Math 39100 K (33191) - Scrambled Solutions

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Spring, 2026

2.$(2x^2y+2x)y' = -(2xy^2+2y)$. Exact, rewrite as $(2x^2y+2x)dy + (2xy^2+2y)dx = 0$.

$$\partial(2x^2y+2x)/\partial x = 4xy+2 = \partial(2xy^2+2y)/\partial y.$$

$$F = \int (2x^2y+2x)dy = x^2y^2 + 2xy + H(x).$$

So $\partial F/\partial x = 2xy^2 + 2y + H'(x) = 2xy^2 + 2y$. With $H(x) = 0$,

$$x^2y^2 + 2xy = C$$

3. $\sin(t)y' + \cos(t)y = e^t$. This is linear, but the left is already $(\sin(t)y)'$. If you don't see that right away, divide to get

$$y' + \frac{\cos(t)}{\sin(t)}y = \frac{e^t}{\sin(t)}.$$

$$\mu = exp(\int \frac{\cos(t)}{\sin(t)}\, dt) \;=\; exp(\ln(\sin(t))) \;=\; \sin(t).$$

Back where you started $(\sin(t)y)' = e^t$ and so $\sin(t)y = e^t + C$.

4. $(2x - y)dy - (4y - 3x)dx = 0$. Not exact. Homogeneous. Rewrite as $\frac{dy}{dx} = \frac{4y-3x}{2x-y}$.

$$x\frac{dz}{dx} = -z + \frac{4z-3}{2-z} = \frac{z^z + 2z - 3}{2-z}.$$

$$\frac{2-z}{(z+3)(z-1)}dz = \frac{dx}{x}.$$

$\frac{2-z}{(z+3)(z-1)} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$. So $z - 2 = A(z-1) + B(z+3)$. With $z = 1, B = 1/4$, and with $z = -3, A = -5/4$.

$$(-5/4)\ln(z+3) + (1/4)\ln(z-1) = \ln(x) + C.$$

$-5(\ln(z+3) + \ln(x)) + (\ln(z-1) + \ln(x)) = C$. So

$$(y+3x)^{-5}(y-x) = C.$$

6. $ty' + y = t(2e^t - y)$. Linear $ty' + (1+t)y = 2te^t$. and so $y' + (\frac{1}{t} + 1)y = 2e^t$.

$$\mu = exp(\int (\frac{1}{t} + 1)dt = exp(t + \ln(t) = te^t.$$

$$te^t y' + (e^t + te^t)y = (te^t y)' = 2te^{2t}.$$

Integrate by parts$te^t y = te^{2t} - e^{2t}/2 + C$.

$$y = e^t - e^t/2t + Ce^{-t}/t.$$

7. $xy' = 1 - y^2$. Variables separable. $\frac{dy}{(1+y)(1-y)} = \frac{dx}{x}$.

$$\int \frac{1/2}{1+y} + \frac{1/2}{1-y} dy = \int \frac{1}{x} dx.$$

$$\frac{(1+y}{(1-y)x^2} = C.$$

8. $(y/x + 6x)dx + (\ln(x) - 2)dy = 0$. Exact with
$F = y \ln|x| + 3x^2 + H(y)$.
So $\partial F/\partial y = \ln(x) + H'(y) = \ln(x) - 2$. and $H(y) = -2y$.

$$y \ln(x) + 3x^2 - 2y = C.$$

12. $t(y' + y - 1) = 1 - y, \qquad y(\ln(2)) = 1.$
Linear $y' + (1 + \frac{1}{t})y = (1 + \frac{1}{t})$. With $\mu = te^t$.
$(te^t y)' = (t+1)e^t$. So $te^t y = te^t + C$. With $t = \ln(2), y = 1$
So $C = 0$ and $y = 1$.

13. $y' = \frac{2x}{y + x^2 y}, \qquad y(0) = -2$. Variables separable.
$ydy = \frac{2xdx}{1+x^2}$. So $y^2/2 = \ln(1 + x^2) + C$. With $x = 0, y = -2$
and $C = 2$.

$$y = -\sqrt{4 + 2\ln(1 + x^2)}.$$

The minus sign on the square root comes from the initial condition.