences**

**Example 18:** In a sample of 250 students, 175 prefer pizza for lunch. The school has 1,200 students. Estimate how many prefer pizza.

Sample proportion: 175/250 = 0.70

Estimate: 0.70 × 1,200 = **840 students**

**Probability**

**Example 19:** A bag contains 5 red, 3 blue, and 2 green marbles. You draw one, replace it, and draw again. Find P(red, then blue).

P(red) = 5/10 = 1/2
P(blue) = 3/10

P(red, then blue) = (1/2) × (3/10) = **3/20 = 0.15 = 15%**

**Example 20:** Two dice are rolled. Find P(sum = 8).

Outcomes with a sum of 8: (2,6), (3,5), (4,4), (5,3), (6,2) = 5 outcomes
Total outcomes: 36

P(sum = 8) = 5/36 ≈ **0.139 or about 13.9%**

✏️ **Practice Problems**

15. Find the mean and MAD of: 10, 14, 16, 18, 22 ____________
16. In a sample of 80 light bulbs, 4 are defective. Predict defects in 2,000 bulbs. ____________
17. A coin is flipped and a die is rolled. Find P(heads and number > 4). ____________
18. From a deck, find P(heart or face card). ____________
16. In a sample of 80 light bulbs, 4 are defective. Predict defects in 2,000 bulbs. ____________
17. A coin is flipped and a die is rolled. Find P(heads and number > ____________ 4).
18. From a deck, find P(heart or face card). ____________

## LESSON 5.5: Linear Relationships Review

### Slope and Equations

**Example 21:** Find the equation of the line through (-2, 5)

and (4, -1). Slope: m = (-1 - 5)/(4 - (-2)) = -6/6 = -1

Using point (4, -1):
y - (-1) = -1(x - 4)
y + 1 = -x + 4
y = -x + 3

**Equation: y = -x + 3**

### Graphing

**Example 22:** Graph the line 3x - 2y = 6.

Find intercepts:
X-intercept (y = 0): 3x = 6, x = 2 → Point (2, 0)
Y-intercept (x = 0): -2y = 6, y = -3 → Point (0, -3)

Plot (2, 0) and (0, -3), and connect with a line.

**Parallel and Perpendicular**

**Example 23:** Write an equation perpendicular to y = 3x - 2 through (6, 4).

Original slope: 3
Perpendicular slope: -1/3

y - 4 = -1/3(x - 6)
y - 4 = -1/3x + 2
y = -1/3x + 6

**Equation: y = -1/3x + 6**
**Systems of Equations**

**Example 24:** A school sells adult and student tickets. Adult tickets cost $8, and student tickets cost $5. They sell 400 tickets for $2,600. How many of each?

Let a = adult, s = student

a + s = 400
8a + 5s = 2,600

From equation 1: s = 400 - a
Substitute: 8a + 5(400 - a) = 2,600
8a + 2,000 - 5a = 2,600
3a = 600
a = 200

s = 400 - 200 = 200

**200 adult tickets and 200 student tickets**

✏️ **Practice Problems**

19. Find the slope and y-intercept of: 4x + 2y = 10 __________________________

20. Write the equation of a line parallel to y = -2x + 5 through (3, 1). __________________________

21. Solve the system: y = 2x - 1 and y = -x + 8 __________________________

22. A rental car costs $35/day plus $0.20/mile. You have $100 for one day. How many miles can you drive? __________________________

## LESSON 5.6: Multi-Step Problem Solving

**Strategy for Complex Problems**

1. **Understand:** What are you trying to find?
2. **Plan:** What concepts and formulas apply?
3. **Solve:** Show all steps clearly
4. **Check:** Does your answer make sense?

**Example 25:** A cylindrical water tank has a radius of 3 meters and a height of 10 meters. Water is pumped in at 2 cubic meters per minute. How long to fill the tank?

