

Math 39100 K (19535)

- Scrambled Solutions

Ethan Akin
Email: eakin@ccny.cuny.edu

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2. $(2x^2y + 2x)y' = -(2xy^2 + 2y)$. Exact, rewrite as $(2x^2y + 2x)dy + (2xy^2 + 2y)dx = 0$.

$$\partial(2x^2y + 2x)/\partial x = 4xy + 2 = \partial(2xy^2 + 2y)/\partial y.$$

$$F = \int (2x^2y + 2x)dy = x^2y^2 + 2xy + H(x).$$

So $\partial F/\partial x = 2xy^2 + 2y + H'(x) = 2xy^2 + 2y$. With $H(x) = 0$,

$$x^2y^2 + 2xy = C$$

3. $\sin(t)y' + \cos(t)y = e^t$. This is linear, but the left is already $(\sin(t)y)'$. If you don't see that right away, divide to get

$$y' + \frac{\cos(t)}{\sin(t)}y = \frac{e^t}{\sin(t)}.$$

$$\mu = \exp\left(\int \frac{\cos(t)}{\sin(t)} dt\right) = \exp(\ln(\sin(t))) = \sin(t).$$

Back where you started $(\sin(t)y)' = e^t$ and so $\sin(t)y = e^t + C$.

4. $(2x - y)dy - (4y - 3x)dx = 0$. Not exact. Homogeneous.
Rewrite as $\frac{dy}{dx} = \frac{4y-3x}{2x-y}$.

$$x \frac{dz}{dx} = -z + \frac{4z-3}{2-z} = \frac{z^2 + 2z - 3}{2-z}.$$

$$\frac{2-z}{(z+3)(z-1)} dz = \frac{dx}{x}.$$

$\frac{2-z}{(z+3)(z-1)} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$. So $z-2 = A(z-1) + B(z+3)$.
With $z=1$, $B=1/4$, and with $z=-3$, $A=-5/4$.

$$(-5/4) \ln(z+3) + (1/4) \ln(z-1) = \ln(x) + C.$$

$-5(\ln(z+3) + \ln(x)) + (\ln(z-1) + \ln(x)) = C$. So

$$(y+3x)^{-5}(y-x) = C.$$

6. $ty' + y = t(2e^t - y)$. Linear $ty' + (1 + t)y = 2te^t$. and so $y' + (\frac{1}{t} + 1)y = 2e^t$.

$$\mu = \exp\left(\int \left(\frac{1}{t} + 1\right)dt\right) = \exp(t + \ln(t)) = te^t.$$

$$te^t y' + (e^t + te^t)y = (te^t y)' = 2te^{2t}.$$

Integrate by parts $te^t y = te^{2t} - e^{2t}/2 + C$.

$$y = e^t - e^t/2t + Ce^{-t}/t.$$

7. $xy' = 1 - y^2$. Variables separable. $\frac{dy}{(1+y)(1-y)} = \frac{dx}{x}$.

$$\int \frac{1/2}{1+y} + \frac{1/2}{1-y} dy = \int \frac{1}{x} dx.$$

$$\frac{(1+y)}{(1-y)x^2} = C.$$

8. $(y/x + 6x)dx + (\ln(x) - 2)dy = 0$. Exact with
 $F = y \ln |x| + 3x^2 + H(y)$.

So $\partial F / \partial y = \ln(x) + H'(y) = \ln(x) - 2$. and $H(y) = -2y$.

$$y \ln(x) + 3x^2 - 2y = C.$$

$$12. \quad t(y' + y - 1) = 1 - y, \quad y(\ln(2)) = 1.$$

Linear $y' + (1 + \frac{1}{t})y = (1 + \frac{1}{t})$. With $\mu = te^t$.

$(te^t y)' = (t + 1)e^t$. So $te^t y = te^t + C$. With $t = \ln(2)$, $y = 1$

So $C = 0$ and $y = 1$.

$$13. \quad y' = \frac{2x}{y+x^2y}, \quad y(0) = -2. \text{ Variables separable.}$$

$ydy = \frac{2xdx}{1+x^2}$. So $y^2/2 = \ln(1 + x^2) + C$. With $x = 0$, $y = -2$
and $C = 2$.

$$y = -\sqrt{4 + 2\ln(1 + x^2)}.$$

The minus sign on the square root comes from the initial condition.