Grade 8 Math · Lesson 10 — The Packaging Proposal

Volume of 3D Figures · ages 12 to 15 · 45 minutes · the workbook, one screen, this page, a pencil and paper

At a Glance

- **The book:** 8th Grade Math Workbook: Real-World Problem Solving, Chapter 10 (The Industrial Designer), by Dr. Gene A Constant.
- **Free tool:** Geometry Quest (distance, area and angles), free, no login, plays in the browser: globalsovereignuniversity.org/geometry-quest.
- **Free class:** Lesson 10 of the free Grade 8 Mathematics class: globalsovereignuniversity.org/grade-8-math-class#lesson-10.
- **Ages and time:** Grade 8 · ages 12 to 15 · 45 minutes.
- **Prep (5 min):** read the Teaching Creed and have the workbook open to Chapter 10. Make a paper cone and find a can or cup with the same radius and height, plus a cup of rice or dry beans.

Why this lesson

Every can, cone, tank and ball holds a volume someone had to compute before it was built. Knowing the three formulas, and breaking odd shapes into simple ones, turns packaging and storage into arithmetic a child can check.

What the learner will be able to do

- Calculate volume of cylinders.
- Calculate volume of cones.
- Calculate volume of spheres.
- Solve real-world problems involving volume.

The 45-Minute Climb

How to read this: the gold box in each step is your lesson, word for word. Run it as written. The notes beneath are optional support. "The group" means your family; "host" means you.

1 · The Hook (5 min)

"Fill a paper cone with rice and pour it into a can of the same radius and height. How many cones fill the can?"

Say it like this: let your child do it with real objects if you can, then ask how they know their answer is right before you check it.

2 · Work It Together (15 min)

Open the workbook to the worked example Find volume: cylinder with radius 5 cm, height 12 cm (Section 10.1) together: Use the formula $V = \pi r^2 h$ Substitute: $V = \pi \times 5^2 \times 12$ Calculate: $V = \pi \times 25 \times 12 = 300\pi$ Approximate: $V \approx 942.48$ cm³. Then read Find volume: cone with radius 6 cm, height 9 cm (Section 10.2) together: Use the formula $V = (1/3)\pi r^2 h$ Substitute: $V = (1/3) \times \pi \times 6^2 \times 9$ Calculate: $V = (1/3) \times \pi \times 36 \times 9 = 108\pi$ Approximate: $V \approx 339.29$ cm³.

Read each step aloud and let your child say the next step before you read it. Then play one round of Geometry Quest.

3 · The Talk (8 min)

1. Why does a cone hold one third of a cylinder with the same base and height? 2. What is the most common mistake with these formulas? 3. How do you find the volume of an odd shape?

You are not fishing for one right answer. You are listening for understanding:

- Q1: Three full cones of rice fill the cylinder exactly, and the formula records that: $\frac{1}{3}\pi r^2 h$.
- Q2: Using the diameter where the formula needs the radius.
- Q3: Break it into cylinders, cones and spheres, then add the parts, or subtract a part that is a hole.

4 · Solo Climb (12 min)

Five problems on your own, with your work shown.
1. Find volume: cylinder with diameter 14 cm, height 8 cm.
2. Find volume: cone with diameter 10 cm, height 15 cm.
3. Find volume: sphere with diameter 20 cm.
4. A sphere has volume 36π in³. Find the radius.
5. A cone (r = 4, h = 6) sits on a cylinder (r = 4, h = 10). Total volume?

Why solo: a test, a job and real life ask you to work alone. Five problems rehearse exactly that. The answers are at the end of this plan.

Keep It Fresh (5 min, the next day)

Two review problems from the book's Extra Practice.
1. Volume of cylinder: r = 7, h = 10. (Leave in terms of π)
2. Volume of cone: r = 9, h = 12.

Ready to move on? Four of the five Solo Climb problems right, with work shown, means this skill is solid. If not, reread the worked examples together and try two more problems from the chapter before the next lesson. Short review of old skills keeps them from fading.

5 · The Takeaway (5 min)

Cylinder: πr²h. Cone: ⅓πr²h. Sphere: 4/3πr³. Break odd shapes into these and add or subtract. · Homework: Measure a real can and a ball. Work out each volume and decide which holds more.

The badge moment: finishing all five steps earns the Bronze ring of this lesson's badge, The Package Designer. Mark it on the Climber's Passport now; the homework is how Silver and Gold are earned during the week.

Common Mistakes to Watch For

- Using the diameter where the formula needs the radius.
- Forgetting the ⅓ for a cone.
- Adding a composite figure's parts when one part should be subtracted, such as a hole.

The Teaching Creed

You are not the expert; the book and the method are. Your job is to ask "how do you know?" one more time than feels natural, and to celebrate wrong answers out loud. In here, a wrong answer is where the lesson lives.

GENO, GSU's free tutor, can explain any step a second way, any hour, in 83 languages, with no login: globalsovereignuniversity.org/free-tutor. A prompt for the host: Ask GENO: "Explain why a cone holds one third of a cylinder with the same base and height."

Adapt It and Measure It

Differentiation

- Struggling: label r and h on a drawing before using a formula.
- Ahead: design a package that holds a given volume with the least material.
- Large co-op: each team measures and computes one container, then the group ranks them.
- Reluctant: measure a favorite cup or ball and find what it holds.

Evidence of learning (no exam)

Look for three things. Tried: finished all five Solo Climb problems with work shown. Noticed: can name the mistake in Common Mistakes that they almost made. Changed: the next day, solves one new problem from the same chapter without help. Those three words are the whole rubric, and they match the badge tiers below.

Gamify It — The Campus Climb

Every GSU lesson is one badge on the Climber's Passport, earned at four depths: Bronze, Silver, Gold and Platinum.

LESSON 10 BADGE — The Package Designer · Bronze — The Climb: finished the 45-minute lesson, all five steps. · Silver — The Try: did the homework and explained it to the host. · Gold — The Proof: completed the chapter's Action Report in the workbook with every answer checked against the Answer Key. · Platinum — The Teacher: ran this lesson as host for someone else. The harvest of a teacher is teachers.

Answers for the Host

1. $392\pi \approx 1{,}231.5$ cm³
2. $125\pi \approx 392.7$ cm³
3. $4000\pi/3 \approx 4{,}188.8$ cm³
4. $r = 3$ in
5. $32\pi + 160\pi = 192\pi \approx 603.2$

Keep It Fresh: 1. 490π · 2. $324\pi \approx 1{,}017.9$

Every answer above comes from the workbook's own Answer Key. The book behind this lesson: 8th Grade Math Workbook: Real-World Problem Solving, Chapter 10 (The Industrial Designer).

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