**Step 1:** Find the volume of the tank. $V = \pi r^2 h = 3.14 \times 9 \times 10 = 282.6\ m^3$

**Step 2:** Find time to fill. Time = Volume ÷ Rate = 282.6 ÷ 2 = **141.3 minutes** (about 2 hours 21 minutes)

**Example 26:** A store marks up items 40% and then offers a 25% discount. What is the overall percent change from cost?

Let cost = $100 (choose an easy number)

After 40% markup: $100 × 1.40 = $140
After 25% discount: $140 × 0.75 = $105

Overall change: ($105 - $100)/$100 = 5% increase

**The overall change is a 5% increase.**

**Example 27:** Two trains leave the same station at the same time, traveling in opposite directions. One travels at 60 mph, the other at 75 mph. When will they be 405 miles apart?

Combined rate: 60 + 75 = 135 mph (moving apart) Time = Distance ÷ Rate = 405 ÷ 135 = **3 hours**

**Example 28:** A rectangular garden is surrounded by a path 2 meters wide. The garden is 12 m by 8 m. Find the area of the path.

Outer rectangle: (12 + 4) × (8 + 4) = 16 × 12 = 192 $m^2$ Inner rectangle (garden): 12 × 8 = 96 $m^2$ Path area: 192 - 96 = **96 $m^2$**

✏️ **Practice Problems**

23. A cone has radius 6 cm and height 10 cm. Find the volume. ($V = 1/3\ \pi r^2 h$) ____________

24. A price is increased by 20% and then decreased by 20%. Is the final price the same as the original? Calculate. ____________

25. Two cars start 300 miles apart and drive toward each other at 55 mph and 45 mph. When do they meet? ____________

26. A circular pond has a diameter of 20 m. A path 3 m wide surrounds it. Find the area of the path. ____________

## COMPREHENSIVE REVIEW PROBLEMS

### Mixed Practice

Solve or simplify:

1. -2/3 × (9/4 - 3/2) ____________
2. What is 15% of 240? ____________
3. 35 is what percent of 140? ____________
4. Simplify: 3(2x - 4) - 2(x + 5)____________
5. Solve: 4x - 7 = 2x + 9 ____________
6. Solve: |3x - 6| = 12 ____________
7. Find the slope through (2, -3) and (-4, 9) ____________
8. Write the equation through (3, 5) with slope -2 ____________
9. A cylinder has r = 5 cm and h = 12 cm. Find V and SA. ____________
10. P(sum = 6) when rolling two dice ____________

### Word Problems

11. A population grew from 25,000 to 28,500. Find the percent increase. ____________
12. You invest $4,000 at 5.5% simple interest. Balance after 4 years? ____________

13. A map scale is 1:250,000. Two cities are 8 cm apart on the map. Actual distance in km? ____________

14. Similar triangles have corresponding sides 5 and 8. The smaller has an area of 30. Find the larger area. ____________

15. In a sample of 150, 45 prefer option A. Predict preference in a population of 2,000. ____________

16. Adult tickets: $12, Child: $7. Sold 350 tickets for $3,200. How many adult tickets? ____________

17. A candle is 15 inches tall and burns at 2 inches per hour. Write an equation for height h after t hours. When will it burn out? ____________

18. Find three consecutive odd integers whose sum is 117. ____________

## �� Challenge Problems

1. A rectangular prism has dimensions that are consecutive integers. The surface area is 148 $cm^2$. Find the dimensions. ____________

2. Two numbers have a sum of 36 and a product of 320. Find the numbers. ____________

3. A boat travels 24 km upstream in 3 hours and returns in 2 hours. Find the boat's speed in still water and the current speed. ____________

4. A circle is inscribed in a square with side 10 cm. Find the area between the square and circle. ____________

5. Three people can paint a house in 6 days. How long would it take 5 people (working at the same rate)? ____________

## Chapter 5 Summary

**Key Formulas Reference**

**Percent Change:** (New - Original)/Original × 100

**Simple Interest:** $I = Prt$

**Circle:** $C = 2\pi r$, $A = \pi r^2$

**Cylinder:** $V = \pi r^2 h$, $SA = 2\pi r^2 + 2\pi rh$

**Slope:** $m = (y_2 - y_1)/(x_2 - x_1)$

**Slope-Intercept:** $y = mx + b$

**Scale:** Drawing/Actual = Scale Factor

**Area scaling:** $k^2$ (for scale factor k)

**Volume scaling:** $k^3$

**Problem-Solving Strategies**

1. Read carefully—identify what's given and what's asked
2. Draw a diagram if helpful
3. Define variables clearly
4. Write equations that model the situation
5. Solve step-by-step
6. Check—does your answer make sense?
7. State your answer in context

