Math 34600 L (33201)
Homework Solutions

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Exercise 1.3.1 - Concerning the system $AX = B$, homogeneous when $B = 0$. True or False?

a. If the system is homogeneous, then every solution is trivial.
False. If there are free variables, then there are be infinitely many solutions.

b. If the system has nontrivial solutions, then it cannot be homogeneous.
False. In fact, this statement is logically equivalent to the one in (a). (It is the *contrapositive*.)

c. If there exists a trivial solution, then the system is homogeneous.
True. If $X = 0$, then $B = AX = 0$.

d. If the system is consistent, then it must be homogeneous.
False. While a homogeneous system is always consistent, there are many consistent systems which are not homogeneous.

Now assume that the system is homogeneous.

e. If there exists a nontrivial solution, then there is no trivial solution.
False. A homogeneous system always has the trivial solution.

f. If there exists a solution, then there are infinitely many solutions.
False. There is always the trivial solution and there are infinitely many solutions if and only if some variables are free.

2.4.2 a Find the inverse of $A = \begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}$.

The adjugate is $\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$

The determinant is $(1)(3) - (-1)(-1) = 2$. So

$$A^{-1} = (1/2) \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$$

2.4.3 d Use the inverse to solve the system, and also compute the determinant of the coefficient matrix:

$$\begin{array}{ccccccc} x_1 & + & 4x_2 & + & 2x_3 & = & 1 \\ 2x_1 & + & 3x_2 & 3x_3 & = & -1 & \\ 4x_1 & + & x_2 & + & 4x_3 & = & \end{array}$$

$$(A|I_3) \quad = \quad \begin{pmatrix} 1 & 4 & 2 & 1 & 0 & 0 \\ 2 & 3 & 3 & 0 & 1 & 0 \\ 4 & 1 & 4 & 0 & 0 & 1 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 4R_1,\ \mu = 1$

$$\begin{pmatrix} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & -5 & -1 & -2 & 1 & 0 \\ 0 & -15 & -4 & -4 & 0 & 1 \end{pmatrix}$$

$R_3 \to R_3 - 3R_2,\ \mu = 1$

$$\begin{pmatrix} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & -5 & -1 & -2 & 1 & 0 \\ 0 & 0 & -1 & 2 & -3 & 1 \end{pmatrix}$$

$R_2 \to -R_2,\ R_3 \to -R_3,\ \mu = -1, \mu = -1$

$$\begin{pmatrix} 1 & 4 & 2 & 1 & 0 & 0 \\ 0 & 5 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -2 & 3 & -1 \end{pmatrix}$$

$R_2 \to R_2 - R_3,\ R_1 \to R_1 - 2R_3,\ \mu = 1$

$$\begin{pmatrix} 1 & 4 & 0 & 5 & -6 & 2 \\ 0 & 5 & 0 & 4 & -4 & 1 \\ 0 & 0 & 1 & -2 & 3 & -1 \end{pmatrix}$$

$R_2 \to (1/5)R_2,\ \mu = 1/5$

$$\begin{pmatrix} 1 & 4 & 0 & 5 & -6 & 2 \\ 0 & 1 & 0 & 4/5 & -4/5 & 1/5 \\ 0 & 0 & 1 & -2 & 3 & -1 \end{pmatrix}$$

$R_1 \to R_1 - 4R_2,\ \mu = 1$

$$\begin{pmatrix} 1 & 0 & 0 & 9/5 & -14/5 & 6/5 \\ 0 & 1 & 0 & 4/5 & -4/5 & 1/5 \\ 0 & 0 & 1 & -2 & 3 & -1 \end{pmatrix}$$

$$A^{-1} = (1/5)\begin{pmatrix} 9 & -14 & 6 \\ 4 & -4 & 1 \\ -10 & 15 & -5 \end{pmatrix}$$

With

$$B = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

With

$$A^{-1}B = (1/5)\begin{pmatrix} 23 \\ 8 \\ -25 \end{pmatrix}$$

To check, multiply

$$A\begin{pmatrix} 23 \\ 8 \\ -25 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix}$$

For the determinant, we have that
$1 = det(I_3) = (-1)(-1)(1/5)det(A)$. Therefore, $det(A) = 5$.