*This chapter demonstrates how mathematical concepts work together to solve complex real-world problems.*

# CHAPTER 6: PREPARING FOR ALGEBRA

## What You'll Learn

Understanding functions and function notation

Exploring exponent rules

Working with square roots and cube roots

Introduction to the Pythagorean theorem

Preview of Algebra 1 concepts

## Why This Matters

This chapter bridges your 7th grade mathematics to Algebra 1. The concepts here—functions, exponents, roots, and the Pythagorean theorem—are foundational tools you'll use throughout high school math and beyond. Mastering these now will give you a strong start in algebra.

## LESSON 6.1: Introduction to Functions

### What Is a Function?

A **function** is a rule that assigns exactly one output to each input.

Think of a function like a machine:

Input goes in

The machine applies a rule

Exactly one output comes out

### Function Notation

**f(x)** is read as "f of x" and represents the output when x is the input.

If $f(x) = 2x + 3$:

$f(1)$ means "plug in 1 for x" $= 2(1) + 3 = 5$

$f(5)$ means "plug in 5 for x" $= 2(5) + 3 = 13$

**Example 1:** If $f(x) = 3x - 4$, find:

a) $f(2) = 3(2) - 4 = 6 - 4 =$ **2**

b) $f(-1) = 3(-1) - 4 = -3 - 4 =$ **-7**

c) $f(0) = 3(0) - 4 = 0 - 4 =$ **-4**

**Example 2:** If $g(x) = x^2 + 1$, find:

a) $g(3) = 3^2 + 1 = 9 + 1 =$ **10**

b) $g(-2) = (-2)^2 + 1 = 4 + 1 =$ **5**

c) $g(0) = 0^2 + 1 =$ **1**

**Identifying Functions**

A relationship is a function if each input has exactly ONE output.

**Example 3:** Is this a function?

| **x** | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| y | 5 | 7 | 9 | 11 |

Each x has exactly one y. **Yes, this is a function.**

**Example 4:** Is this a function?

| **x** | **1** | **2** | **1** | **3** |
|---|---|---|---|---|
| y | 4 | 5 | 6 | 7 |

The input $x = 1$ has two different outputs (4 and 6). **No, not a function.**

**Vertical Line Test**

On a graph, a relation is a function if no vertical line intersects the graph more than once.

**Example 5:** Is $y = x^2$ a function?
Draw any vertical line—it hits the parabola at most once. **Yes, it's a function.**

**Example 6:** Is $x^2 + y^2 = 25$ (a circle) a function?

A vertical line can hit the circle twice. **No, not a function.**

**Domain and Range**

**Domain:** All possible input values (x-values)

**Range:** All possible output values (y-values)

**Example 7:** Find the domain and range of: {(1, 4), (2, 5), (3, 6), (4, 7)}

Domain: {1, 2, 3, 4}
Range: {4, 5, 6, 7}

✏️ **Try This**

1. If $f(x) = 4x - 1$, find f(3) and f(-2). ____________
2. If $g(x) = x^2 - 2x$, find g(4) and g(-1). ____________
3. Is this a function? {(2, 3), (4, 5), (2, 7), (6, 9)} ____________
4. Find domain and range: {(-1, 2), (0, 4), (1, 6), (2, 8)} ____________

## LESSON 6.2: Linear vs. Nonlinear Functions

**Linear Functions**

A **linear function** has:

Constant rate of change (slope)
Graph is a straight line

Equation form: $f(x) = mx + b$

**Example 8:** Is $f(x) = 3x + 2$ linear?

Yes—it's in the form $f(x) = mx + b$ with $m = 3$, $b = 2$.

**Linear**

**Nonlinear Functions**

A **nonlinear function** does NOT have a constant rate of change.

**Type Example Shape**

| |
|---|
| $f(x) = x^2$ |
| $f(x) = 2^x$ |

Quadratic Parabola Exponential Curve

Absolute value f(x) = x

**Example 9:** Is this table linear or nonlinear?

**x 0 1 2 3**

y 1 3 9 27

Check rate of change:

3 - 1 = 2

9 - 3 = 6

27 - 9 = 18

Rate of change is NOT constant.

**Nonlinear**

(This is $y = 3^x$, an exponential function.)

**Example 10:** Is this linear or nonlinear? ____________

**x 0 2 4 6**

y 5 11 17 23

Rate of change:

11 - 5 = 6

17 - 11 = 6

23 - 17 = 6

Constant rate of change = 6 ÷ 2 = 3. **Linear** (f(x) = 3x + 5)

✏️ **Try This**

Linear or nonlinear?

1. $f(x) = x^2 + 3$ ____________
2. f(x) = -2x + 7 ____________

## LESSON 14.3: Exponent Rules

**Review of Exponents**

**a^n** means multiply a by itself n times.

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

**Product Rule**

**a^m × a^n = a^(m+n)**

When multiplying with the same base, ADD exponents.

**Example 11:** Simplify: $x^3 \times x^5$

$x^3 \times x^5$ = x^(3+5) = $\mathbf{x^8}$

**Example 12:** Simplify: $2^4 \times 2^3$

$2^4 \times 2^3$ = 2^(4+3) = $2^7$ = **128**

**Quotient Rule**

**a^m ÷ a^n = a^(m-n)**

When dividing with the same base, SUBTRACT exponents.

**Example 13:** Simplify: $y^8 \div y^3$

$y^8 \div y^3 = y^{(8-3)} = \mathbf{y^5}$

**Example 14:** Simplify: $5^6/5^2$

$5^6/5^2 = 5^{(6-2)} = 5^4 =$ **625**

**Power Rule**

**(a^m)^n = a^(m×n)**

When raising a power to a power, MULTIPLY

exponents. **Example 15:** Simplify: $(x^2)^4$

$(x^2)^4 = x^{(2\times4)} = \mathbf{x^8}$

**Example 16:** Simplify: $(3^2)^3$

$(3^2)^3 = 3^{(2\times3)} = 3^6 =$ **729**

**Zero Exponent**

$\mathbf{a^0 = 1}$ (for any $a \neq 0$)

**Example 17:** Simplify: $7^0 =$ **1**

**Example 18:** Simplify: $(-5)^0 =$ **1**

**Negative Exponents**

**a^(-n) = 1/a^n**

A negative exponent means "take the reciprocal."

**Example 19:** Simplify: 2^(-3)

2^(-3) = $1/2^3$ = **1/8**

**Example 20:** Simplify: x^(-4)

x^(-4) = $\mathbf{1/x^4}$

**Example 21:** Simplify: (1/3)^(-2)

(1/3)^(-2) = $(3/1)^2 = 3^2 =$ **9**

✏️ **Try This**

Simplify:

1. $x^4 \times x^6$ ____________
2. $y^9 \div y^4$ ____________
3. $(a^3)^2$ ____________
4. $5^0$ ____________
5. 3^(-2) ____________
6. $(2^3 \times 2^2)/2^4$ ____________

## LESSON 6.4: Square Roots and Cube Roots

### Square Roots

The **square root** of a number n is a value that, when multiplied by itself, equals n.

√n means "what number times itself equals n?"

**Example 22:** Find √49

√49 = 7 because 7 × 7 = 49

**Example 23:** Find √121

√121 = 11 because 11 × 11 = 121

### Perfect Squares

Memorize these:

| **n** | **1** | **4** | **9** | **16** | **25** | **36** | **49** | **64** | **81** | **100** | **121** | **144** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| √n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

### Estimating Square Roots

**Example 24:** Estimate √50

√49 = 7 and √64 = 8
50 is between 49 and 64, closer to 49.

√50 ≈ **7.1** (actual: 7.071...)

**Example 25:** Between which two integers is $\sqrt{75}$?

$\sqrt{64} = 8$ and $\sqrt{81} = 9$
$64 < 75 < 81$

$\sqrt{75}$ is between **8 and 9** (closer to 9, actually $\approx 8.66$)

**Cube Roots**

The **cube root** of n is a value that, when cubed, equals n.

$\sqrt[3]{n}$ means "what number cubed equals n?"

**Example 26:** Find $\sqrt[3]{27}$

$\sqrt[3]{27} = 3$ because $3^3 = 27$

**Example 27:** Find $\sqrt[3]{-8}$

$\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$

Note: Cube roots can be negative!

**Perfect Cubes**

| n | 1 | 8 | 27 | 64 | 125 | 216 |
|---|---|---|---|---|---|---|
| $\sqrt[3]{n}$ | 1 | 2 | 3 | 4 | 5 | 6 |

✏️ **Try This**

1. $\sqrt{81}$ ____________
2. $\sqrt{144}$ ____________
3. Estimate $\sqrt{40}$ to one decimal place ____________
4. Between which integers is $\sqrt{90}$? ____________
5. $\sqrt[3]{64}$ ____________
6. $\sqrt[3]{-125}$ ____________

# LESSON 6.5: The Pythagorean Theorem

### The Theorem

In a **right triangle**, the square of the hypotenuse equals the sum of the squares of the other two sides.
$\mathbf{a^2 + b^2 = c^2}$

Where:

- a and b are the legs (shorter sides)
- c is the hypotenuse (longest side, opposite the right angle)

### Finding the Hypotenuse

**Example 28:** A right triangle has legs 3 and 4. Find the hypotenuse.

$a^2 + b^2 = c^2$ $3^2 + 4^2 = c^2$ $9 + 16 = c^2$ $25 = c^2$ $c = \sqrt{25} = \mathbf{5}$

**Example 29:** Find the hypotenuse of a right triangle with legs 5 and 12.

$5^2 + 12^2 = c^2$ $25 + 144 = c^2$ $169 = c^2$ $c = \sqrt{169} = \mathbf{13}$

### Finding a Leg

**Example 30:** A right triangle has hypotenuse 10 and one leg 6. Find the other leg.

$a^2 + b^2 = c^2$ $a^2 + 6^2 = 10^2$ $a^2 + 36 = 100$ $a^2 = 64$ $a = \sqrt{64} = \mathbf{8}$

**Example 31:** Find the missing leg if $c = 15$ and one leg $= 9$.

$a^2 + 9^2 = 15^2$ $a^2 + 81 = 225$ $a^2 = 144$ $a = \sqrt{144} = \mathbf{12}$

### Pythagorean Triples

Some right triangles have all integer sides. These sets are called

**Pythagorean triples**. Common triples:

- 3, 4, 5
- 5, 12, 13
- 8, 15, 17
- 7, 24, 25

Multiples also work:

6, 8, 10 (double of 3, 4, 5)

9, 12, 15 (triple of 3, 4, 5)

**Real-World Applications**

**Example 32:** A ladder leans against a wall. The base is 5 feet from the wall, and the ladder reaches 12 feet up.

How long is the ladder?

This forms a right triangle. $5^2 + 12^2 = c^2$ $25 + 144 = c^2$ $169 = c^2$ $c =$ **13 feet**

**Example 33:** A TV screen is 40 inches wide and 30 inches tall. What is the diagonal measurement? $40^2 + 30^2 = d^2$ $1600 + 900 = d^2$ $2500 = d^2$ $d = \sqrt{2500} =$ **50 inches**

**Converse of the Pythagorean Theorem**

If $a^2 + b^2 = c^2$ for a triangle's sides, then the triangle is a right triangle.

**Example 34:** Is a triangle with sides 6, 8, and 10 a right triangle?

Check: $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

Yes, **it is a right triangle.**

**Example 35:** Is a triangle with sides 7, 8, and 12 a right triangle?

Check: $7^2 + 8^2 = 49 + 64 = 113$
$12^2 = 144$

$113 \neq 144$, so it is **not a right triangle.**

✏️ **Try This**

1. Find the hypotenuse: legs 6 and 8 ____________
2. Find the hypotenuse: legs 9 and 12 ____________
3. Find the missing leg: hypotenuse 17, one leg 8 ____________
4. Is a triangle with sides 5, 6, 8 a right triangle? ____________
5. A rectangular field is 80 m by 60 m. What is the diagonal distance across? ____________

# LESSON 6.6: Preview of Algebra 1

**What's Coming in Algebra 1**

**Solving more complex equations:**

Literal equations (solving for any variable)

Systems of equations (all methods)

Quadratic equations

**Working with polynomials:**

Adding, subtracting, multiplying polynomials

Factoring expressions

Polynomial long division

**Graphing:**

Quadratic functions (parabolas)

Exponential functions

Transformations of functions

**Radicals and rational expressions:**

Simplifying square roots

Operations with radicals

Working with rational expressions

**Skills You're Ready For**

Based on 7th grade, you can already:

✓ Solve multi-step equations
✓ Work with variables on both sides
✓ Graph linear equations
✓ Understand slope and y-intercept
✓ Work with exponents and roots
✓ Apply the Pythagorean theorem
✓ Understand functions

**A Glimpse Ahead**

**Example 36:** (Algebra 1 preview) Solve by factoring: $x^2 - 5x + 6 = 0$ Factor: $(x - 2)(x - 3) = 0$
Set each factor equal to 0:

$x - 2 = 0 \rightarrow x = 2$

$x - 3 = 0 \rightarrow x = 3$

Solutions: $x = 2$ and $x = 3$

**Example 37:** (Algebra 1 preview) Simplify: $\sqrt{50}$
$\sqrt{50} = \sqrt{(25 \times 2)} = \sqrt{25} \times \sqrt{2} = \mathbf{5\sqrt{2}}$

✏️ **Try This**

Simplify (preview):

1. $\sqrt{72}$ ____________
2. $\sqrt{32}$ ____________

## Chapter 6 Practice Problems

**Section A: Functions**

1. If $f(x) = 5x - 3$, find $f(4)$. ____________
2. If $g(x) = x^2 + 2x$, find $g(-3)$. ____________
3. If $h(x) = |x - 4|$, find $h(1)$ and $h(7)$. ____________
4. Is {(1, 2), (2, 4), (3, 2), (4, 5)} a function? ____________
5. Find domain and range: {(0, 3), (1, 5), (2, 7), (3, 9)} ____________

**Section B: Linear vs. Nonlinear**

Identify as linear or nonlinear:

6. $f(x) = -4x + 9$ ____________
7. $f(x) = x^3$ ____________

**Section C: Exponent Rules**

Simplify:

8. $a^5 \times a^3$ ____________
9. $b^7 \div b^2$ ____________

10. $(c^4)^3$ _____________

11. $4^0 + 4^1$ _____________

12. 2^(-4) _____________

13. $(x^3 \times x^5)/(x^2)$_____________

**Section D: Square and Cube Roots**

14. $\sqrt{196}$ _____________

15. $\sqrt{169}$ _____________

16. Estimate $\sqrt{55}$ (between which integers? approximate value?) _____________

17. $\sqrt[3]{1000}$ _____________

18. $\sqrt[3]{-64}$ _____________

19. $\sqrt{(144/49)}$ _____________

**Section E: Pythagorean Theorem**

20. Find c: a = 9, b = 12 _____________

21. Find a: b = 24, c = 25 _____________

22. Is 9, 40, 41 a Pythagorean triple? _____________

23. A 20-foot ladder reaches 16 feet up a wall. How far is its base from the wall? _____________

24. Find the diagonal of a rectangle 15 cm by 8 cm. _____________

**Section F: Mixed Review**

25. Write the equation of a line with slope 3 through (2, 7). _____________

26. Solve: 3(x - 4) = 2(x + 1) - 5 _____________

27. Find the surface area of a cylinder with r = 4, h = 10. _____________

28. P(sum = 9) when rolling two dice _____________

29. In similar triangles, sides are 5, 12, 13 and 10, 24, x. Find x. _____________

## �� Challenge Problems

1. If $f(x) = 2x + 1$, find x when $f(x) = 15$. ____________

2. Simplify: $(3^2 \times 3^4)/(3^3 \times 3^2)$ ____________

3. The diagonal of a square is 10 cm. Find the side length. ____________

4. A rectangular box has dimensions $3 \times 4 \times 5$. Find the length of the space diagonal (corner to opposite corner through the box). ____________

5. If $f(x) = x^2 - 3x + 2$, find all values of x where $f(x) = 0$. (Hint: Factor)____________

## Chapter 6 Summary

### Functions

**Function:** Each input has exactly one output

**Notation:** f(x) means output when input is x

**Domain:** All possible x-values

**Range:** All possible y-values

**Vertical line test:** If any vertical line hits the graph more than once, it's not a function

### Exponent Rules

**Rule Formula Example**

| |
|---|
| $a^m \times a^n = a^{m+n}$ |
| $a^m \div a^n = a^{m-n}$ |
| $(a^m)^n = a^{mn}$ |
| $a^0 = 1$ |

Product $x^3 \times x^4 = x^7$ Quotient $x^5 \div x^2 = x^3$ Power $(x^2)^3 = x^6$

Zero $5^0 = 1$

Negative $a^{-n} = 1/a^n$ $2^{-3} = 1/8$

**Roots**

**Square root:** $\sqrt{n} \times \sqrt{n} = n$

**Cube root:** $\sqrt[3]{n} \times \sqrt[3]{n} \times \sqrt[3]{n} = n$

Perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100...

Perfect cubes: 1, 8, 27, 64, 125, 216...

**Pythagorean Theorem**

**$a^2 + b^2 = c^2$**

Use to find missing side of right triangle

Converse: If $a^2 + b^2 = c^2$, then it's a right triangle

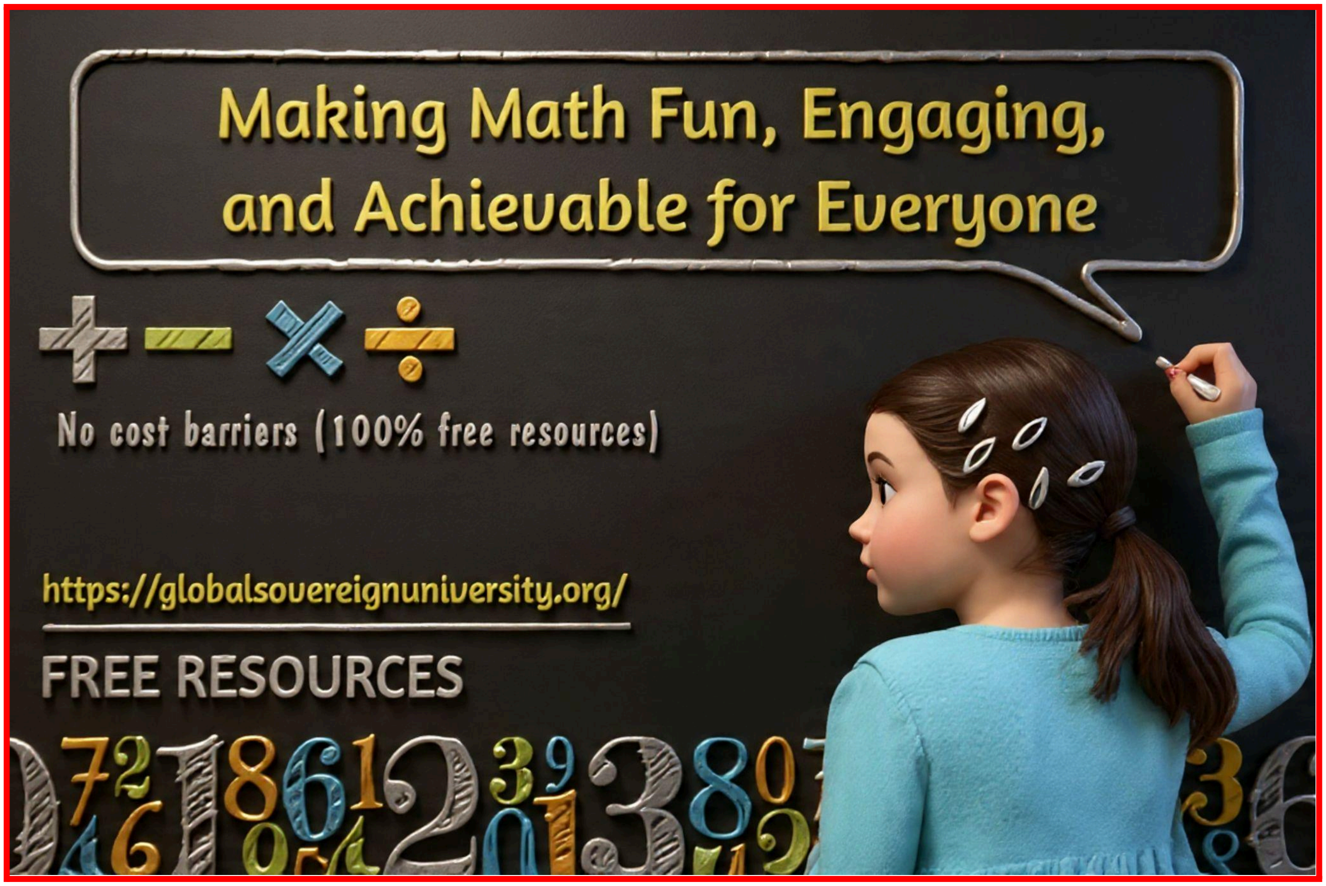
Common triples: 3-4-5, 5-12-13, 8-15-17, 7-24-25

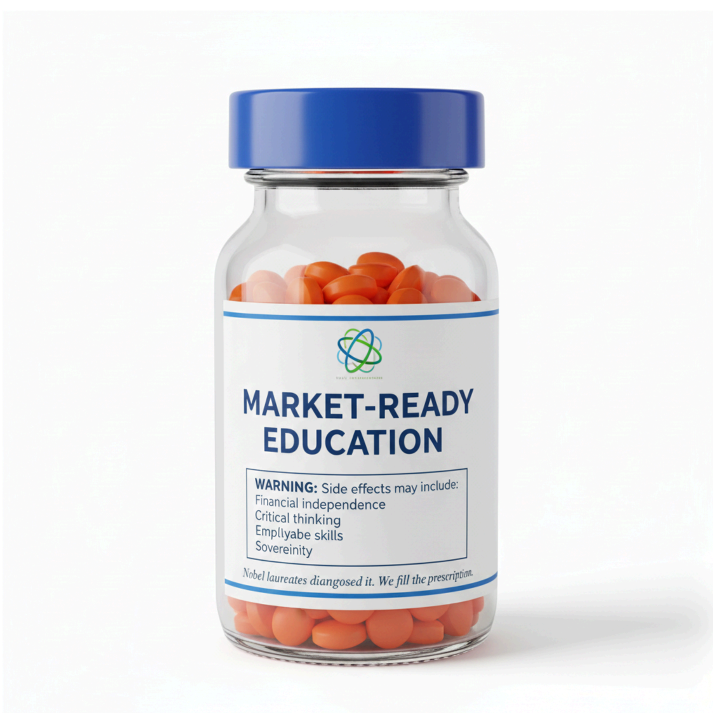
## Congratulations!

You've completed 7th Grade Mathematics! You now have:

✓ Strong number sense with rational numbers
✓ Proportional reasoning skills
✓ Equation and inequality solving abilities
✓ Geometric understanding of angles, triangles, and circles
✓ Statistical literacy and probability foundations
✓ Linear relationship expertise
✓ Introduction to functions and advanced algebra concepts

**You are ready for Algebra 1!**


Making Math Fun, Engaging,
and Achievable for Everyone
No cost barriers (100% free resources)
https://globalsovereignuniversity.org/
FREE RESOURCES


MARKET-READY
EDUCATION
WARNING: Side effects may include:
Financial independence
Critical thinking
Empllyabe skills
Sovereinity
Nobel laureates diangosed it. We fill the prescription.